a place of mind

FACULTY OF EDUCATION

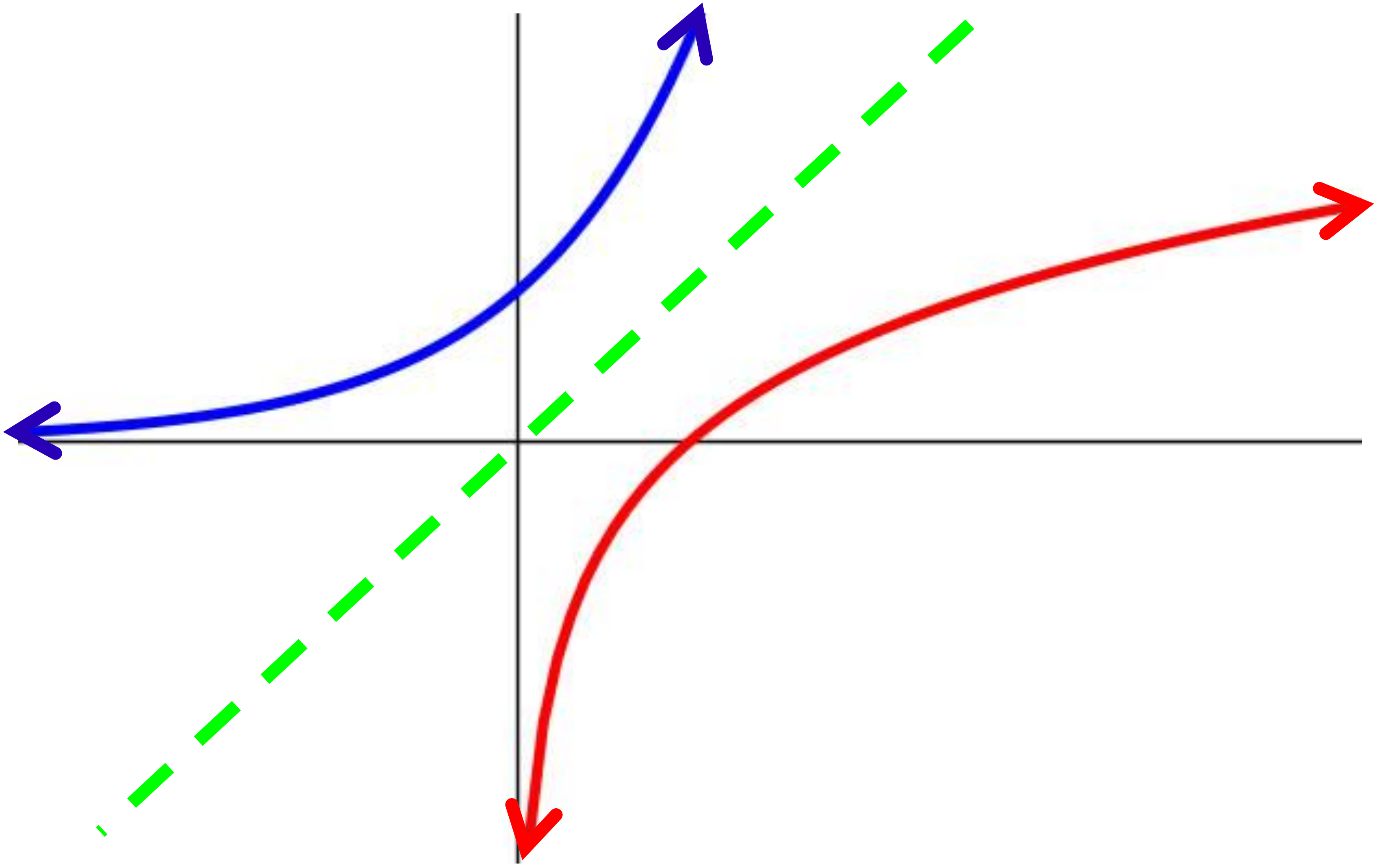
Department of
Curriculum and Pedagogy

Mathematics
Functions: Logarithms

Science and Mathematics
Education Research Group

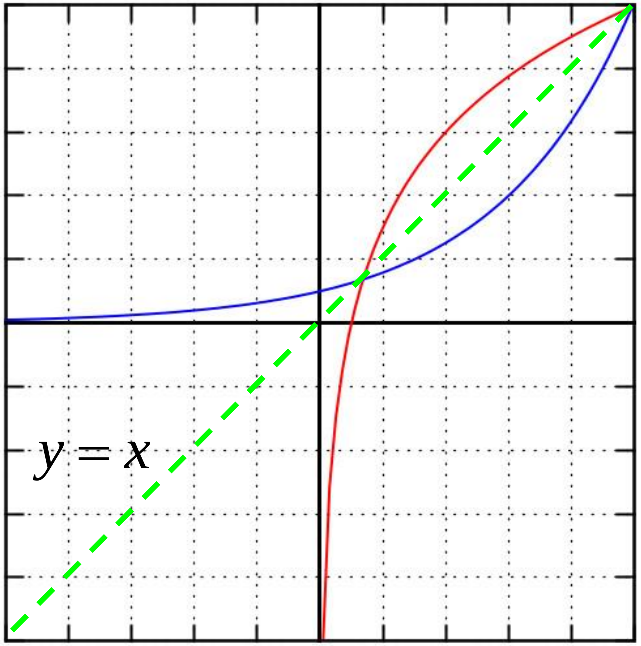
Supported by UBC Teaching and Learning Enhancement Fund 2012-2014

Log Functions



Review of Inverse Functions

This question set expects students to be comfortable with transformations and finding the inverse of functions. Students should also know the basic properties of logarithms and exponents.



Review: Reflect a graph in the line $y = x$ to find the graph of the inverse.

The inverse of an equation in the form

$$y = f(x)$$

can be found by interchanging x and y, then solving the equation for y.

Log Functions I

We are given the function $y = a^x$.

What is the equation of its inverse?

A. $y = x^a$

B. $y = -a^x$

C. $y = \log_x a$

D. $y = \log_a x$

E. The function does not have an inverse

Press for hint

Solve for y: $x = a^y$

Solution

Answer: D

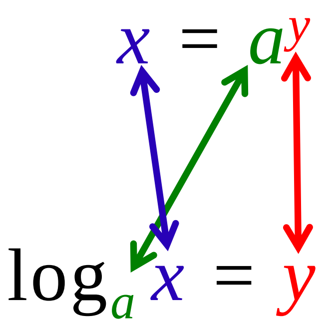
Justification: We find the inverse of the function by interchanging x and y values, then solving for y:

$$y = a^x$$

$$x = a^y \qquad \text{interchange } x \text{ and } y$$

$$y = \log_a x$$

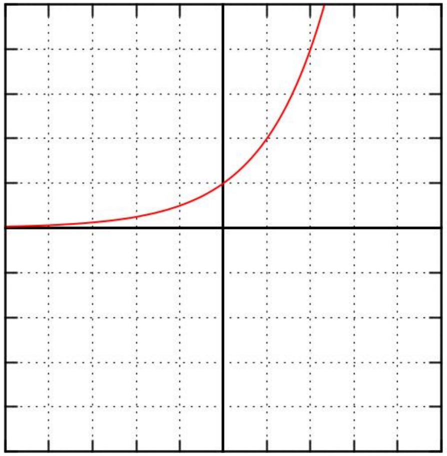
Recall the definition of logarithms:



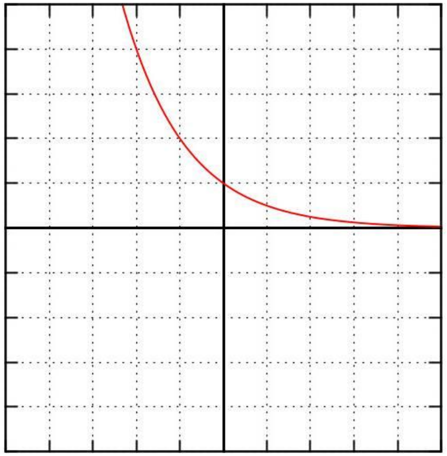
Log Functions II

What is the graph of the function $y = a^x$ if $a > 1$.

