a place of mind

FACULTY OF EDUCATION

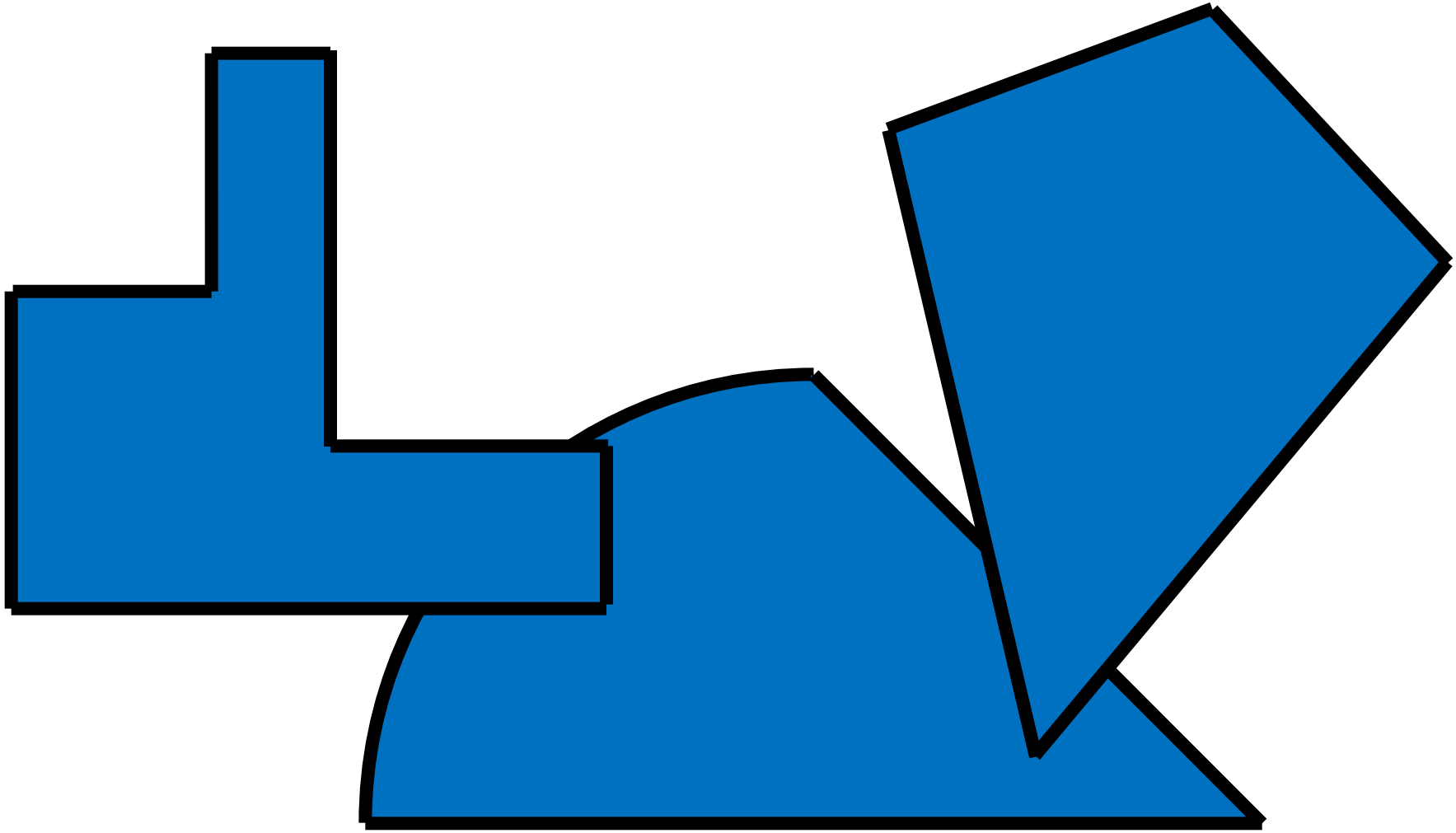
Department of
Curriculum and Pedagogy

Mathematics
Shape and Space: Area

Science and Mathematics Education Research Group

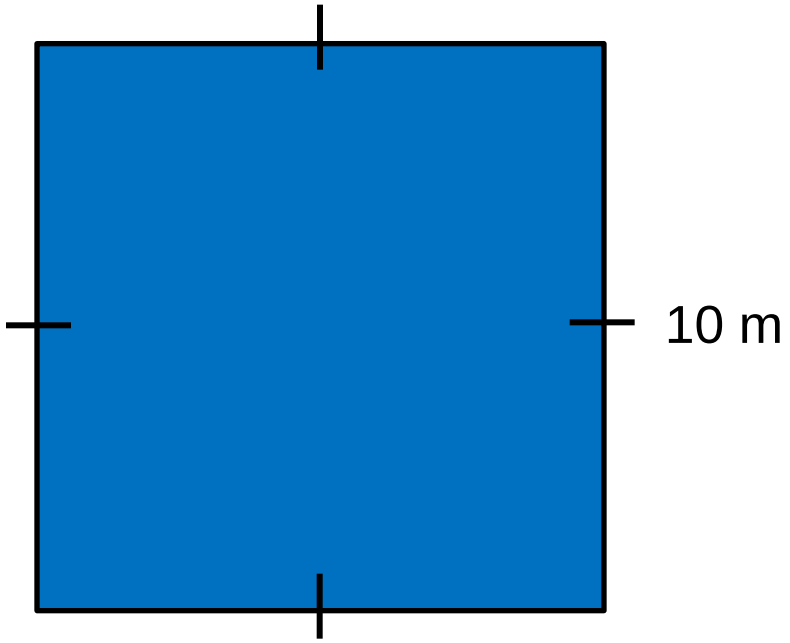
Supported by UBC Teaching and Learning Enhancement Fund 2012-2013

Areas with Rectangles, Triangles, and Circles



Area I

What is the area of the square below?



