a place of mind

FACULTY OF EDUCATION

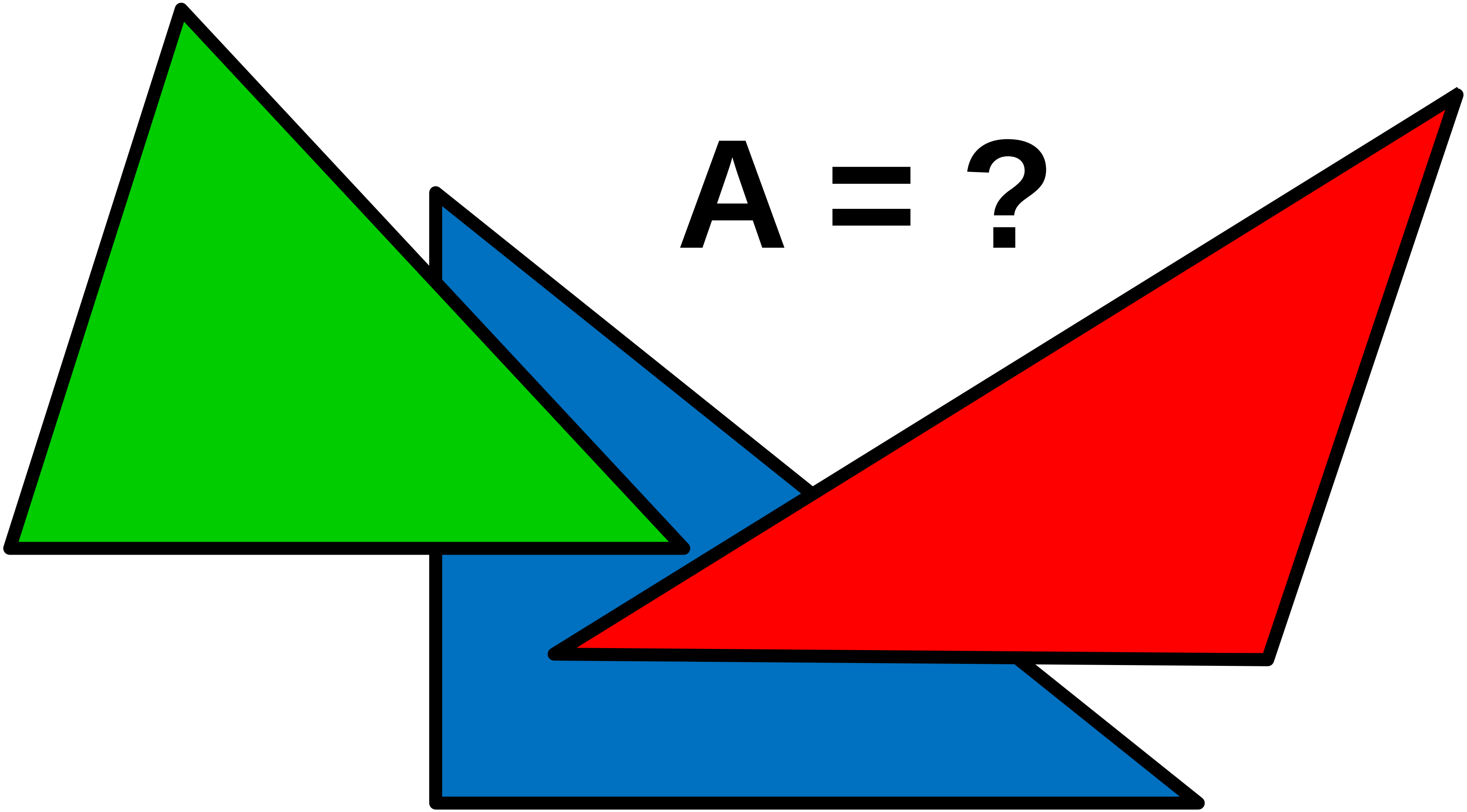
Department of
Curriculum and Pedagogy

Mathematics
Shape and Space: Area of Triangles

Science and Mathematics
Education Research Group

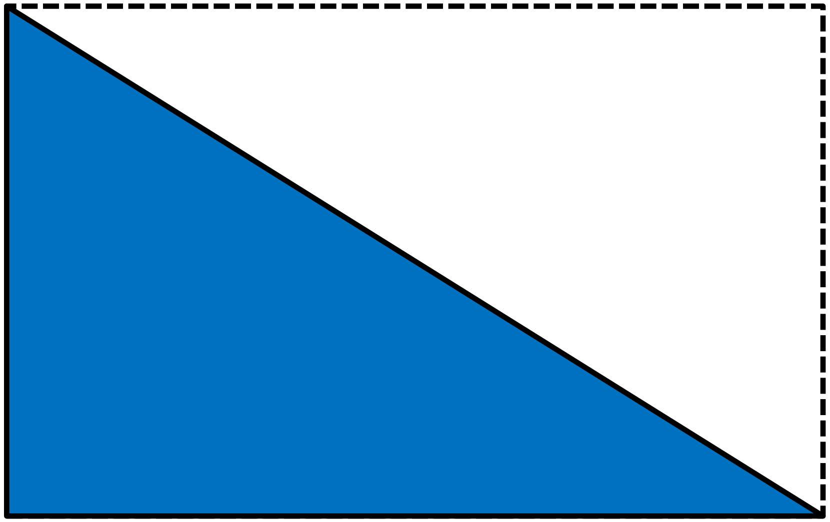
Supported by UBC Teaching and Learning Enhancement Fund 2012-2013

Deriving the Area of Triangles



Area of Triangles I

Consider a triangle drawn by connecting two opposite corners of a rectangle. What percent of the rectangle's area does the triangle cover?