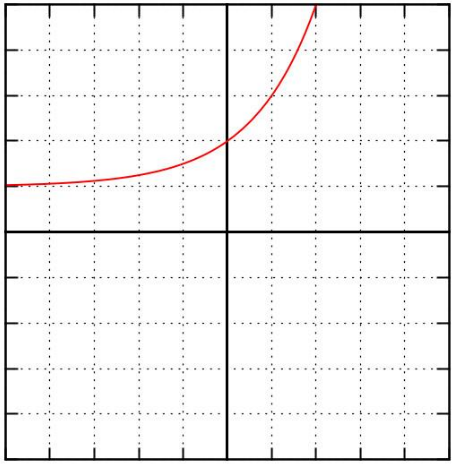
A.



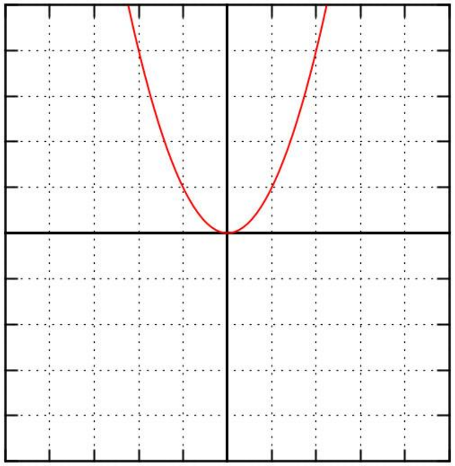
B.



C.



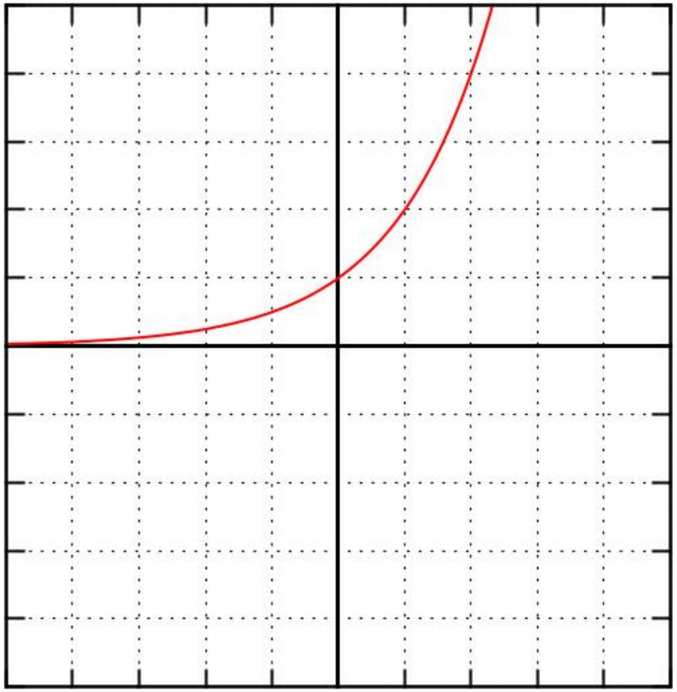
D.



Solution

Answer: A

Justification: This is one of the basic graphs you should know how to quickly sketch.



If $a > 1$, all functions in the form $y = a^x$ cross the y-axis at the point (0, 1).

As x goes to positive infinity, the graph grows exponentially.

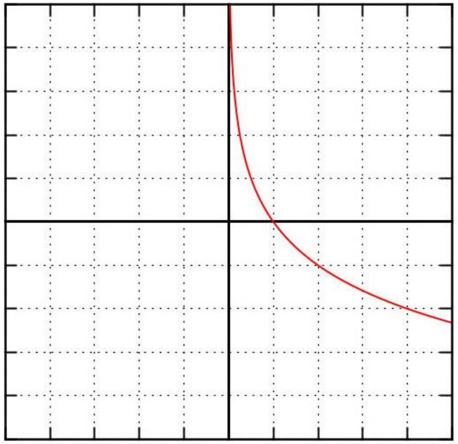
As x goes to negative infinity, the graph is still positive but decays exponentially to $y = 0$.

Log Functions III

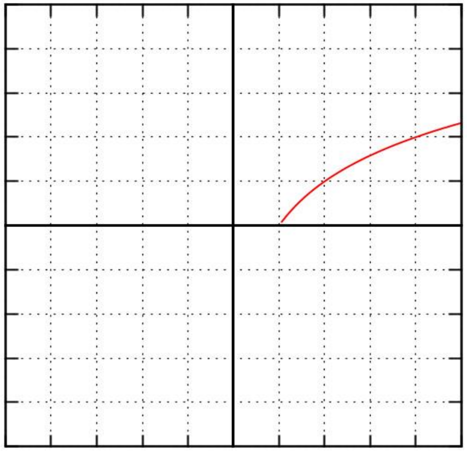
The function $y = \log_a x$ is the inverse of the function $y = a^x$.

What is the graph of the function $y = \log_a x$ when $a > 1$?

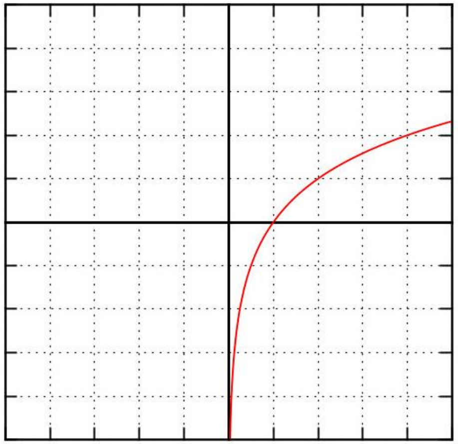
A.



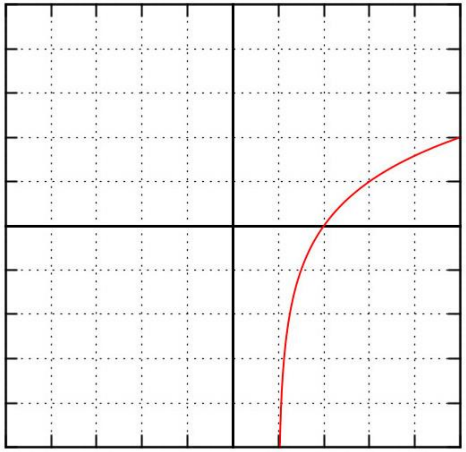
B.



C.