A. 10 m

B. 10 m^2

C. 100 m

D. 100 m^2

E. Not enough information

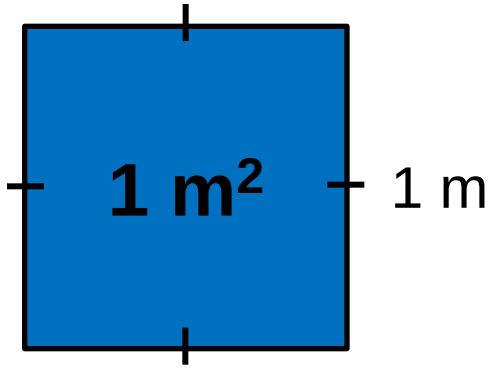
Solution

Answer: D

Justification: A square has 4 sides with equal length. Multiplying the sides together give:

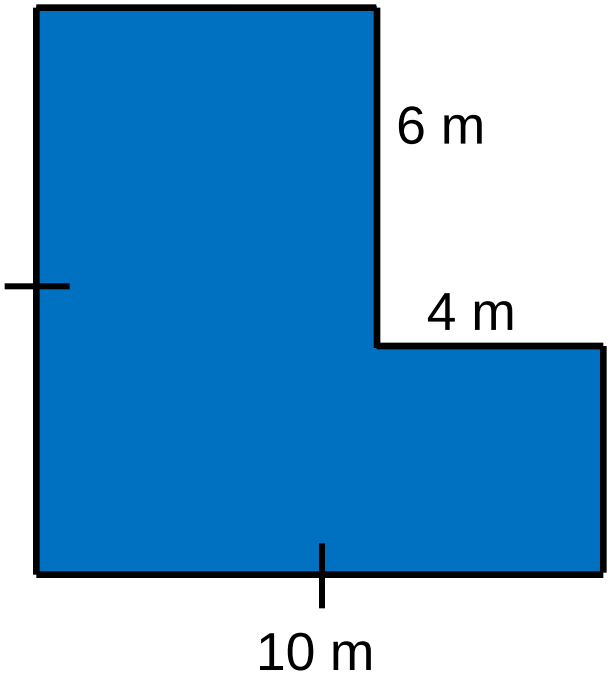
$$A = 10\ m \times 10\ m = 100\ m^2$$

(Note: $m^2 = m \times m$, $1\ m^2$ represents the area of a 1 m by 1 m square)



Area II

What is the area of the figure below?



A. 24 m^2

B. 76 m^2

C. 100 m^2

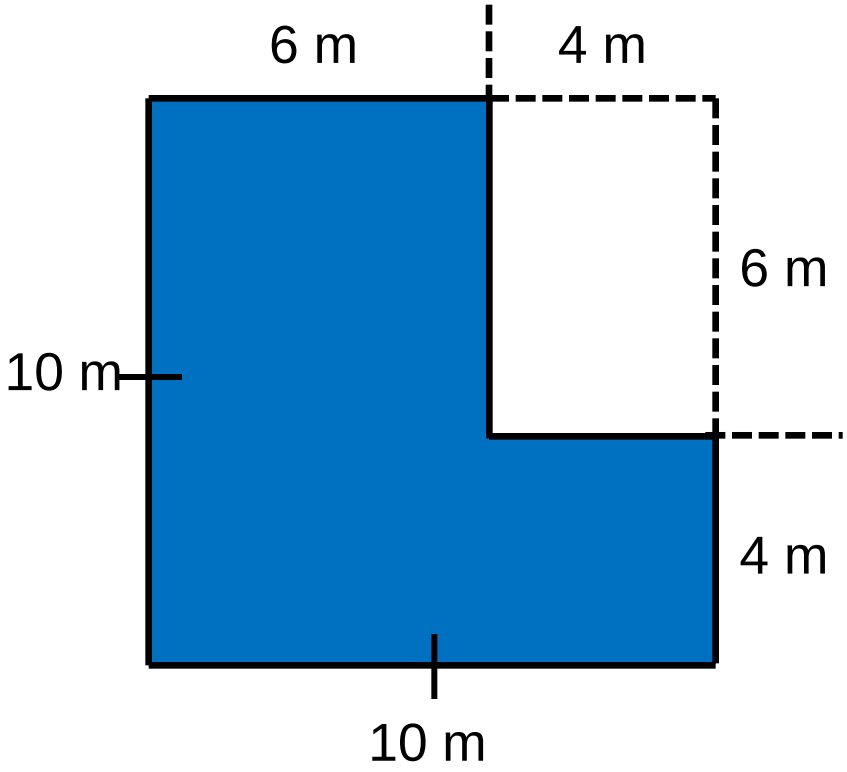
D. 124 m^2

E. Not enough information

Solution

Answer: B

Justification: One way to find the area is to imagine a 10 m by 10 m square and subtracting a 4 m by 6 m rectangle.

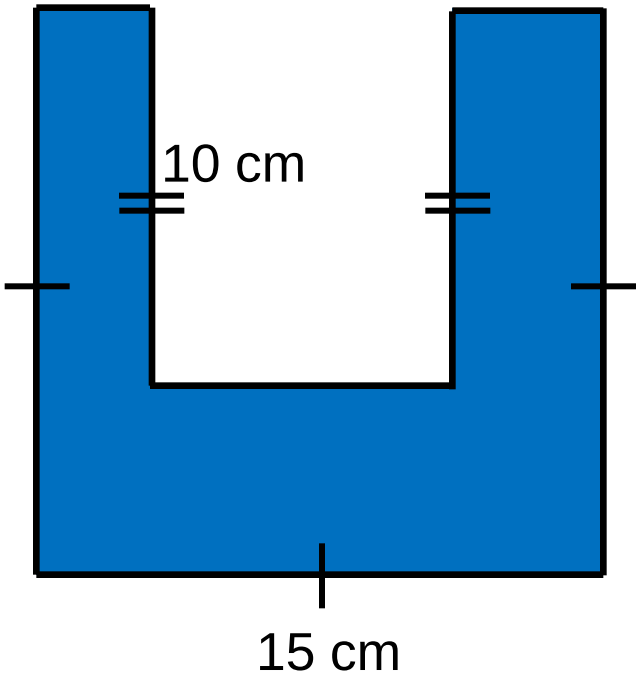


$A = 10\ m \times 10\ m - 4\ m \times 6\ m = 76\ m^2$

Alternatively, the shape's area can be found by dividing it into 2 rectangles and adding the areas together.

Area / Perimeter III

Which of the following statements correctly describe the figure?