A. 25%

B. 50%

C. 75%

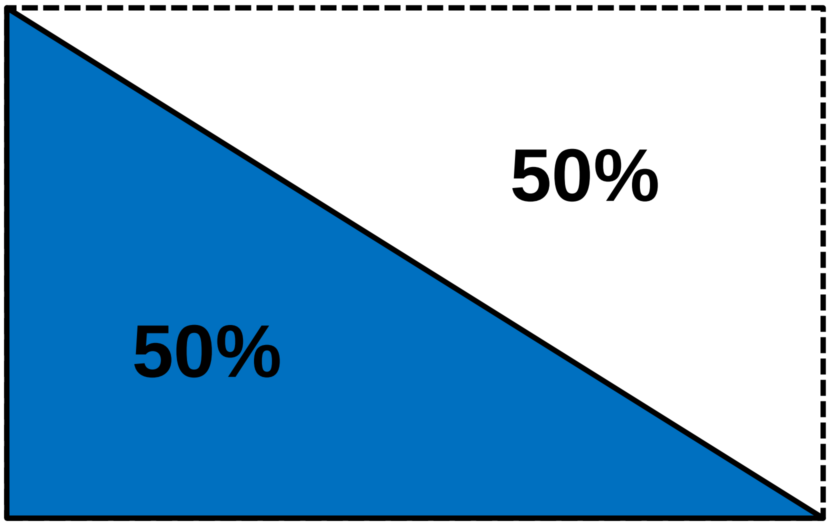
D. None of the above (but can still be determined)

E. Not enough information

Solution

Answer: B

Justification: Exactly half of the rectangle's area is covered by the triangle. The area of the triangle should therefore be 50% of the area of the rectangle.



Area of Triangles II

The area of the rectangle formed by the dashed line is 40 m^2. What is the area of the blue triangle?

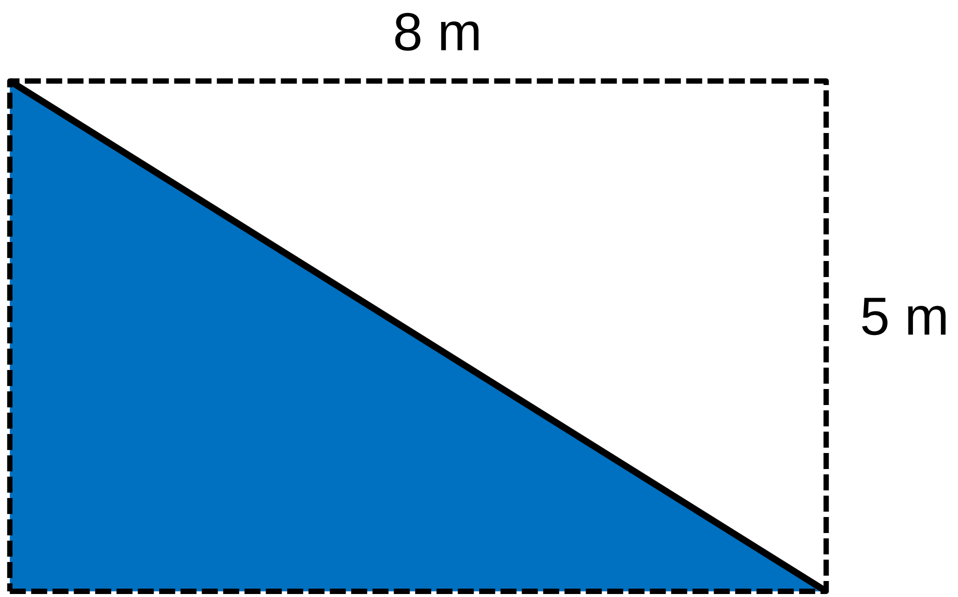
A. 20 m^2

B. 40 m^2

C. 60 m^2

D. 80 m^2

E. Not enough information

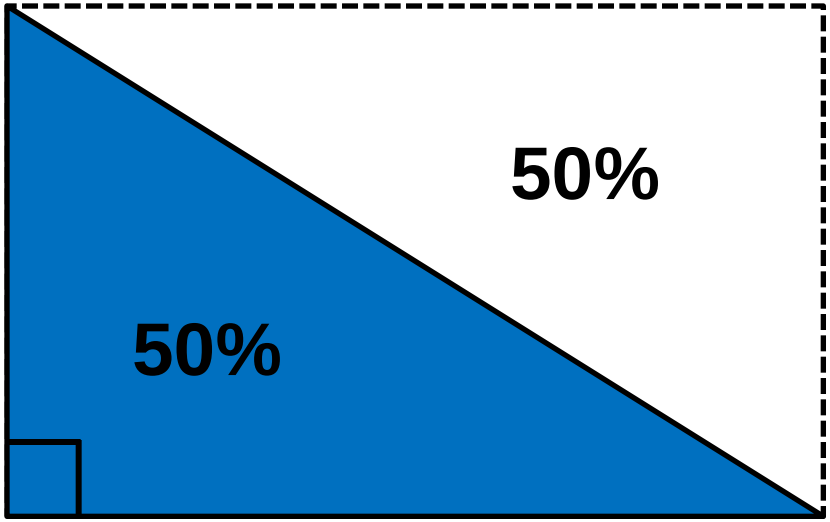


Solution

Answer: A

Justification: The area of the triangle must be half of the area of the rectangle.

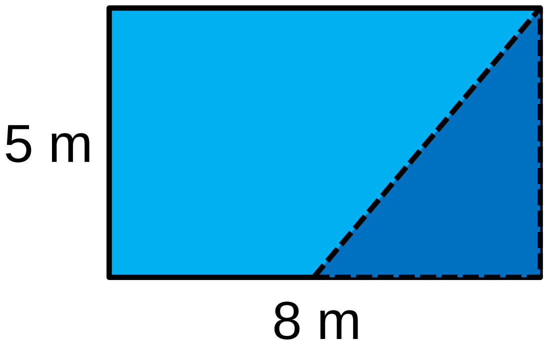
$$A = \frac{1}{2}(40\,\mathrm{m}^2) = 20\,\mathrm{m}^2$$

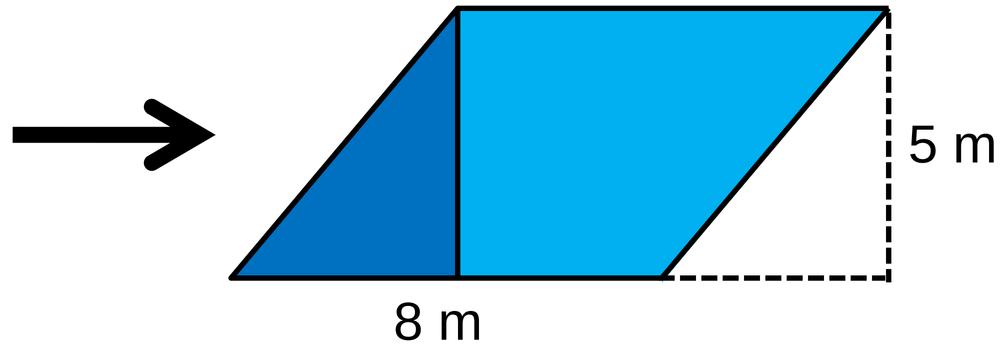


Note: The blue triangle is called a "right triangle" because it contains a right angle (90°).