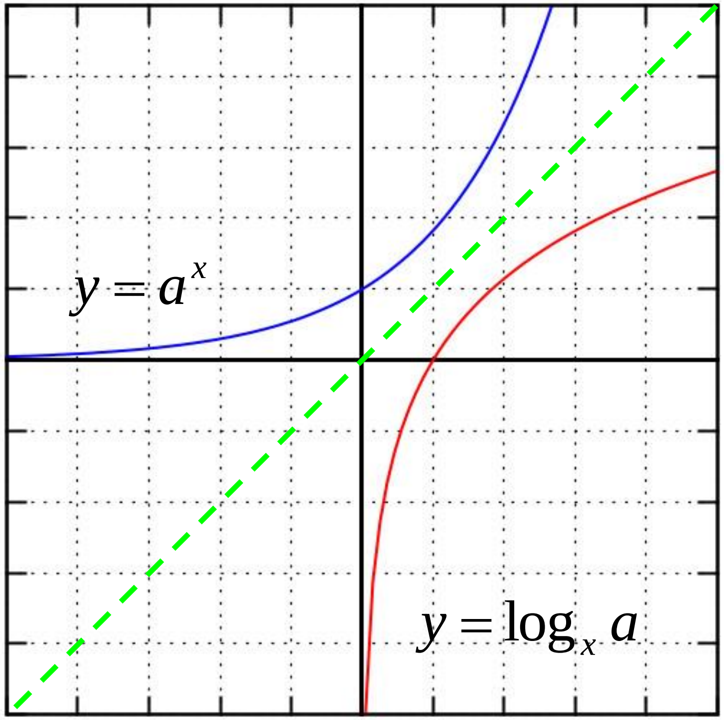
D.



Solution

Answer: C

Justification: To find the graph of $y = \log_a x$, reflect the graph $y = a^x$ across the line $y = x$ because the two are inverses of each other.



Graph B is incorrect because negative exponents are never returned by the log function.

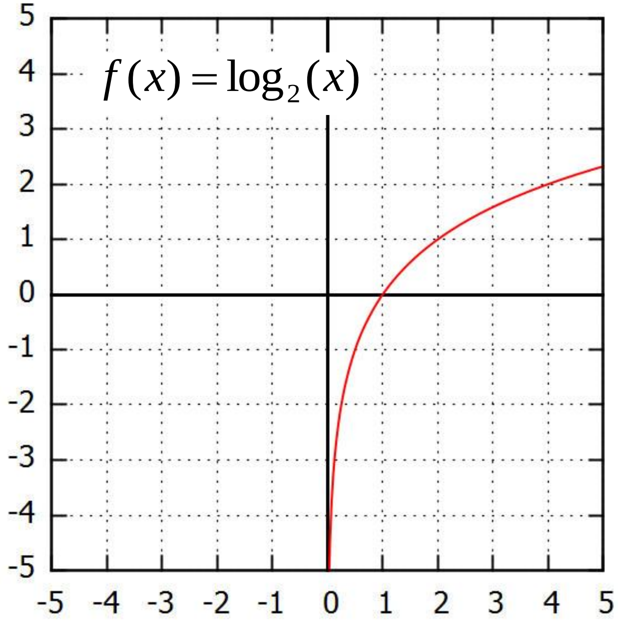
Graph D is incorrect because the log function should be able to return values when x is between 0 and 1.

Graph A is the graph of $y = \log_a x$ when $0 < a < 1$. What does $y = a^x$ look like when $0 < a < 1$?

Log Functions IV

What are the domain and range of
$f(x) = \log_2(x)$?

	Domain	Range
A.	$x > 0$	$y \geq 0$
B.	$x > 0$	All real numbers
C.	$x \geq 0$	$y \geq 0$
D.	$x \geq 0$	All real numbers
E.	All real numbers	All real numbers



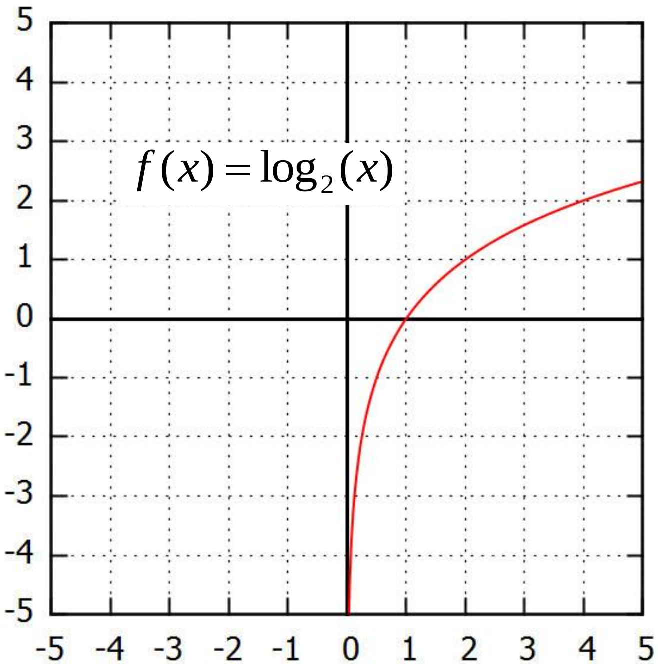
Press for hint

Solution

Answer: B

Justification: The domain of the log function is the range of an exponential function since these two functions are inverses. Notice that we do not include 0 in the domain because exponential functions do not return the value 0.

The range of the log function is the domain of the exponential function. There are no restrictions on the exponent on a number, so the log function can return any number.



Domain: $x > 0$

Range: All real numbers

Log Functions V

What point exists on all graphs in the form $y = \log_a x$, where $a > 0$?

A. $(0, 1)$

B. $(1, 0)$

C. $(1, 1)$

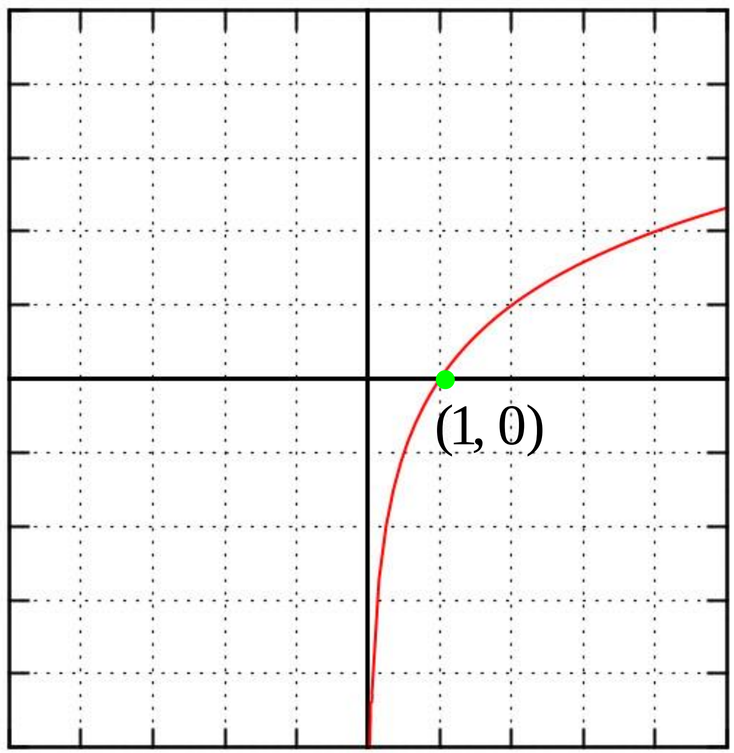
D. $(1, a)$

E. No such point exists

Solution

Answer: B

Justification: The graph of $y = \log_a x$ must pass through the point (1, 0). This is because $\log_a 1 = 0$ for all $a > 0$.