A. Both the area and perimeter can be found

B. Only the perimeter can be found

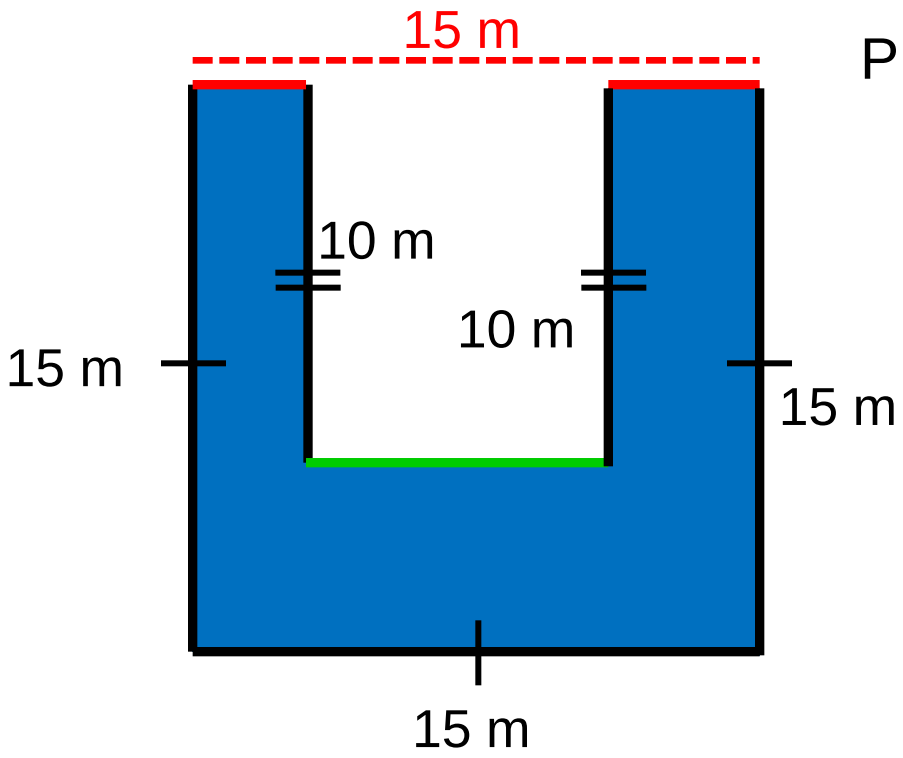
C. Only the area can be found

D. Neither the area nor the perimeter can be found

Solution

Answer: B

Justification: It is possible to find the perimeter of the figure since we can determine the combined length of the red sides.

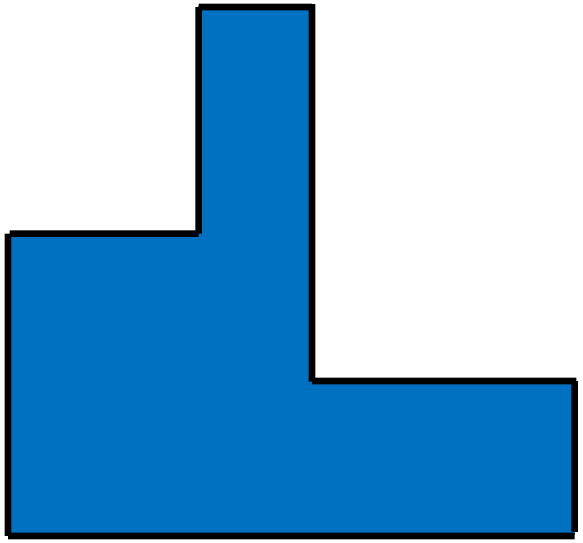


$P = (15\text{ m} \times 4) + (10\text{ m} \times 2) = 80\text{ m}$

It is not possible to find the area of the figure. The length of the green side or the length of the 2 red sides must be known in order to calculate the area.

Area IV

The shape below has no sides with equal length. What is the minimum number of measurements needed in order to determine the area of the shape?



A. 4

B. 5

C. 6

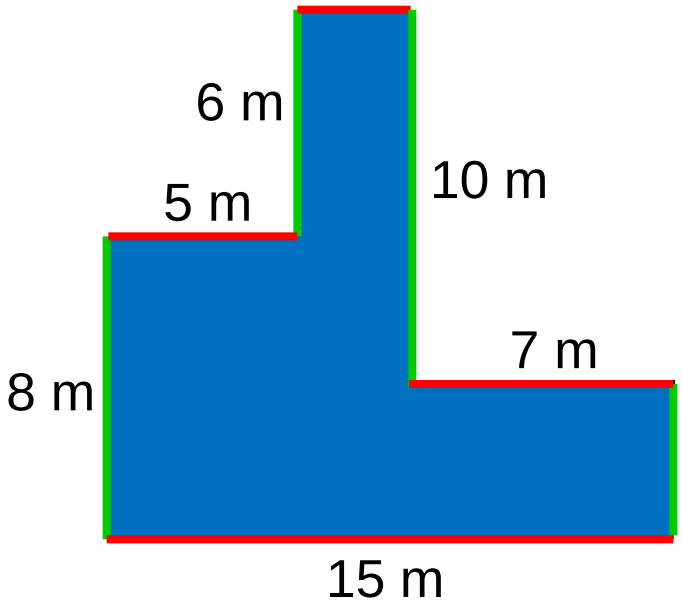
D. 7

E. 8

Solution

Answer: C

Justification: There are 4 **horizontal** sides and 4 **vertical** sides. As soon as any 3 of the horizontal sides and any 3 vertical sides are known, it is possible to calculate the length of the remaining sides.