Area of Triangles III

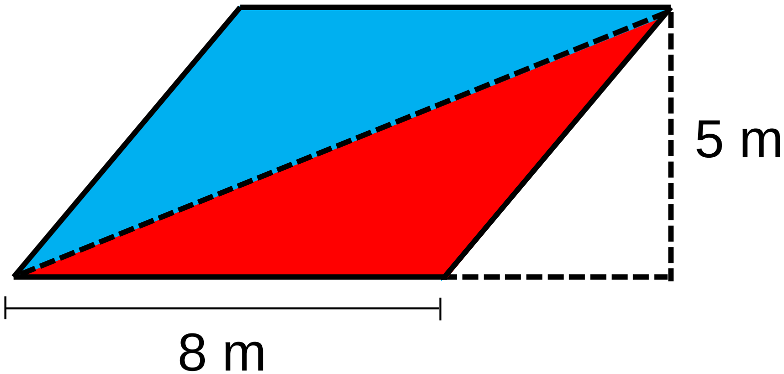
A triangle is cut from the side of an 8 m by 5 m rectangle and glued to the other side as shown.





A diagonal is now drawn from the two furthest apart corners.

What is the area of the red triangle?



A. Between 0 m^2 and 20 m^2

B. Exactly 20 m^2

C. Between 20 m^2 and 40 m^2

D. Exactly 40 m^2

E. Not enough information

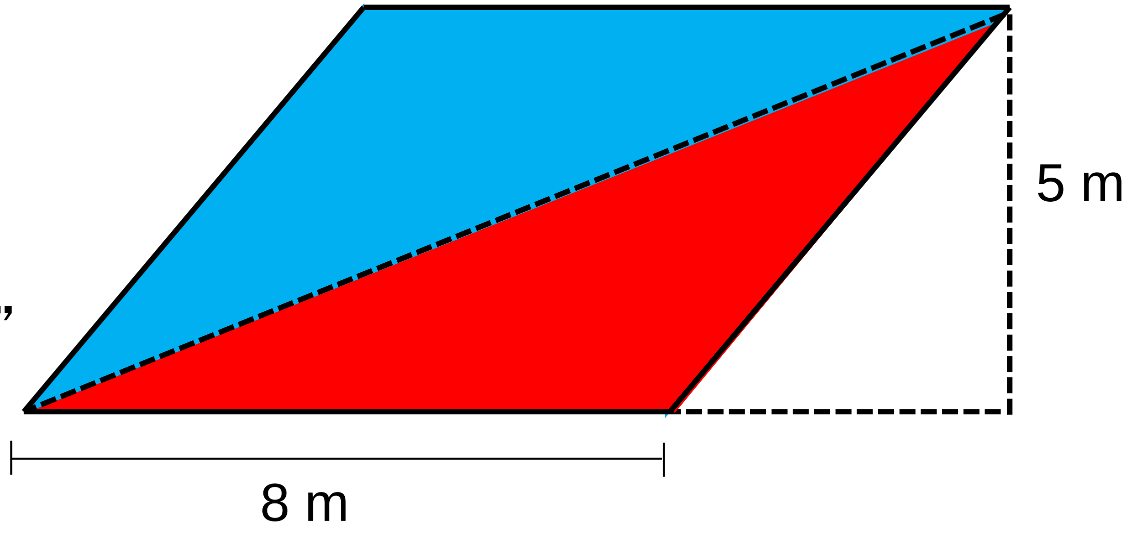
Solution

Answer: B

Justification: The area of the original rectangle was 40 m^2. No area is lost when pieces of the rectangle are moved around. Since the diagonal line cuts the figure in half, the area of the triangle should be half the area of the rectangle.

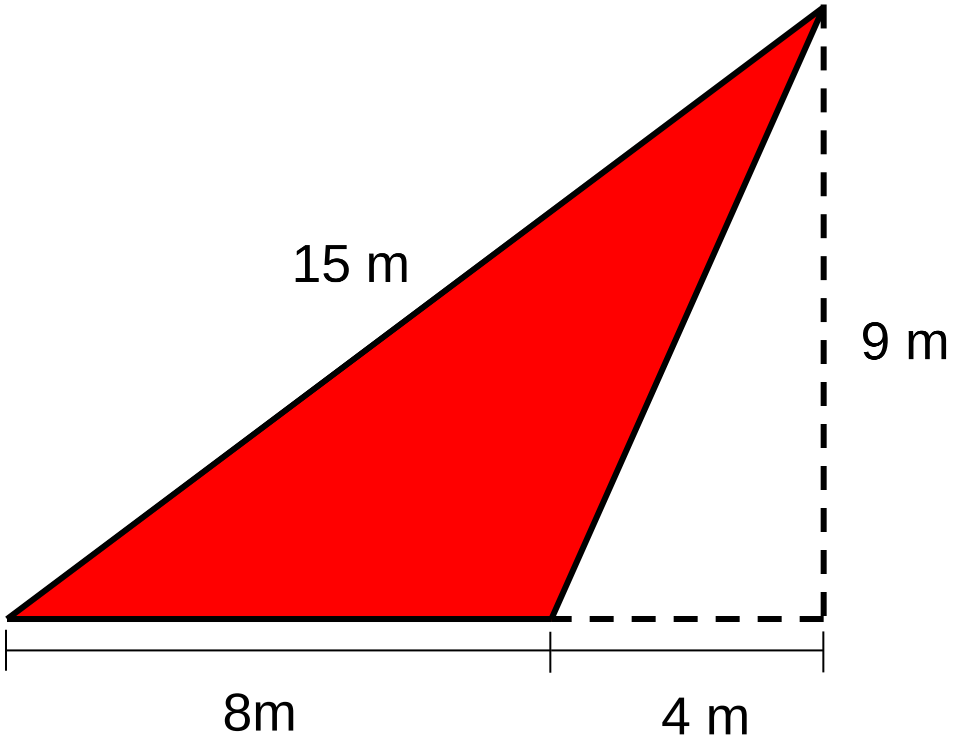
$$A = \frac{1}{2}(40\,\mathrm{m}^2) = 20\,\mathrm{m}^2$$

Note: The red triangle is called an "obtuse triangle" because it contains an obtuse angle (an angle greater than 90°).