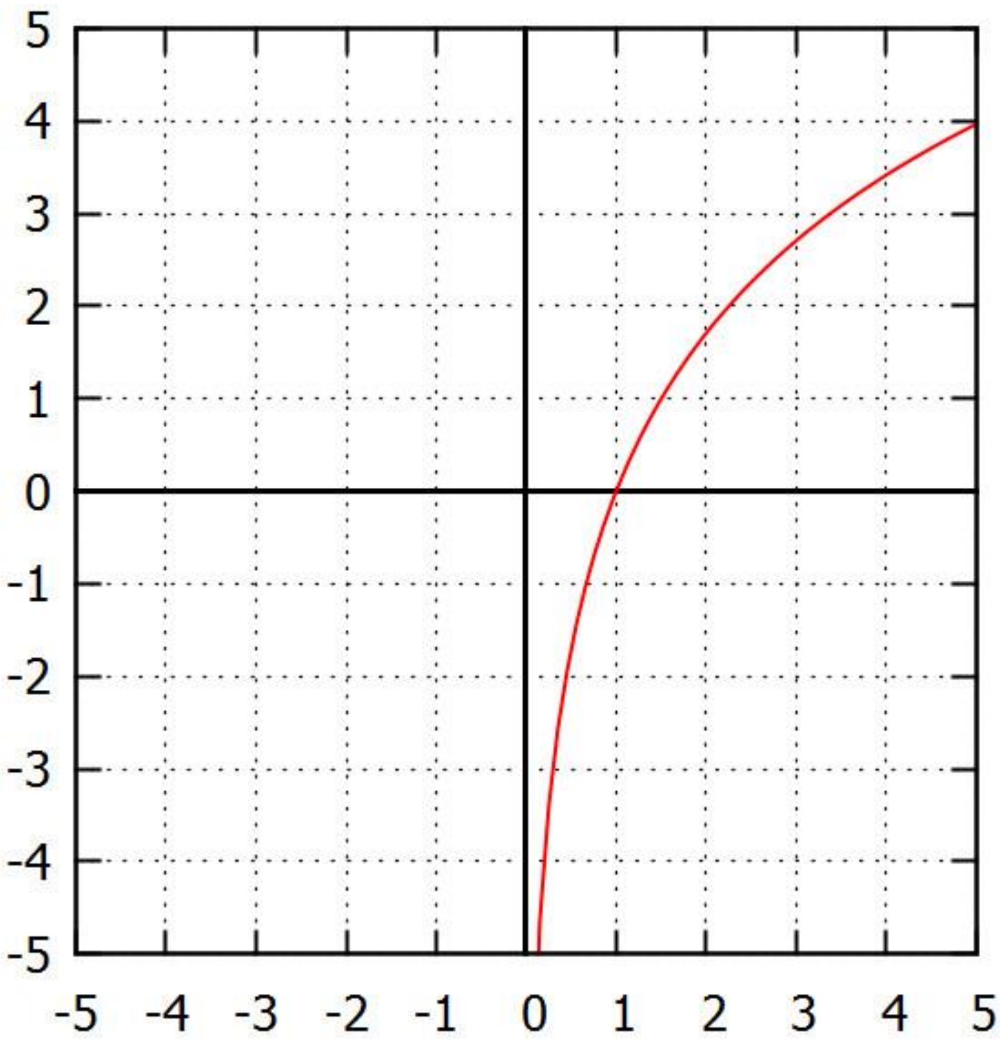


This is similar to the rule that the graph $y = a^x$ always passes through the point (0, 1), because $a^0 = 1$.

Notice how the point (0, 1) on the exponential function is the inverse of the point on the logarithmic function, (1, 0).

Log Functions VI

The graph shown to the right is in the form $y = \log_a x$. What is the value of a.



A. 1

B. 2

C. $\frac{3}{2}$

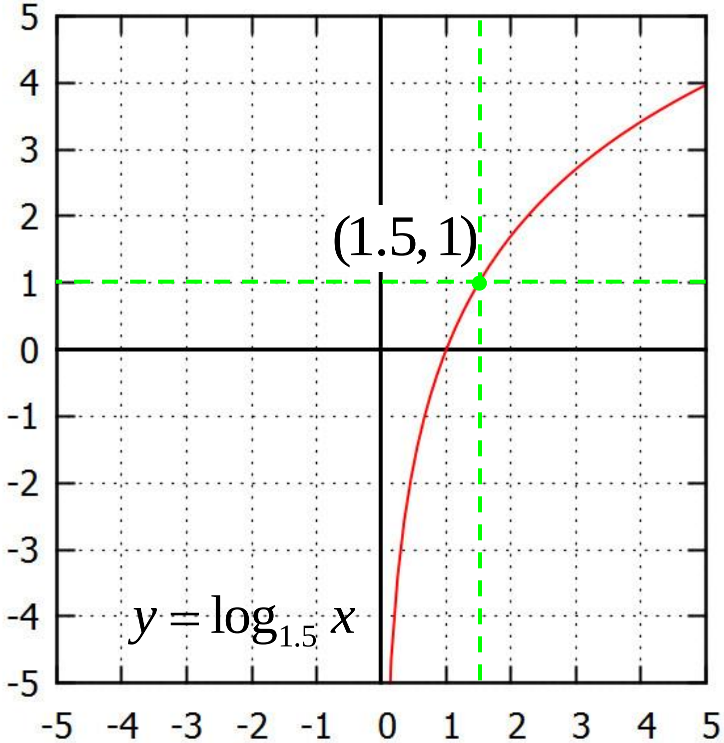
D. 4

E. Cannot be determined

Solution

Answer: C

Justification: Use the property that $\log_a a = 1$. This means that all graphs in the form $y = \log_a x$ pass through the point $(a, 1)$.



Look at the graph and find where it crosses the line $y = 1$. The x-value of this point is the base of the log function.

In this question, the graph crosses the line $y = 1$ at $(1.5, 1)$. The equation of the graph is therefore:

$$y = \log_{1.5} x$$

Log Functions VII

The graph shown to the right is in the form $y = \log_a x$. What is the value of a.