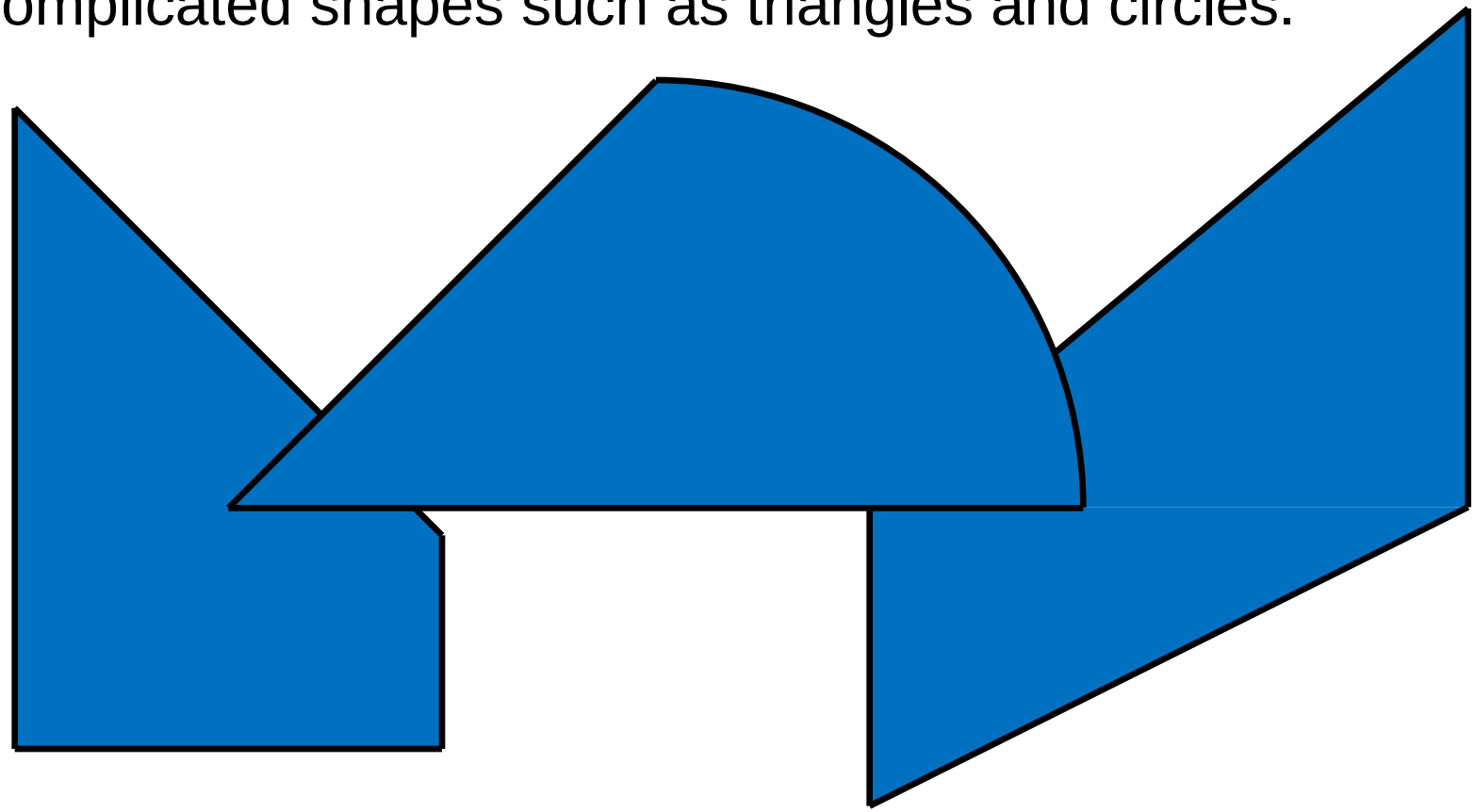
Note that it is also possible to calculate the area of the figure using only 3 horizontal and 3 vertical sides, without having to determine the lengths of the missing sides (see diagram).

A = (15 m x (8 + 6) m) – (5 m x 6 m) – (7 m x 10 m)

(Note: the answer shown is not the only way to determine the area of the shape)

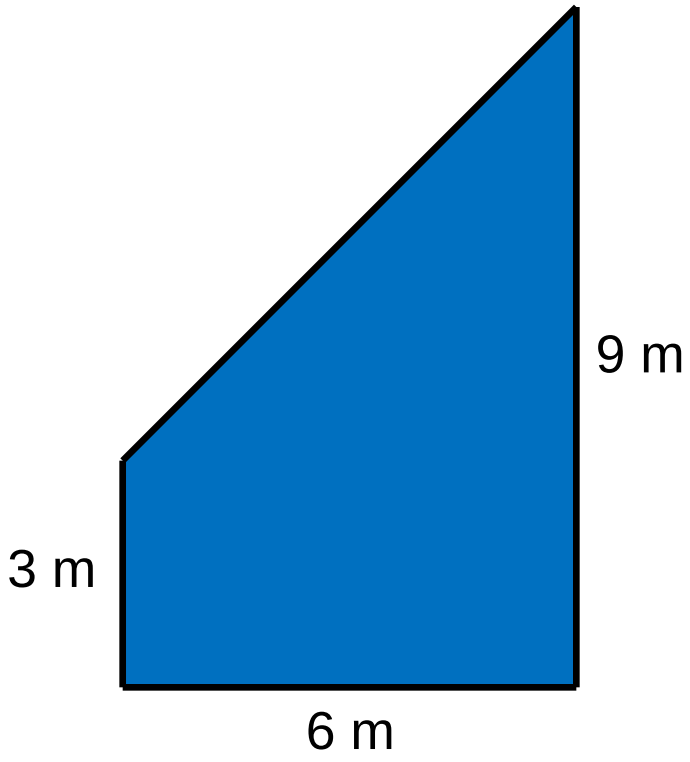
Areas Involving Triangles and Circles

The next set of questions involve the areas of more complicated shapes such as triangles and circles.



Area V

What is the area of the figure below?



A. 27 m^2

B. 36 m^2

C. 45 m^2

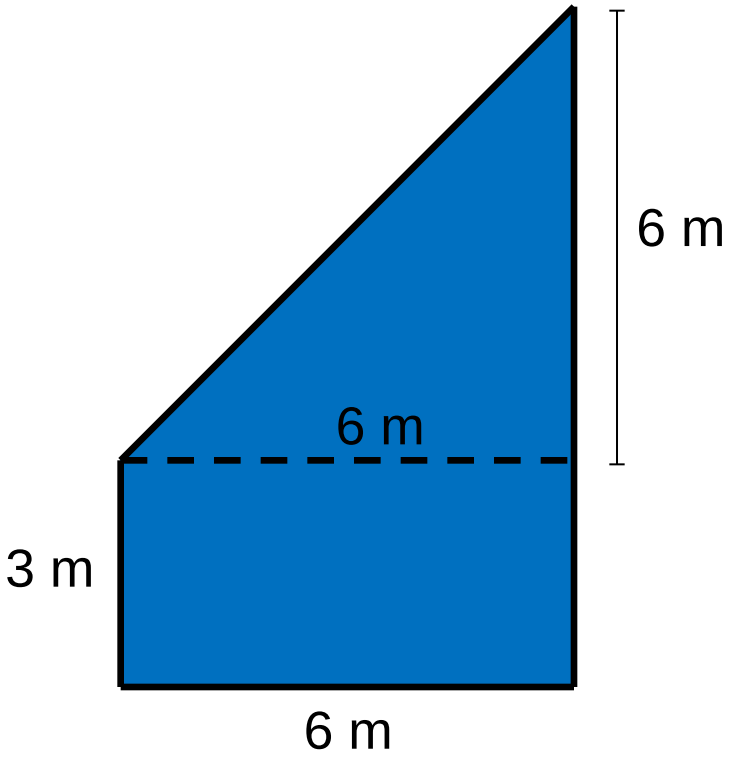
D. 54 m^2

E. Not enough information

Solution

Answer: B

Justification: The figure consists of a 6 m by 3 m rectangle and a triangle with a 6 m base and 6 m height.