Area of Triangles IV

What is the area of the red triangle?



A. 36 m^2

B. 54 m^2

C. 60 m^2

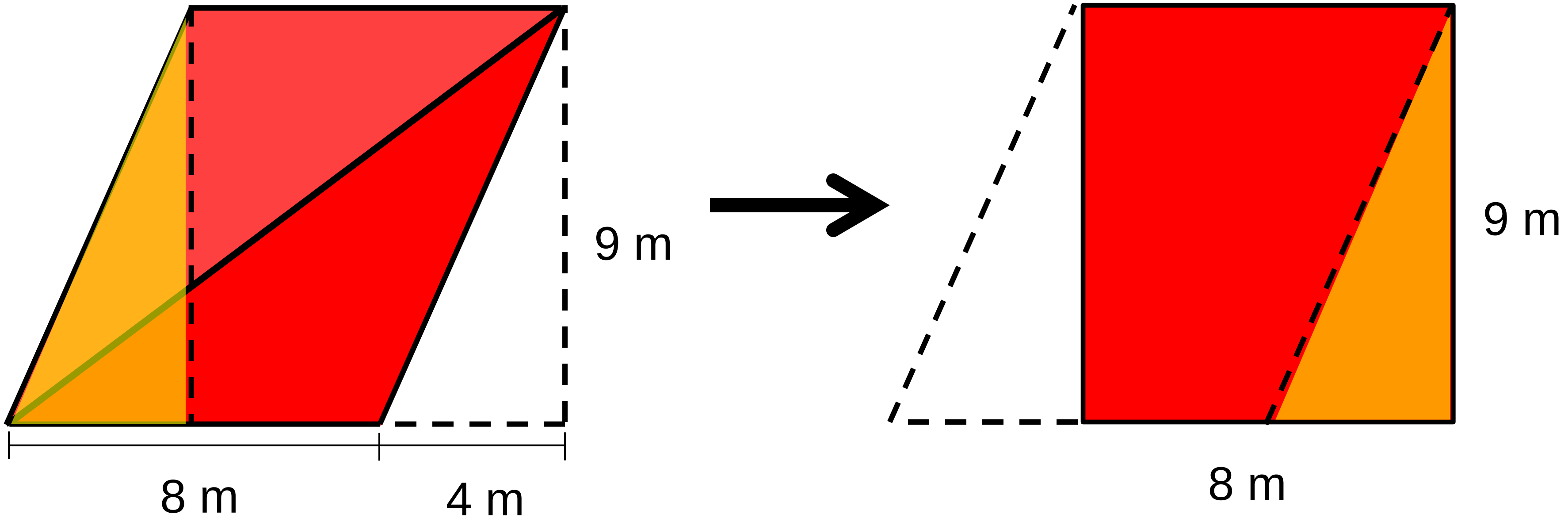
D. 72 m^2

E. 108 m^2

Solution

Answer: A

Justification: The triangle can be represented as half the area of a rectangle.

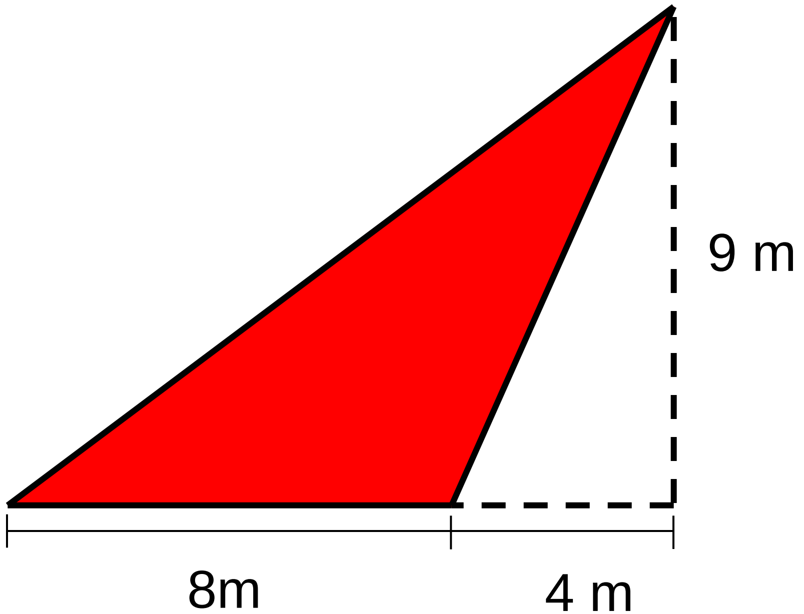


$$A = \frac{1}{2}(8\text{ m})(9\text{ m}) = 36\,\text{m}^2$$

Solution

Answer: A

Justification: Imagine a triangle with a 12 m base and 9 m height. Find the area of this triangle, then subtract the missing 4 m by 9 m triangle to find the area of the red triangle.



$$A_{\text{full triangle}} = \frac{9 \times 12}{2} = 54\ \text{m}^2$$

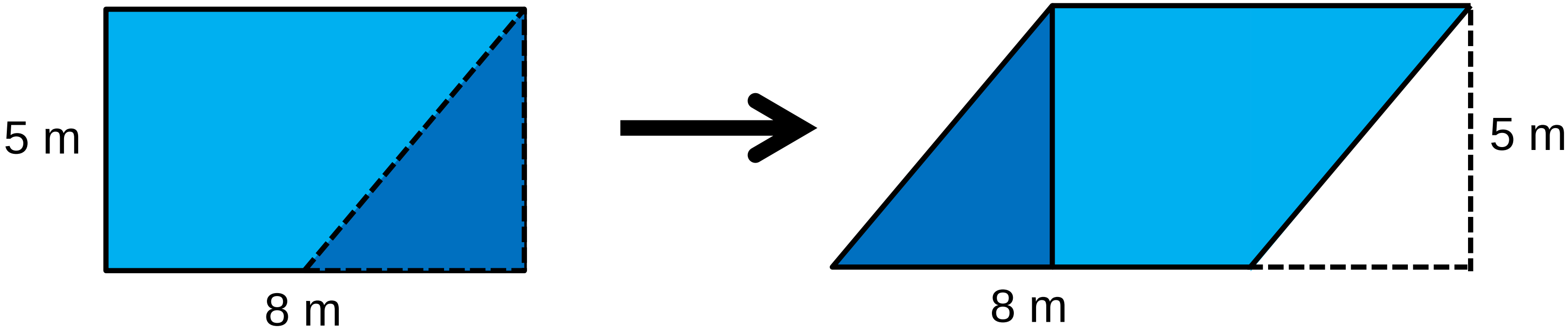
$$A_{\text{white triangle}} = \frac{4 \times 9}{2} = 18\ \text{m}^2$$

$$A_{\text{red triangle}} = A_{\text{full triangle}} - A_{\text{white triangle}}$$