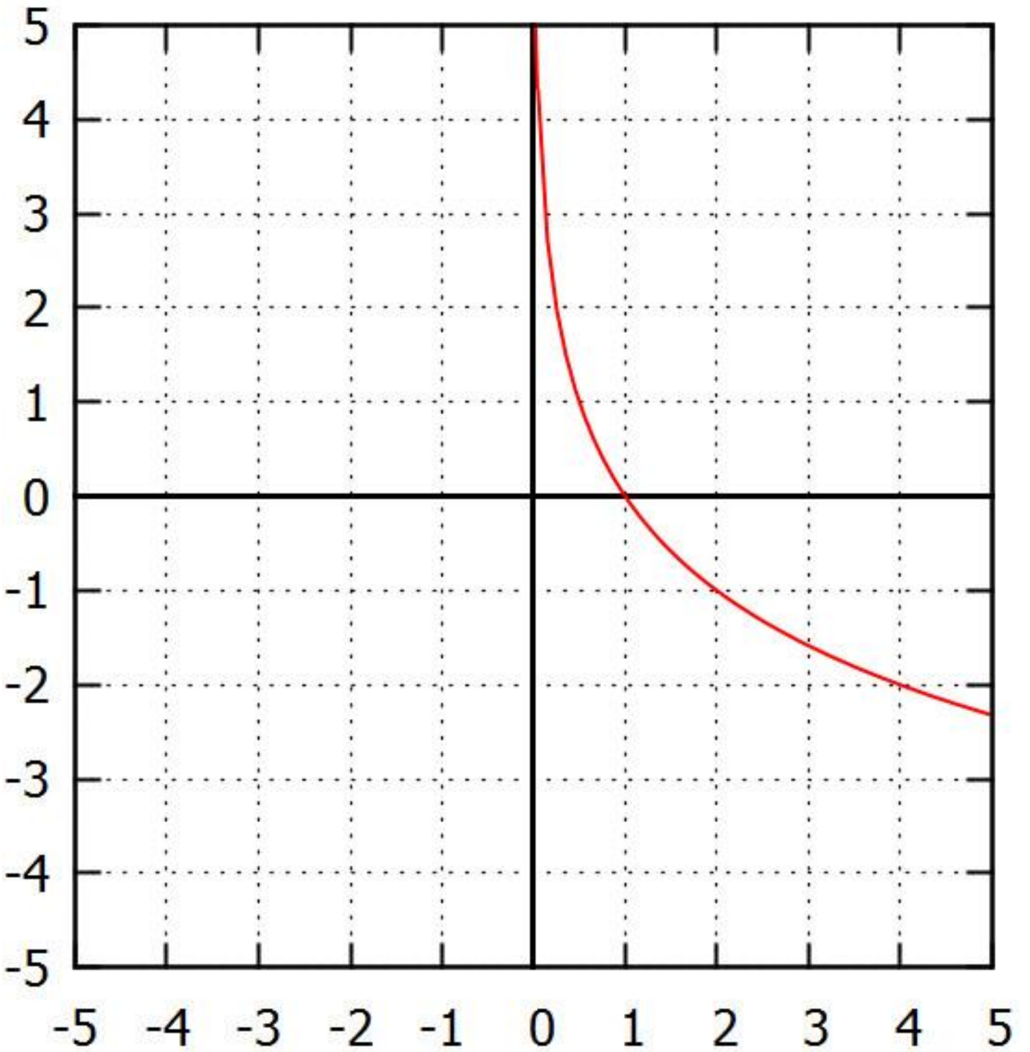
A. -2

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. 2

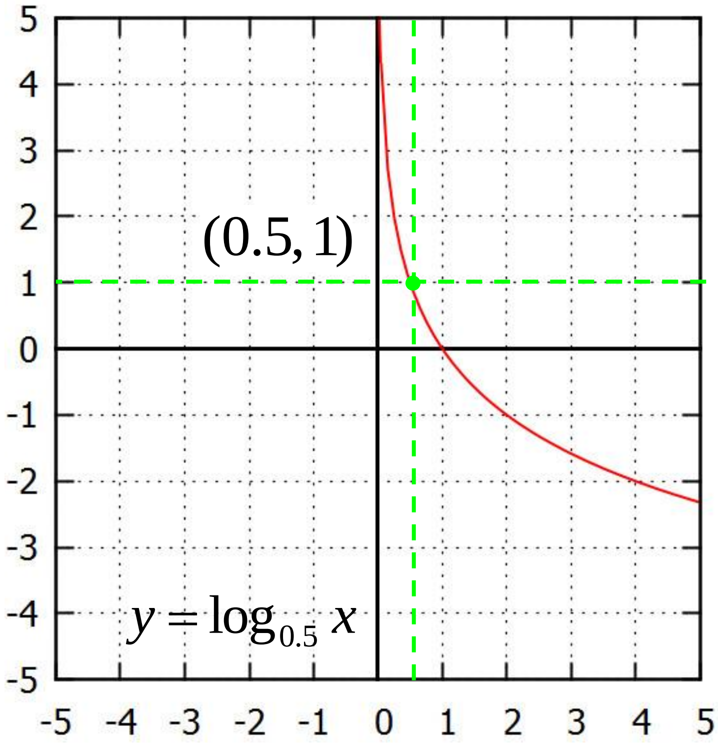
E. Cannot be determined



Solution

Answer: C

Justification: Use the same technique explained in the previous question to find the value of a.



Look at the graph and find where it crosses the line $y = 1$. The x-value of this point is the base of the log function.

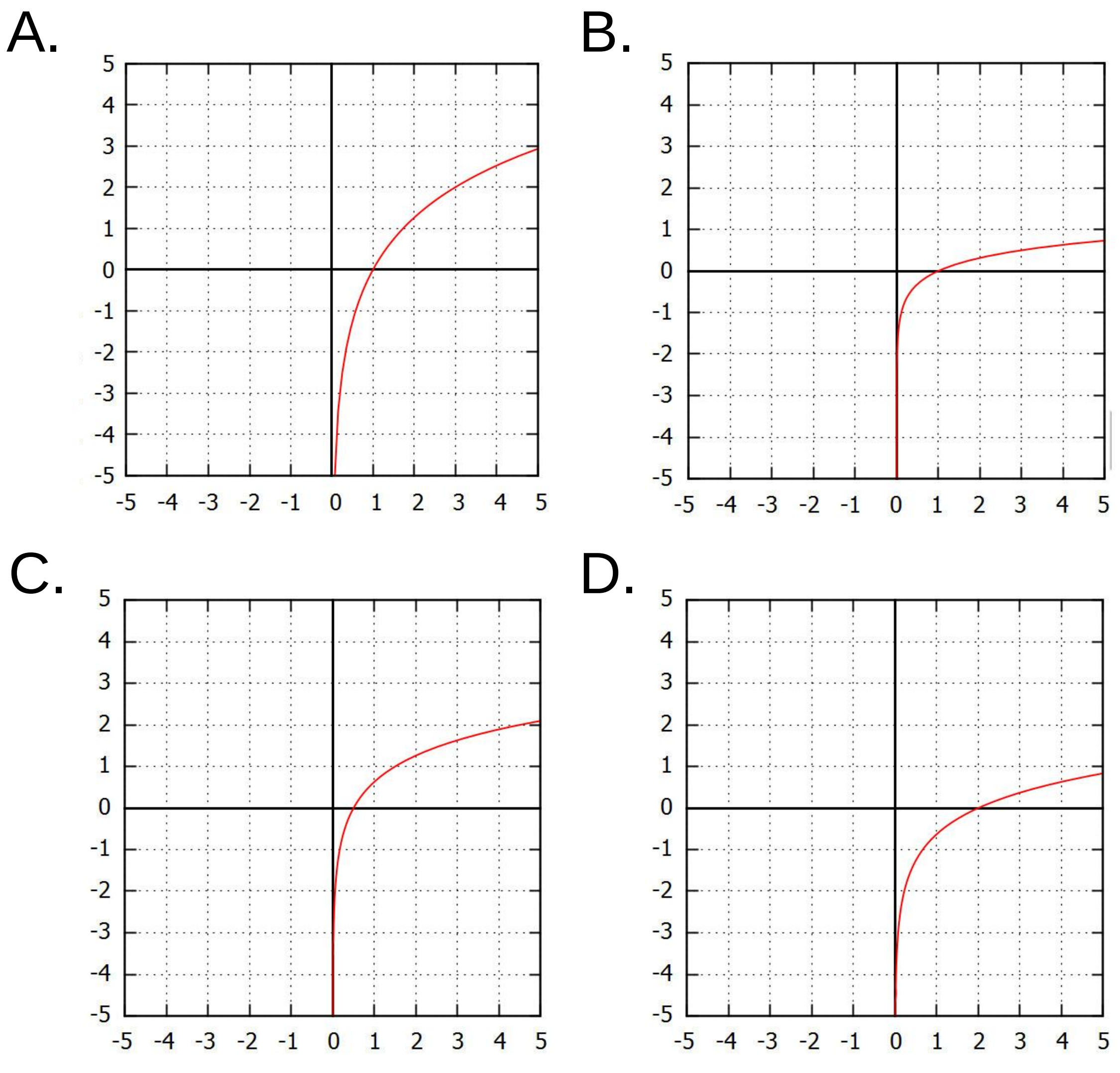
In this question, the graph crosses the line $y = 1$ at $(0.5, 1)$. The equation of the graph is therefore:

$$y = \log_{0.5} x$$

Log Functions VIII

What is the correct graph of the following function?

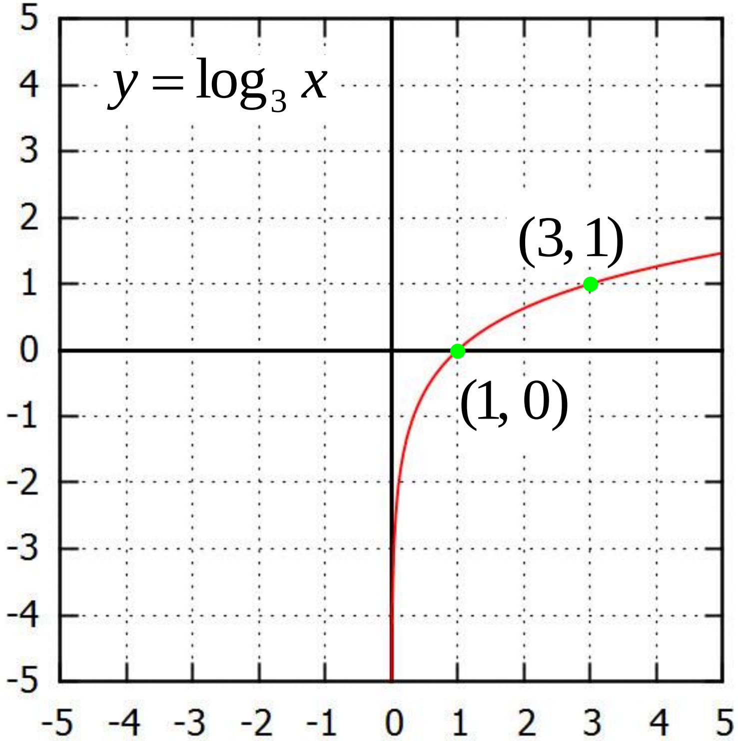
$$f(x) = 2\log_3(x)$$

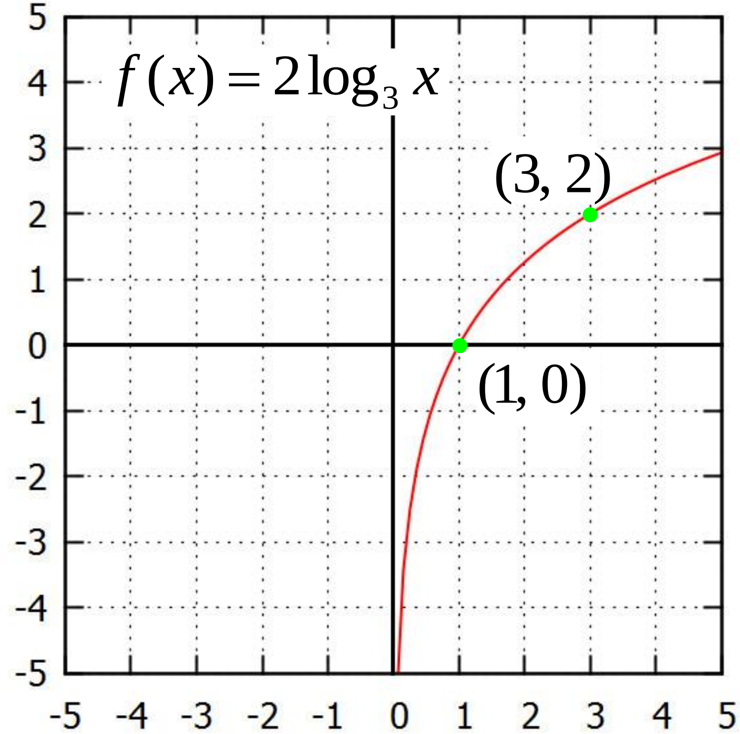


Solution

Answer: A

Justification: We know the shape of the graph of $y = \log_3 x$. The graph of $f(x) = 2\log_3 x$ vertically stretches this graph by 2:



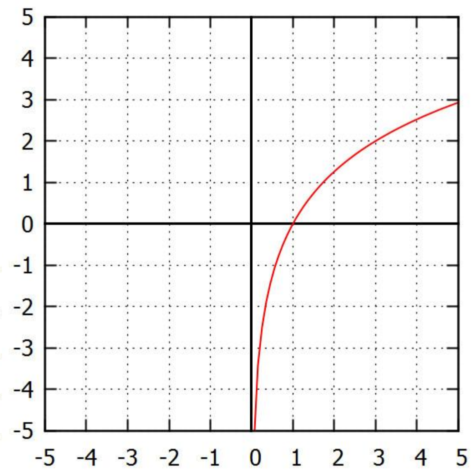


Log Functions IX

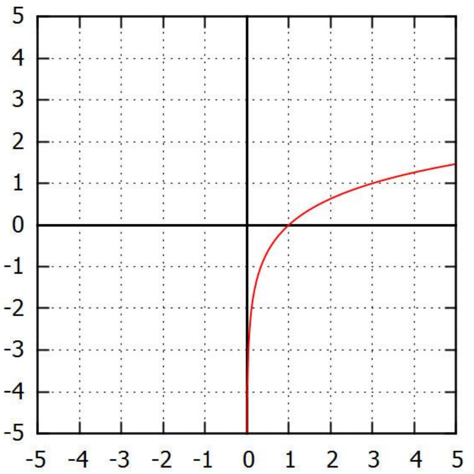
What is the correct graph of the following function?

$$f(x) = \log_3(x^2)$$

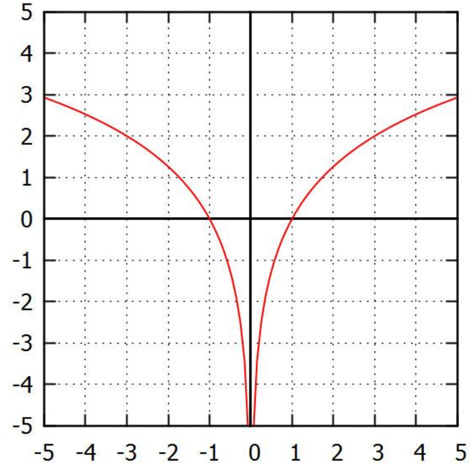
A.