$$A = A_{\text{triangle}} + A_{\text{rectangle}}$$

$$= \frac{(\text{base}) \times (\text{height})}{2} + (\text{length}) \times (\text{width})$$

$$= \frac{6\ \text{m} \times 6\ \text{m}}{2} + 3\ \text{m} \times 6\ \text{m}$$

$$= 18\ \text{m}^2 + 18\ \text{m}^2$$

$$= 36\ \text{m}^2$$

Alternative Solution

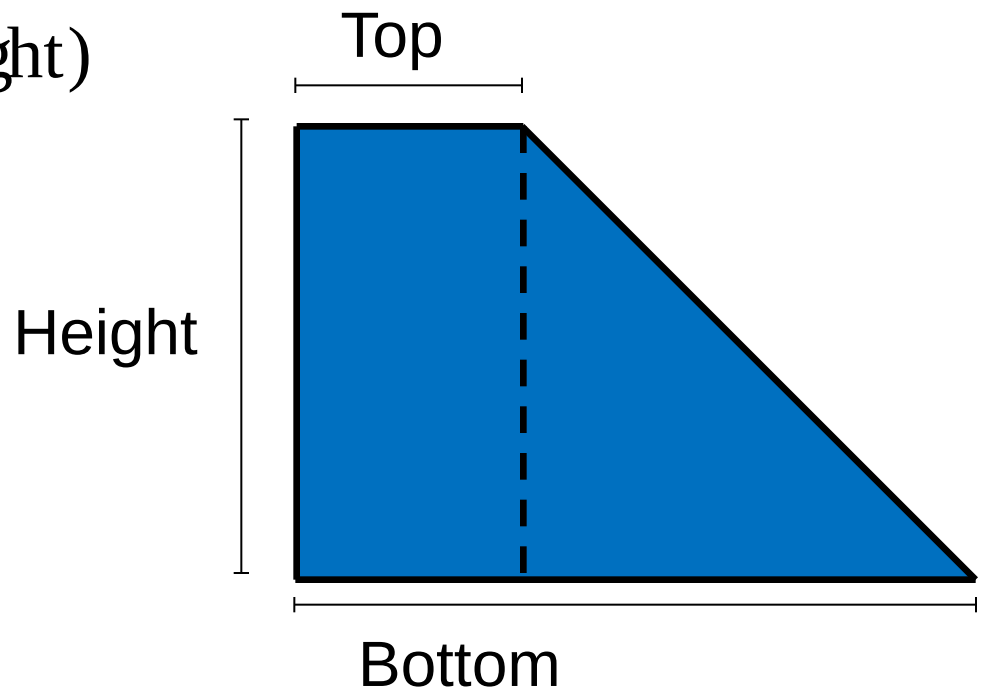
Answer: B

Justification: The area can also be found directly using the formula for the area of a trapezoid:

$$A = \frac{(\text{top}) + (\text{bottom})}{2} \times (\text{height})$$

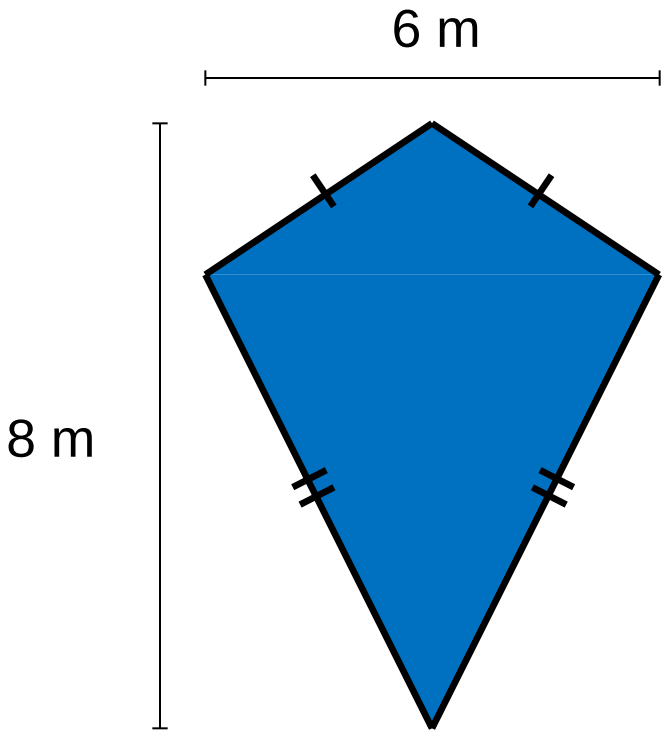
$$= \frac{3\,\text{m} + 9\,\text{m}}{2} \times 6\,\text{m}$$

$$= 36\,\text{m}^2$$



Area VI

What is the area of the figure below?



A. 24 m^2

B. 27 m^2

C. 48 m^2

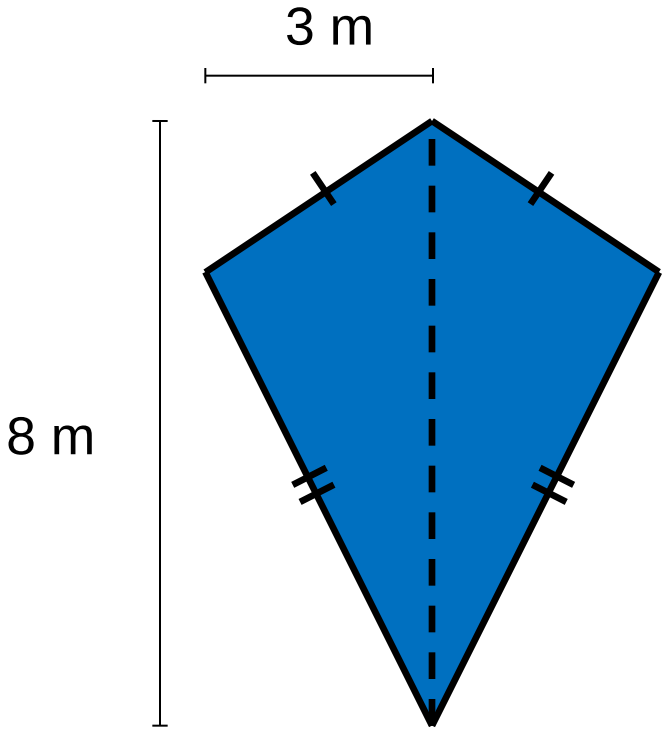
D. 54 m^2

E. Not enough information

Solution

Answer: A

Justification: The figure can be divided into 2 triangles, each with an 8 m base and a 3 m height.