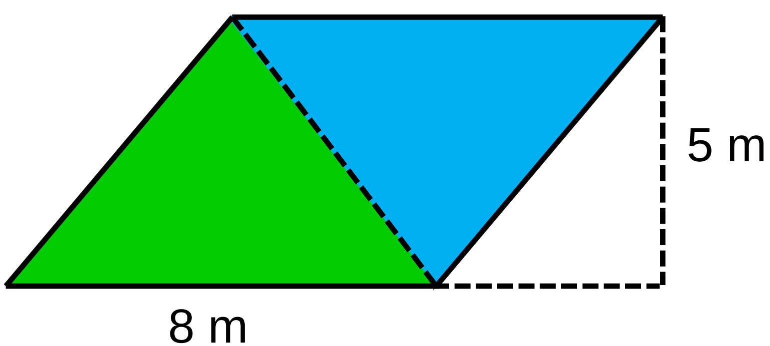
$$= 36\ \text{m}^2$$

Area of Triangles V

A triangle is cut from the side of an 8 m by 5 m rectangle and glued to the other side as shown.



A diagonal is drawn between the two closest opposite corners.
What is the area of the green triangle?



A. Between 0 m^2 and 20 m^2

B. Exactly 20 m^2

C. Between 20 m^2 and 40 m^2

D. Exactly 40 m^2

E. Not enough information

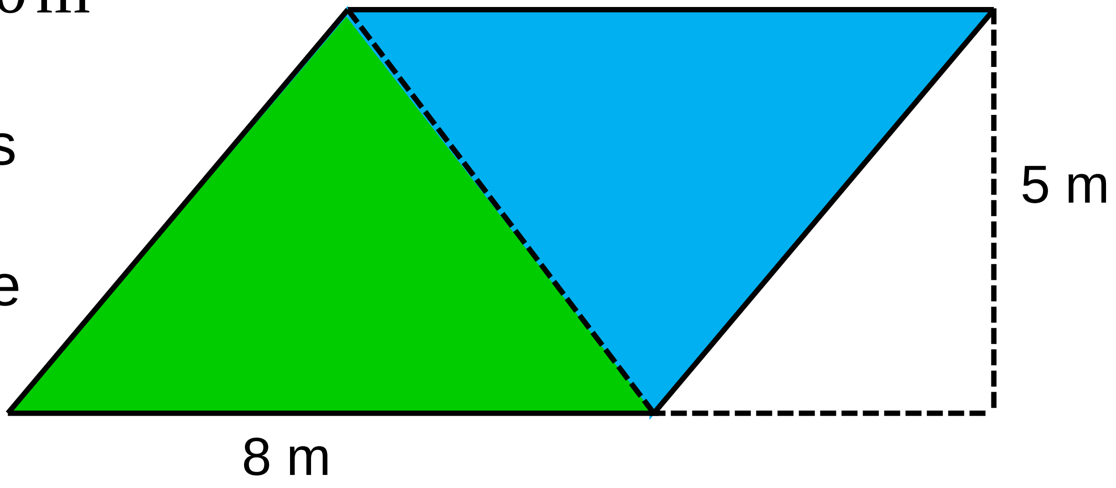
Solution

Answer: B

Justification: The area of the original rectangle was 40 m^2. No area is lost when pieces of the rectangle are moved around. Since the diagonal line cuts the figure in half, the area of the triangle should be half the area of the rectangle.

$$A = \frac{1}{2}(40\,\text{m}^2) = 20\,\text{m}^2$$

Note: The green triangle is called an "acute triangle" because it contains 3 acute angles (all angles are less than 90°).



Area of Triangles VI

What is the area of the green triangle?

