a place of mind

FACULTY OF EDUCATION

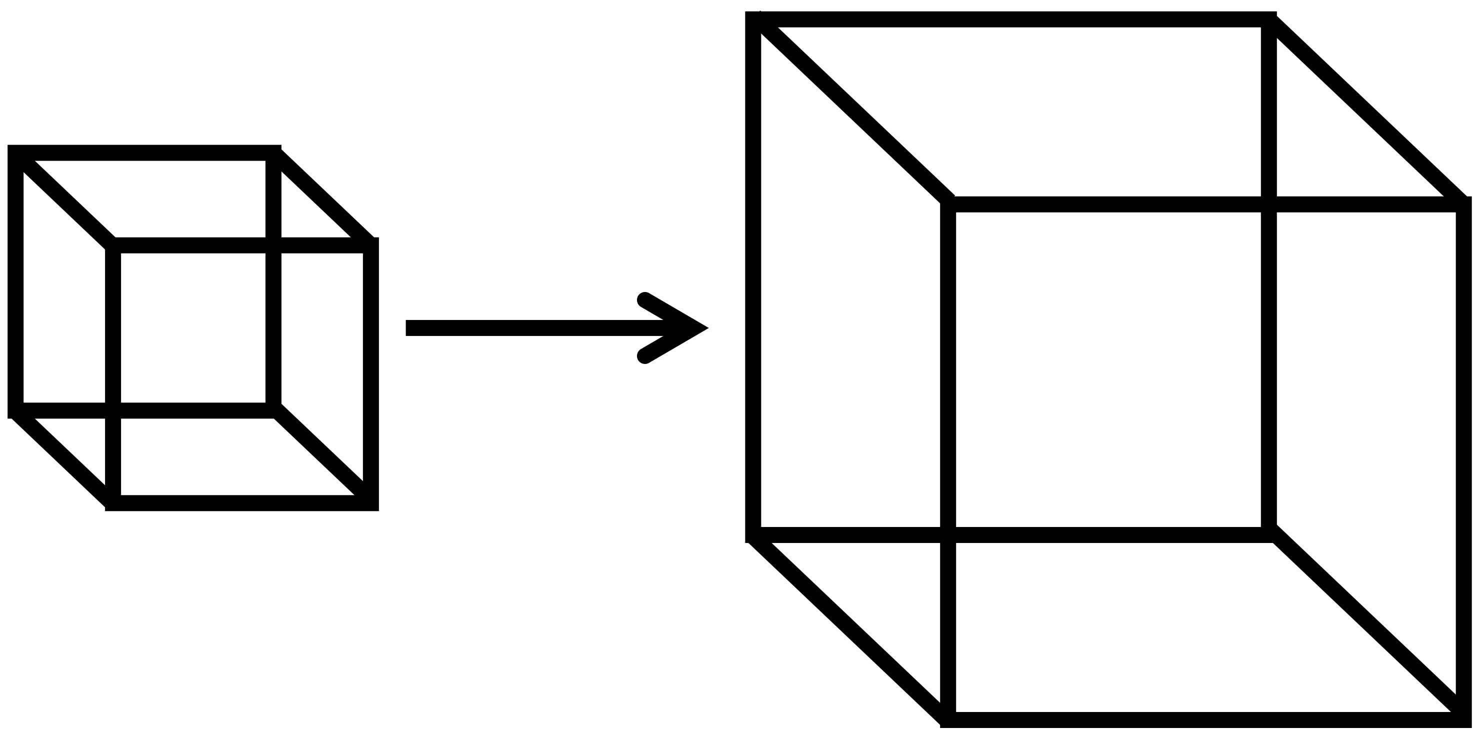
Department of
Curriculum and Pedagogy

Mathematics
Geometry: Dimensions

Science and Mathematics
Education Research Group

Supported by UBC Teaching and Learning Enhancement Fund 2012-2013

Dimension



Dimensions I

The length of a line segment is x. The line is then doubled in length. What is the length of the new line segment?