$$A_{\text{triangle}} = \frac{(\text{base}) \times (\text{height})}{2}$$

$$A = \frac{8\,\text{m} \times 3\,\text{m}}{2} + \frac{8\,\text{m} \times 3\,\text{m}}{2}$$

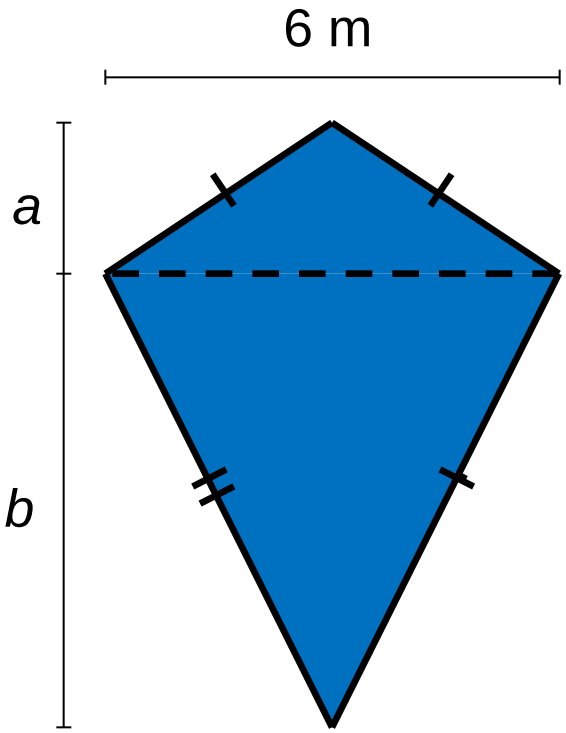
$$= 12\,\text{m}^2 + 12\,\text{m}^2$$

$$= 24\,\text{m}^2$$

Alternative Solution

Answer: A

Justification: The figure can also be divided horizontally, forming two triangles with base 6 m and different heights.



$$a + b = 8$$

$$A = \frac{6 \times a}{2} + \frac{6 \times b}{2}$$

$$= 3a + 3b$$

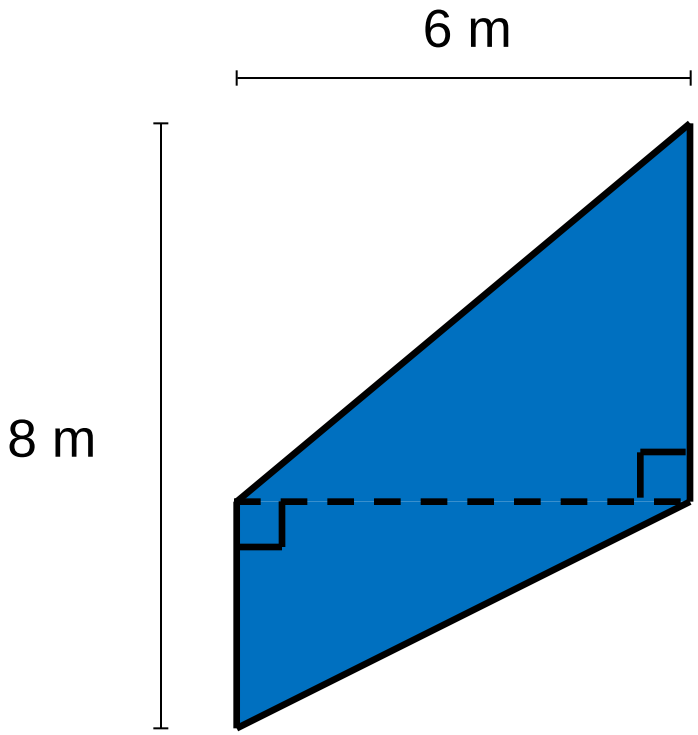
$$= 3(a + b)$$

$$= 3(8) \quad \text{since } a + b = 8$$

$$= 24 \text{ m}^2$$

Area VII

What is the area of the figure below?



A. 24 m^2

B. 27 m^2

C. 48 m^2

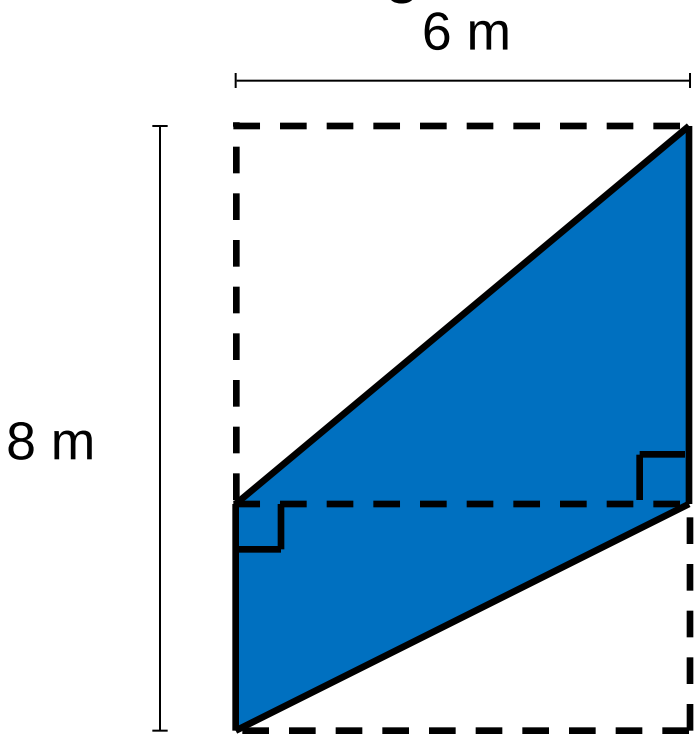
D. 54 m^2

E. Not enough information

Solution

Answer: A

Justification: Double both right triangles as shown to form a 6 m by 8 m rectangle. The area of the figure is then half the area of the rectangle.