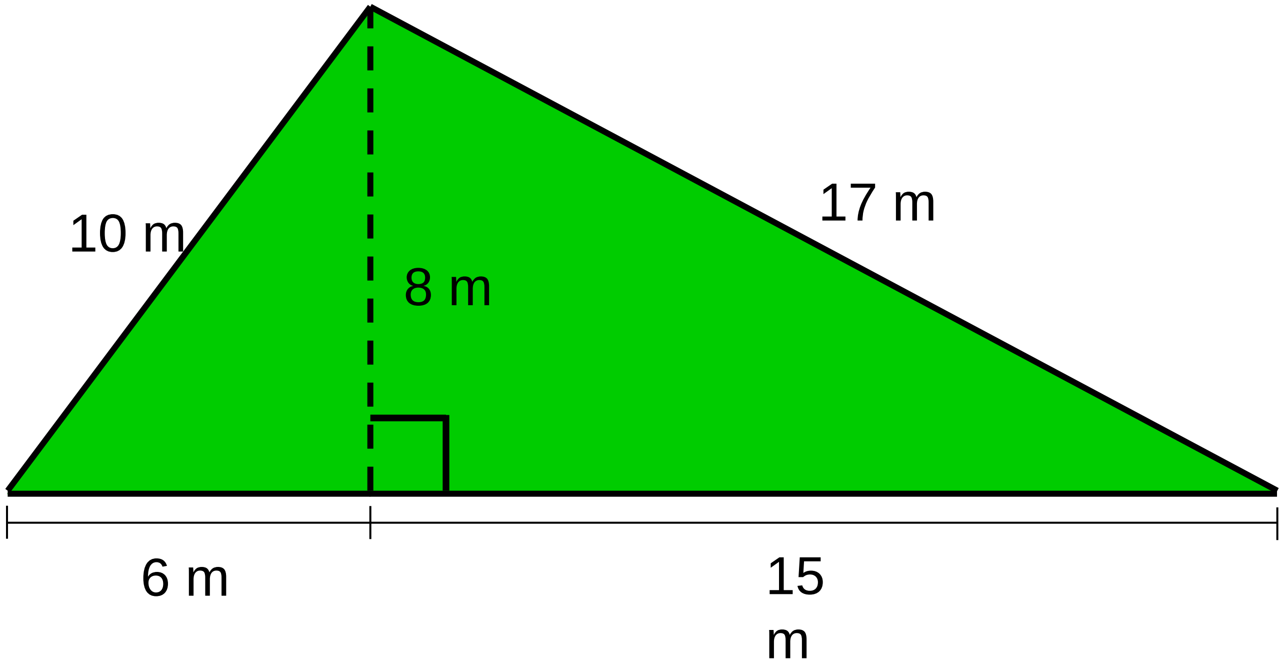
A. 84 m^2

B. 85 m^2

C. 105 m^2

D. 158 m^2

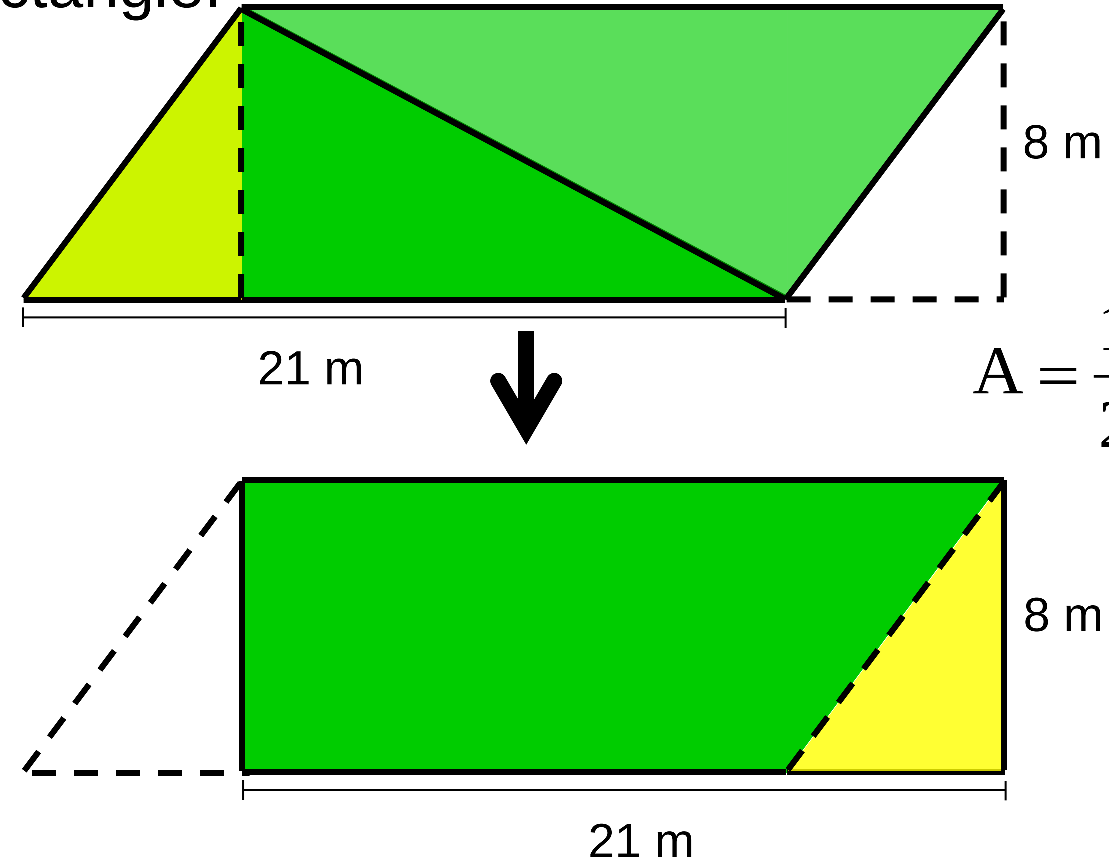
E. 179 m^2



Solution

Answer: A

Justification: The triangle can be represented as the area of half a rectangle.

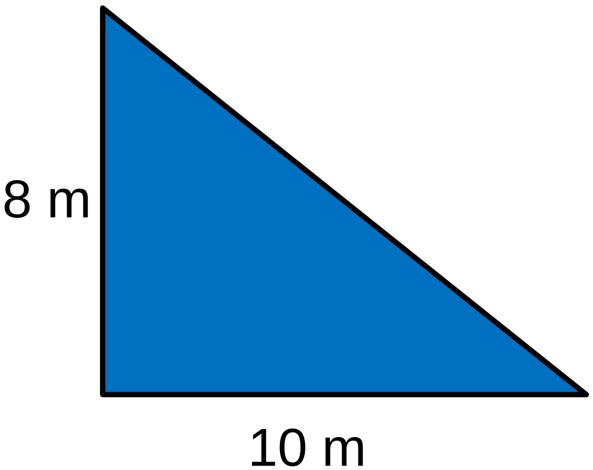


$$A = \frac{1}{2}(21\text{ m})(8\text{ m}) = 84\text{ m}^2$$

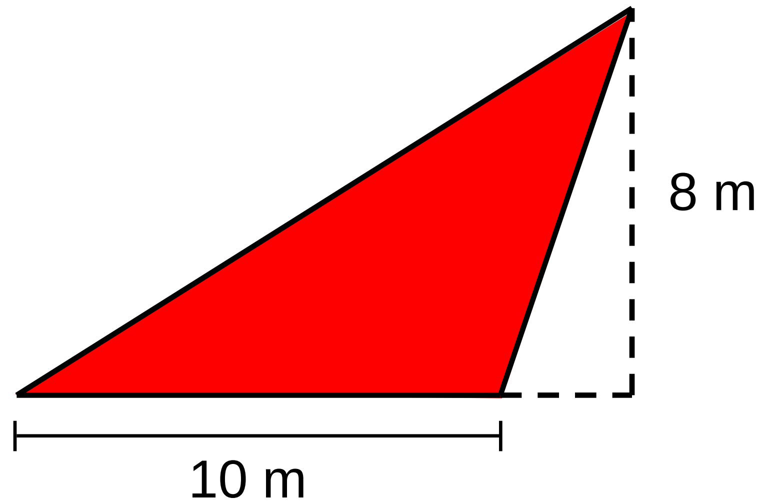
Area of Triangles VII

Which triangle has the largest area?