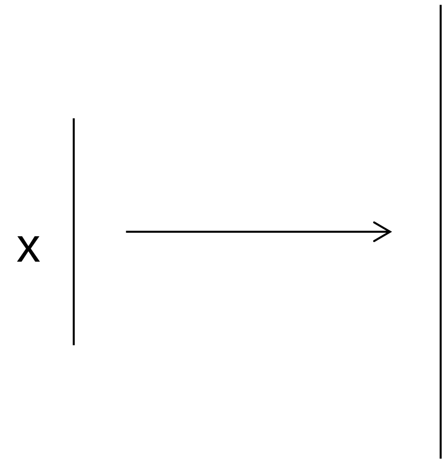
A. 2x

B. x

C. $\frac{x}{2}$

D. 1

E. No Idea



Solution

Answer: A

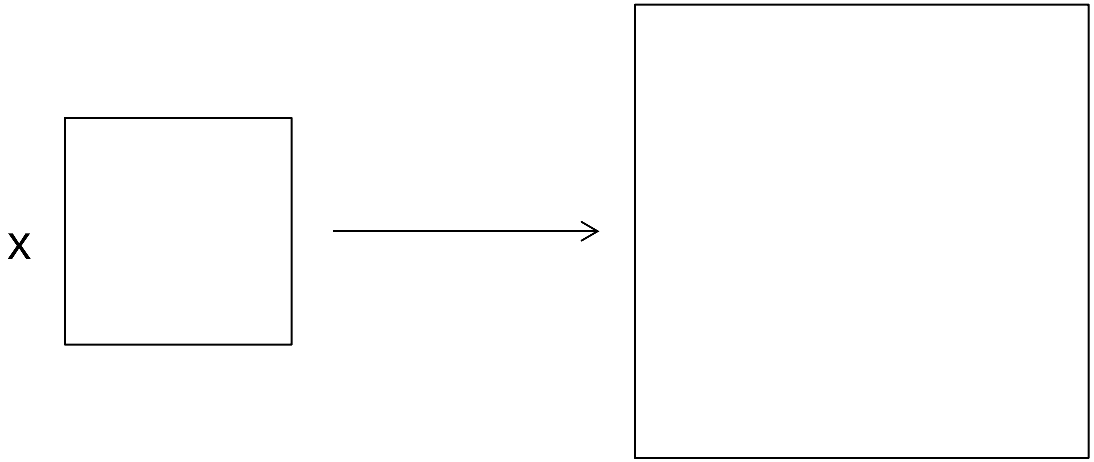
Justification: The length of the line is doubled. Therefore, we must double the original length to obtain the new, expanded length.

$$2 \times x = 2x$$

Dimensions II

The side length of a square is x and therefore has a perimeter of 4x. Each of the side lengths in the square are then doubled. What is the perimeter of the new square?

A. x
B. 2x
C. 4x
D. 8x
E. 16x



Solution

Answer: D

Justification: Each of the side lengths have been doubled, so each side now has a length of 2x. There are four sides, so the perimeter is going to be $4 \times 2x = 8x$

Alternatively, the original perimeter is 4x. Since each side is doubled, the perimeter is also doubled, giving 8x .

Dimensions III

A square with side length x has area x^2. Each of the side lengths in the square are then doubled. What is the area of the new square?