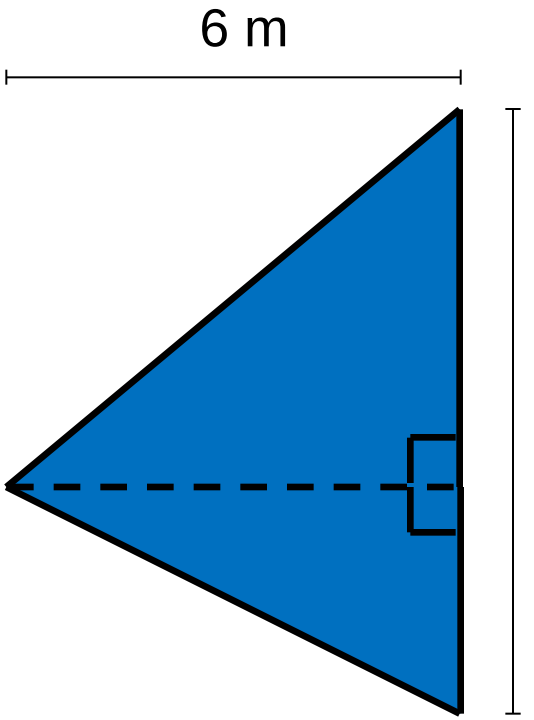


$$A = \frac{8\,\text{m} \times 6\,\text{m}}{2} = 24\,\text{m}^2$$

Alternative Solution

Answer: A

Justification: Since the figure is built with 2 right triangles with equal height, you can imagine flipping one to construct a single larger triangle (as shown in the diagram).



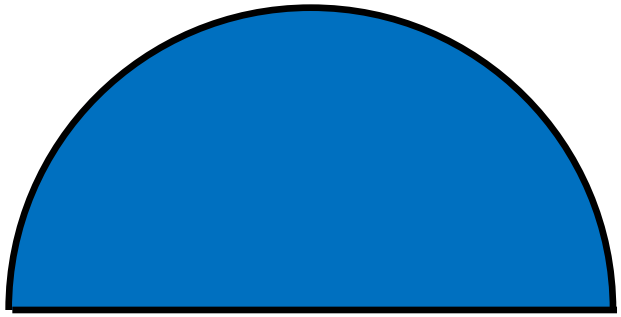
$$A = \frac{(\text{base}) \times (\text{height})}{2}$$

$$= \frac{8\,\text{m} \times 6\,\text{m}}{2}$$

$$= 24\,\text{m}^2$$

Area VIII

What is the area of the semicircle?



8 m

A. 8π m^2

B. 16π m^2

C. 32π m^2

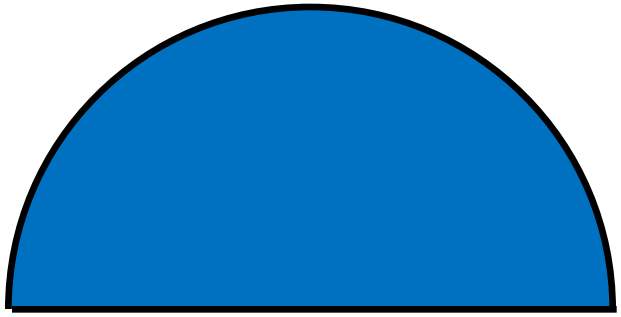
D. 64π m^2

E. Not enough information

Solution

Answer: A

Justification: The area of the semicircle is half the area of a circle. The diameter of the circle is 8 m, so the radius is 4 m.



8 m

$$A_{circle} = \pi r^2$$

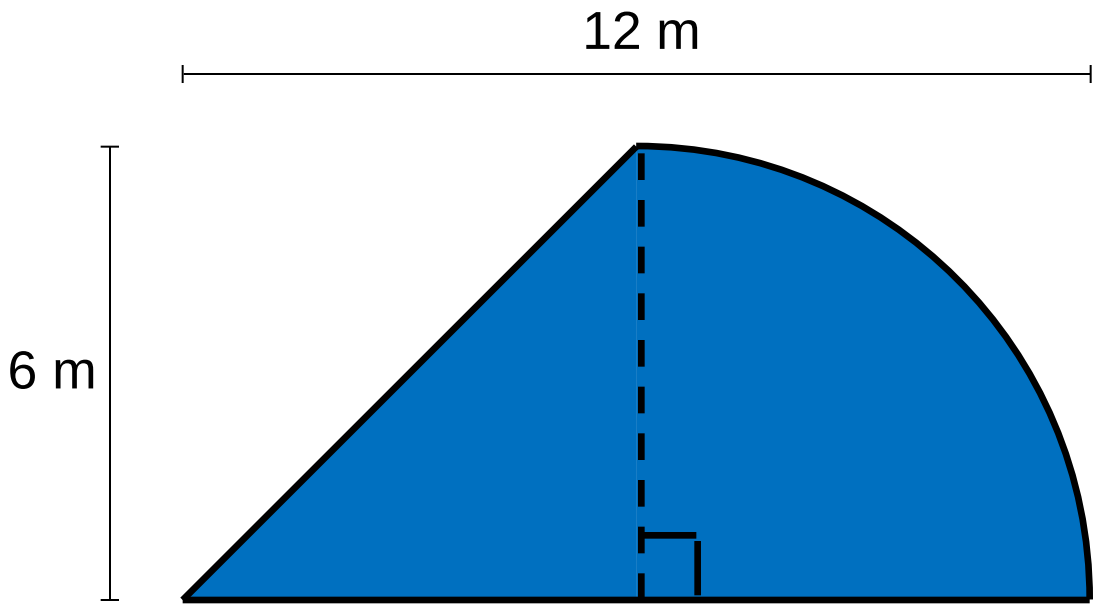
$$A = \frac{\pi r^2}{2}$$

$$= \frac{\pi(4\text{ m})^2}{2}$$

$$= 8\pi\text{ m}^2$$

Area IX

What is the area of the figure below?