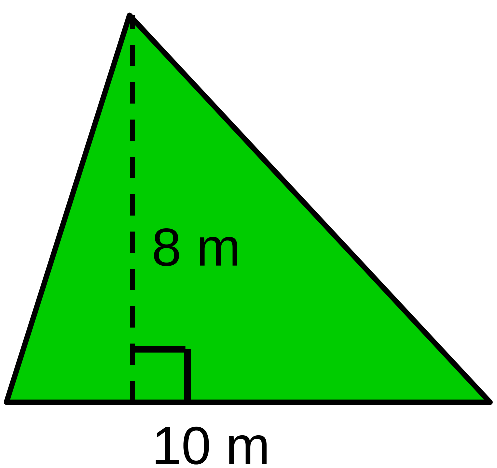
A.



B.



C.

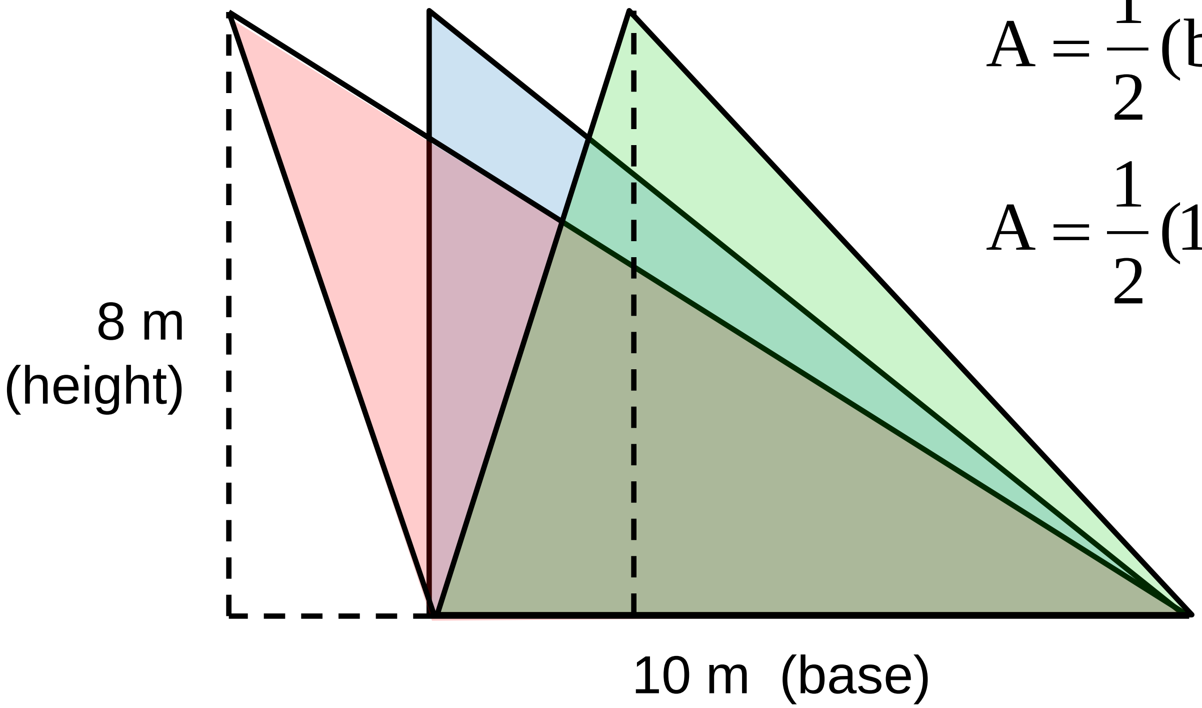


D. All 3 triangles have the same area

Solution

Answer: D

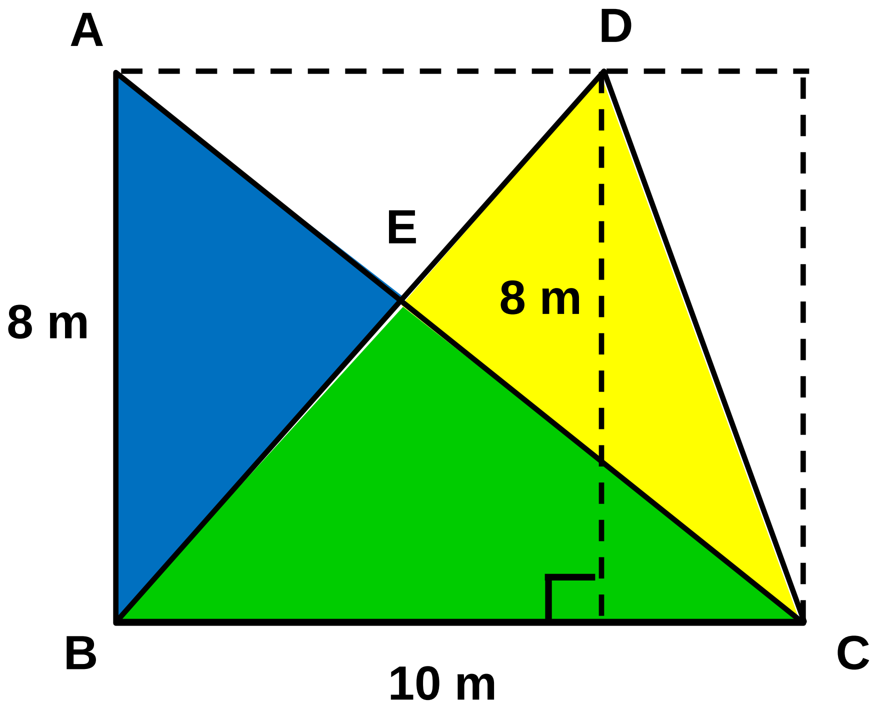
Justification: All 3 triangles have the same base and height. The formula to find the area of any triangle is:



$$A = \frac{1}{2}(\text{base})(\text{height}) = 40\,\text{m}^2$$

$$A = \frac{1}{2}(10\text{ m})(8\text{ m}) = 40\,\text{m}^2$$

Area of Triangles VIII



Jeremy says that ΔABE has a larger area than ΔCDE.

Kevin says that ΔCDE has a larger area than ΔABE.

Marina says that both ΔABE and ΔCDE have the same area.