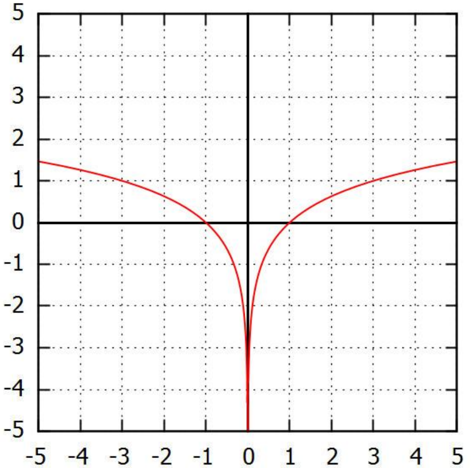
B.



C.



D.



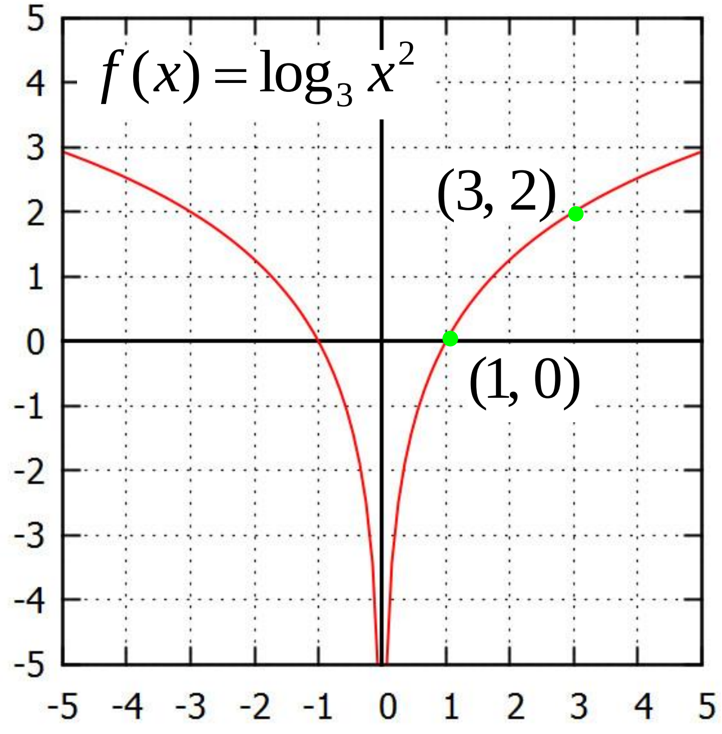
Solution

Answer: C

Justification: The value of $f(x) = \log_3 x^2$ is equivalent to $y = 2\log_3 x$ (review the previous question to see its graph).

Recall that $y = \log_3(x)$ is only defined when x is positive. The domain of the log function is $x > 0$.

Since x^2 is always positive, the log function $f(x) = \log_3(x^2)$ is defined for all values of x. Since x^2 is symmetric across the y-axis, so is $f(x) = \log_3 x^2$.



Log Functions X

Given the graph $y = \log_2(x^2)$, how many solutions does the following equation have?

$$x = \log_2(x^2)$$

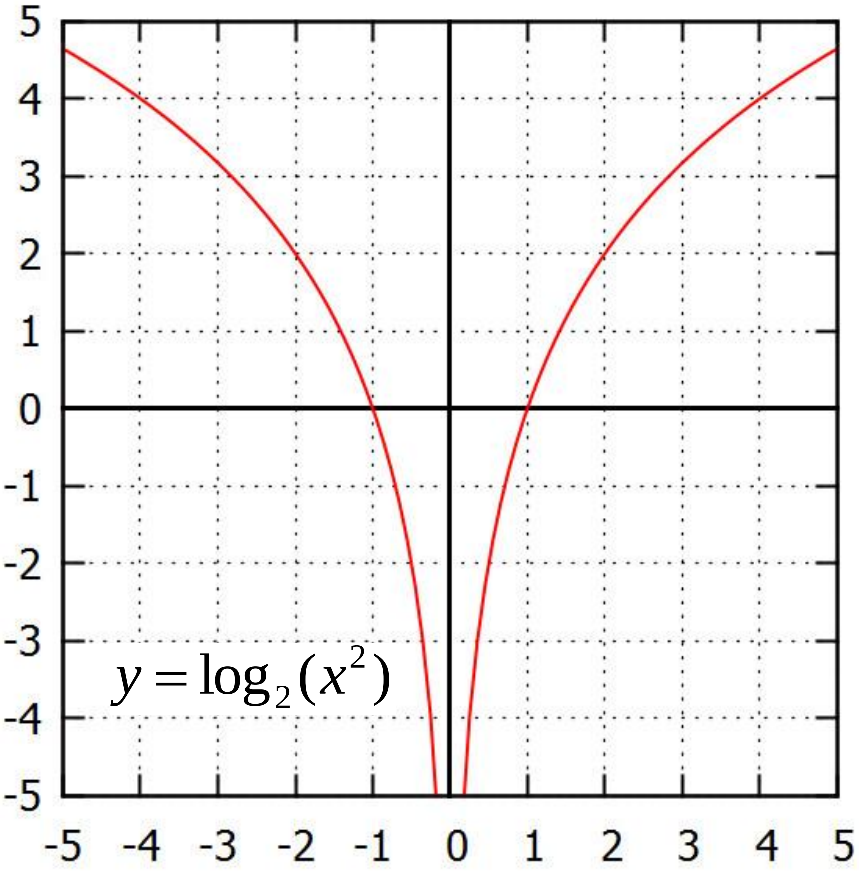
A. 0

B. 1

C. 2

D. 3

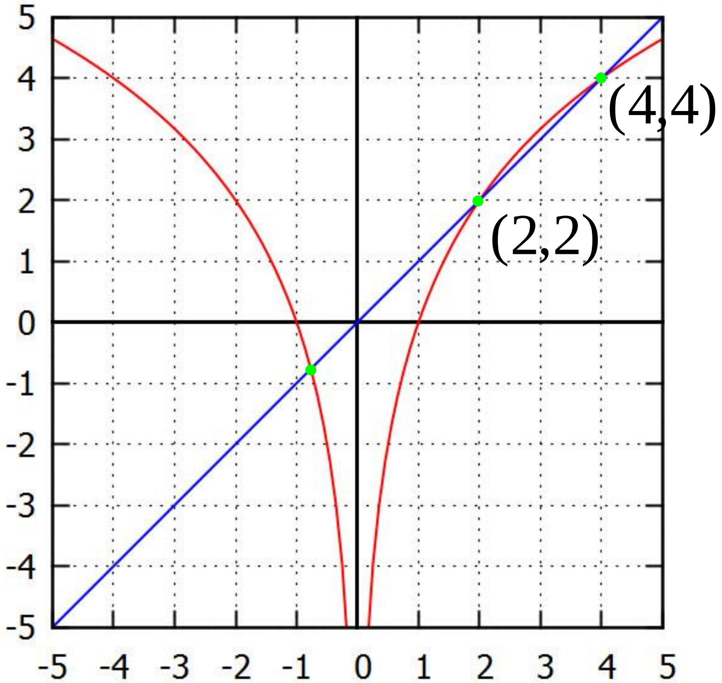
E. Cannot be determined



Solution

Answer: C

Justification: The equation given is hard to solve by working only with equations. Since the graph of $y = \log_2(x^2)$ is given, find the number of points where $y = x$ intersects this graph:

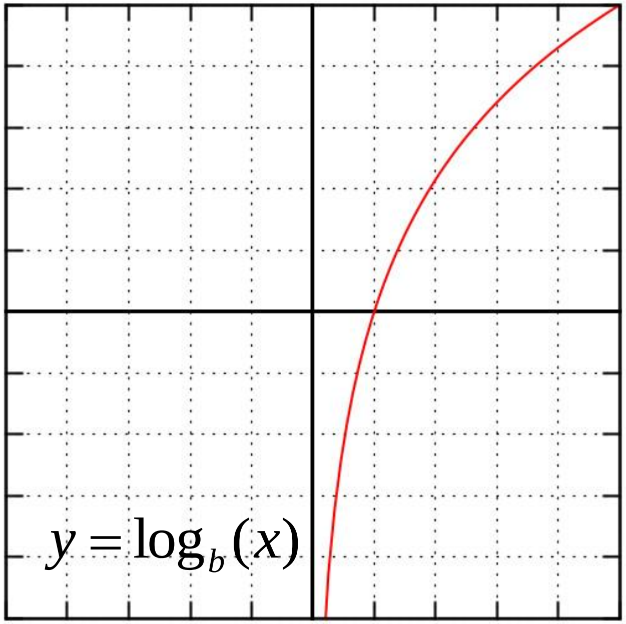


There are two integer solutions at $x = 2$ and $x = 4$. The third solution is between 0 and -1.

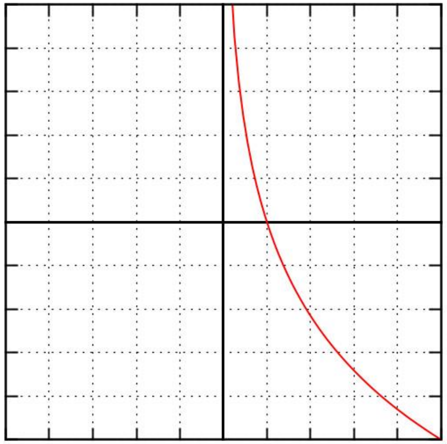
We do not expect the graph to cross the line $y = x$ more times when $x > 5$ since the log function increases much more slowly than $y = x$.

Log Functions XI

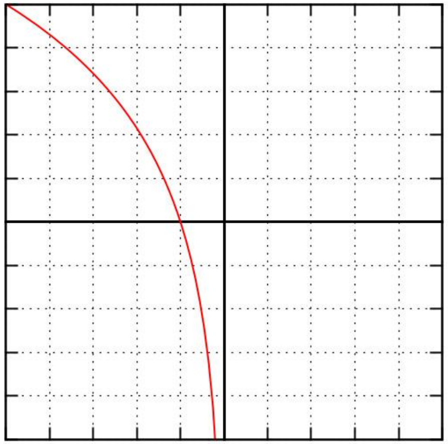
The graph of $y = \log_b x$ is given below. What is the graph of $y = \log_{1/b}(x)$?



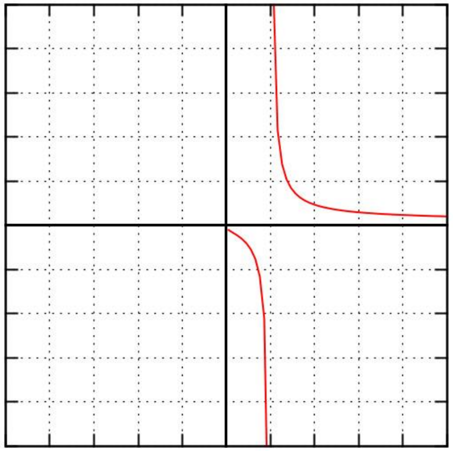
A.



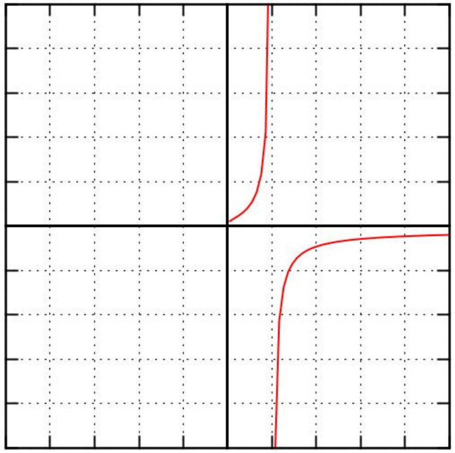
B.



C.



D.



Solution

Answer: A

Justification: The first step is to rewrite logarithm in terms of a graph we are more familiar with, such as $y = \log_b x$.

$$y = \log_{1/b}(x)$$

$$= \frac{\log x}{\log \frac{1}{b}}$$ Change of base property (to base 10)

$$= \frac{\log x}{\log 1 - \log b}$$ Since $\log \frac{a}{b} = \log a - \log b$

$$= \frac{\log x}{0 - \log b}$$ Since $\log_b 1 = 0$

$$= -\log_b x$$ Change of base property

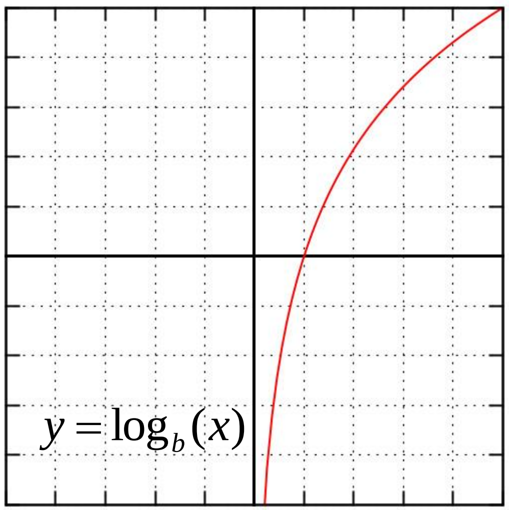
Change of base property:

$$\log_b a = \frac{\log_c a}{\log_c b}$$

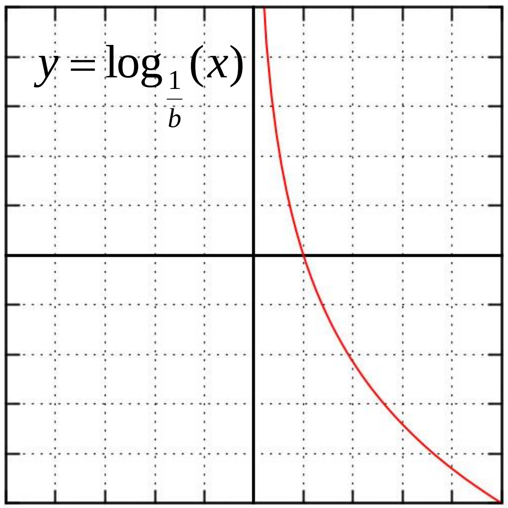
Solution Continued

$$y = \log_{1/b}(x) = -\log_b x$$

This is a reflection of the graph $y = \log_b x$ across the x-axis.



Reflect in x-axis



Recall that negative exponents takes the reciprocal of its base. This explains the change in sign of the exponent returned by the log function.

$$\left(\frac{1}{b}\right)^{-y} = b^y$$