A. $(6\pi + 18)\ m^2$

B. $(9\pi + 36)\ m^2$

C. $(6\pi + 36)\ m^2$

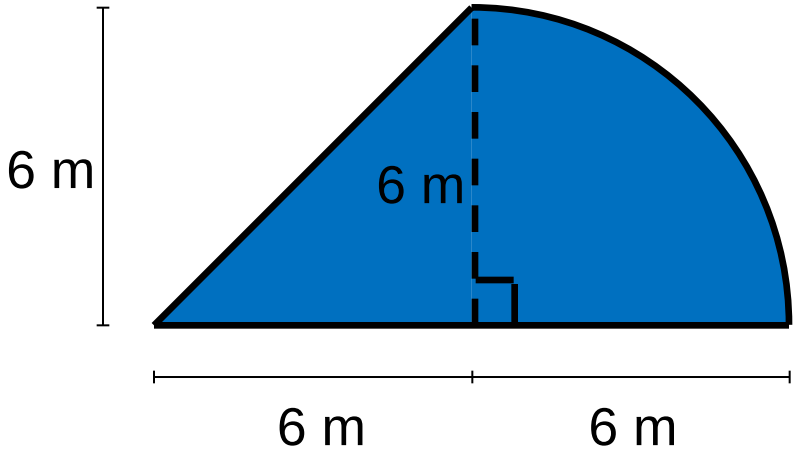
D. $(9\pi + 18)\ m^2$

E. Not enough information

Solution

Answer: D

Justification: The figure consists of a right triangle and a quarter of a circle. The area of the quarter circle is found by dividing the area of a full circle with radius 6 m by 4.



$$A_{\text{circle}} = \pi r^2$$

$$A = \frac{A_{\text{Circle}}}{4} + A_{\text{triangle}}$$

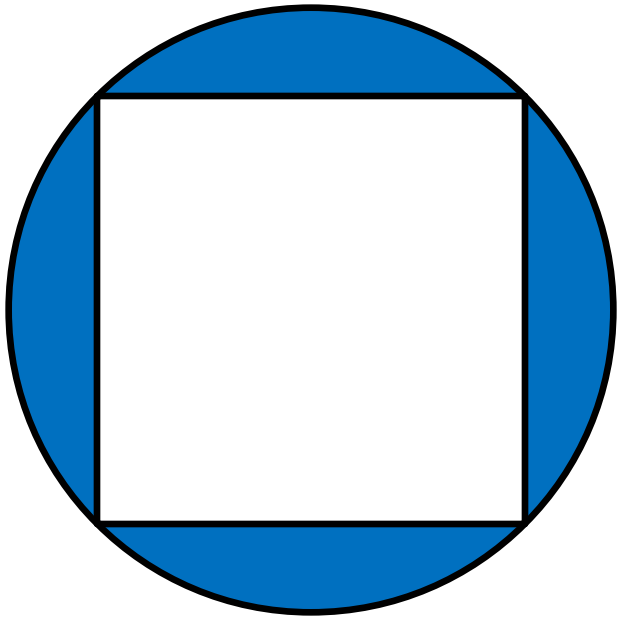
$$= \frac{\pi(6\,\text{m})^2}{4} + \frac{6\,\text{m} \times 6\,\text{m}}{2}$$

$$= (9\pi + 18)\,\text{m}^2$$

Area X (Hard)

A 32 m^2 square is removed from a circle as shown.

What is the remaining area of the circle?



A. $(4\pi - 32)\ m^2$

B. $(8\pi - 32)\ m^2$

C. $(16\pi - 32)\ m^2$

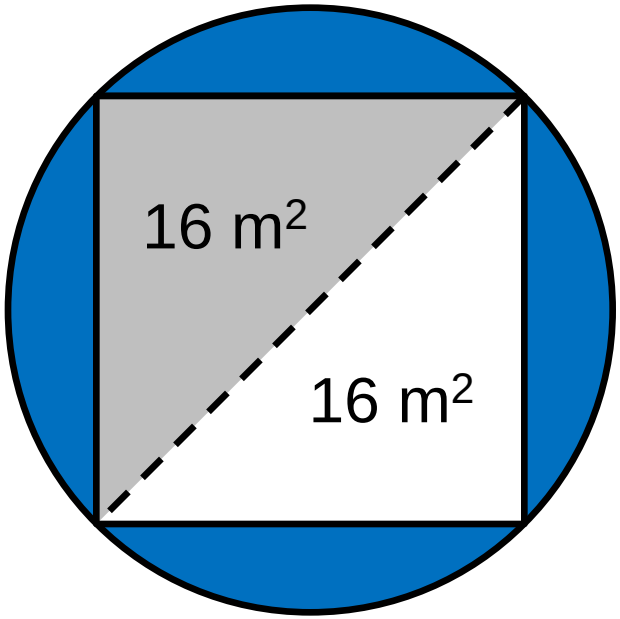
D. $(32\pi - 32)\ m^2$

E. Not enough information

Solution

Answer: C

Justification: The area of the figure is the area of the circle minus the area of the square.



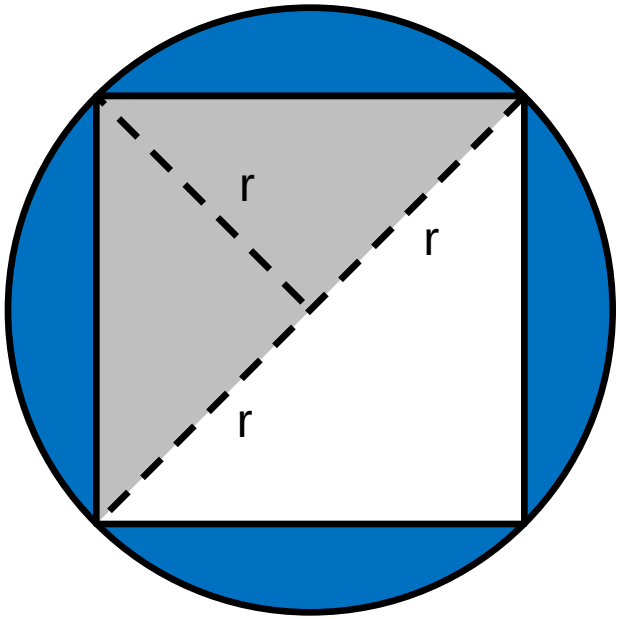
In order to determine the area of the circle, we need to find its radius.

The square can be divided into 2 right triangles as shown. The area of each triangle is $16m^2$, half the area of the $32m^2$ square.

Answer continues on the next slide

Solution Continued

The height of the highlighted triangle has a length equal to the radius of the circle, and the base is equal to the diameter of the circle (twice the radius).



Since we know the area of the triangle, we can use this to solve for the radius of the circle:

$$16\,\mathrm{m}^2 = \frac{2\mathrm{r}\times\mathrm{r}}{2}$$

$$\mathrm{r}^2 = 16\,\mathrm{m}^2$$

Subtract the square from the area of the circle:

$$\mathrm{A} = \mathrm{A}_{\text{circle}} - \mathrm{A}_{\text{square}}$$

$$= \pi\mathrm{r}^2 - 32\,\mathrm{m}^2$$

$$= (16\pi - 32)\,\mathrm{m}^2$$