A. Jeremy is correct.

B. Kevin is correct.

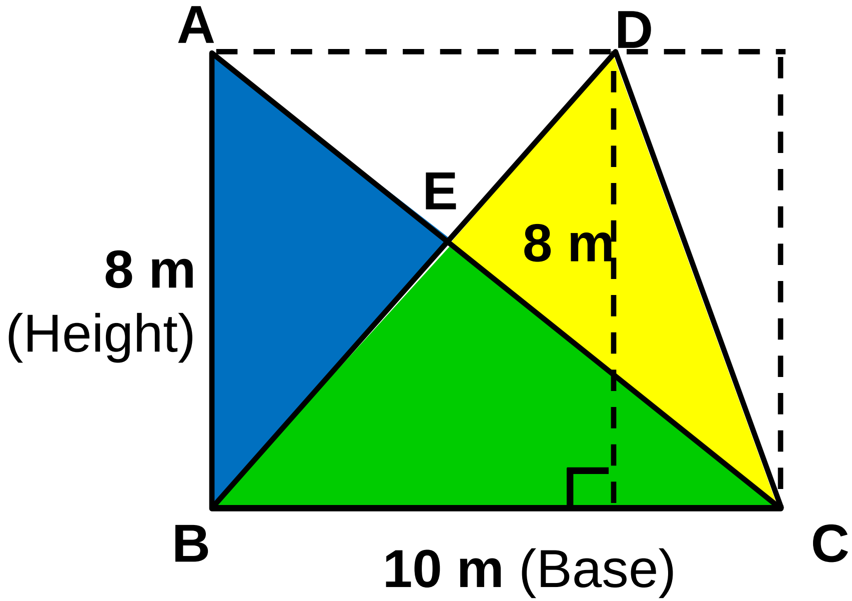
C. Marina is correct.

D. Everyone is guessing because there is not enough information.

Solution

Answer: C

Justification: The blue and yellow triangle should have the same area.



Notice that ΔABC has the same area as ΔBCD since they both have a base of 10 m and a height of 8 m.

Area of ΔABC = Area of ΔBCD

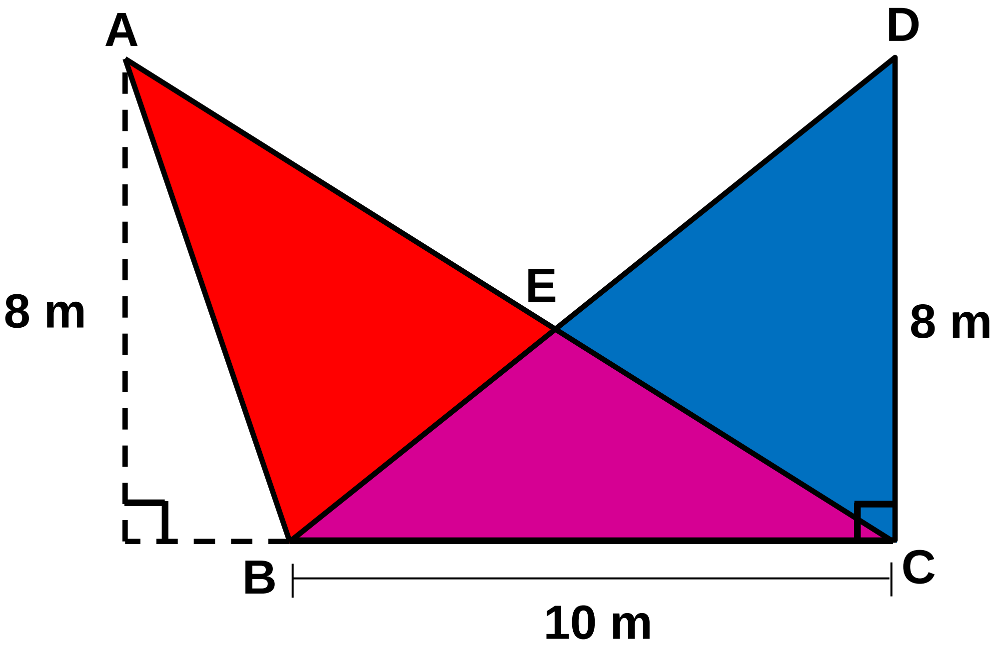
ΔABC = ΔABE + ΔBCE

ΔBCD = ΔCDE + ΔBCE

Therefore: Area of ΔABE + ~~ΔBCE~~ = Area of ΔCDE + ~~ΔBCE~~

Area of ΔABE = Area of ΔCDE

Area of Triangles IX



Jeremy says that ΔABE has a larger area than ΔCDE.

Kevin says that ΔCDE has a larger area than ΔABE.

Marina says that both ΔABE and ΔCDE have the same area.

A. Jeremy is correct.

B. Kevin is correct.

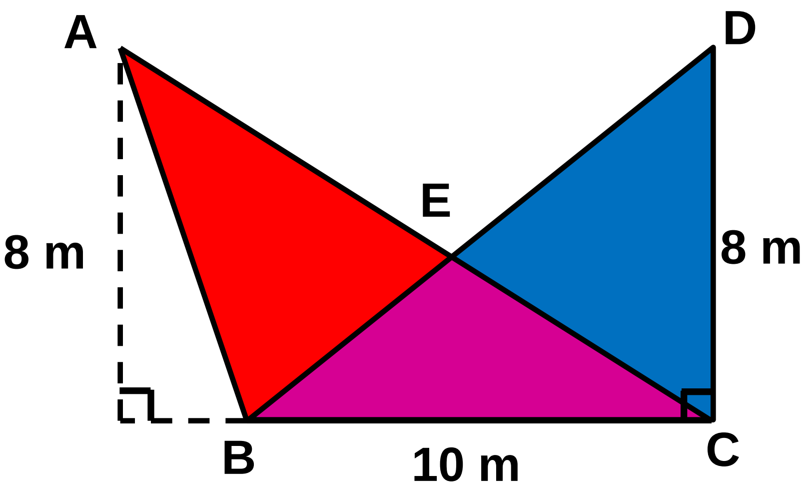
C. Marina is correct.

D. Everyone is guessing because there is not enough information.

Solution

Answer: C

Justification: The red and blue triangle should have the same area.



This question is almost exactly the same as the previous, except an obtuse triangle is used instead of an acute triangle. Remember that two questions ago we determined the type of triangle is not important, base and height are.

Area of ΔABC = Area of ΔBCD

Area of ΔABE + ~~ΔBCE~~ = Area of ΔCDE + ~~ΔBCE~~

Area of ΔABE = Area of ΔCDE