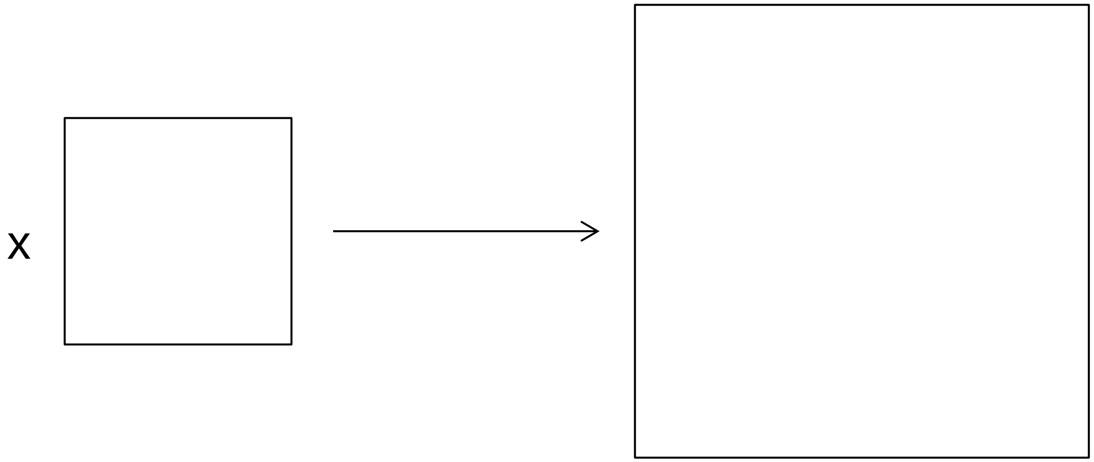
A. $(4x)^2$

B. $4x^2$

C. $2x^2$

D. x^2

E. $4x$



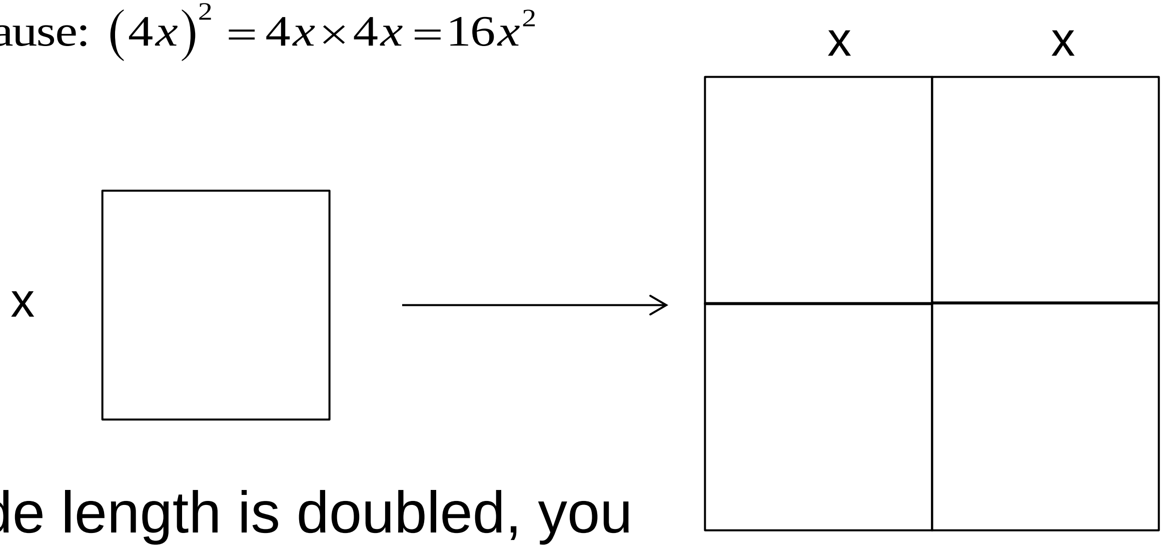
Solution

Answer: B

Justification: The doubled side length is 2x. The area of the square is side length squared: $(2x)^2 = 2x \times 2x = 4x^2$

Therefore, $4x^2$ is the area of the new square. Notice,

$(4x)^2 \neq 4x^2$, because: $(4x)^2 = 4x \times 4x = 16x^2$

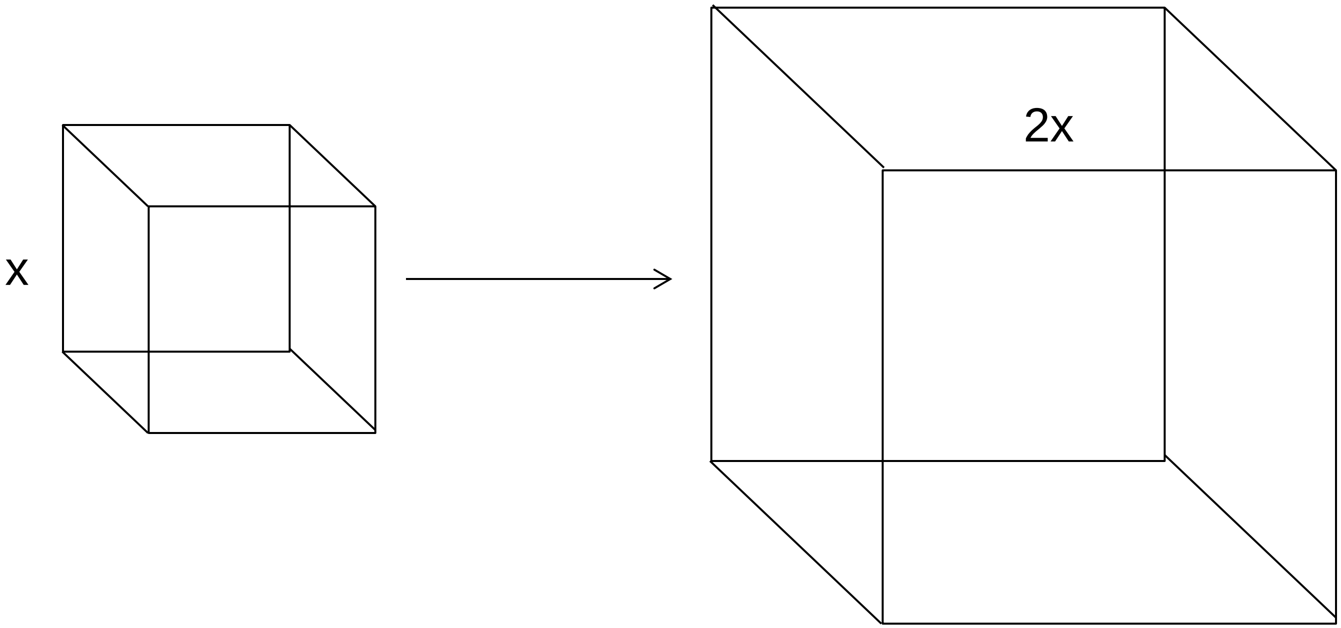


When each side length is doubled, you can “fill” the larger square with four squares the same size as the original.

Dimensions IV

The edges of a cube are length x. A cube has 12 edges, so the total edge length is 12x. What is the total edge length, if each edge is doubled?

A. 48x
B. 36x
C. 24x
D. 12x
E. $6x^2$



Solution

Answer: C

Justification: Each of the edge lengths doubles, so the total length of all the edges together must also double.

$$2 \times 12x = 24x$$

Alternatively, each side length doubles, and there are 12 sides, so:

$$12 \times (2x) = 24x$$

Dimensions V

The edge length of a cube is x, and each face has an area of x^2. Each of the edge lengths in the cube are then doubled. What is the total surface area of the new cube?

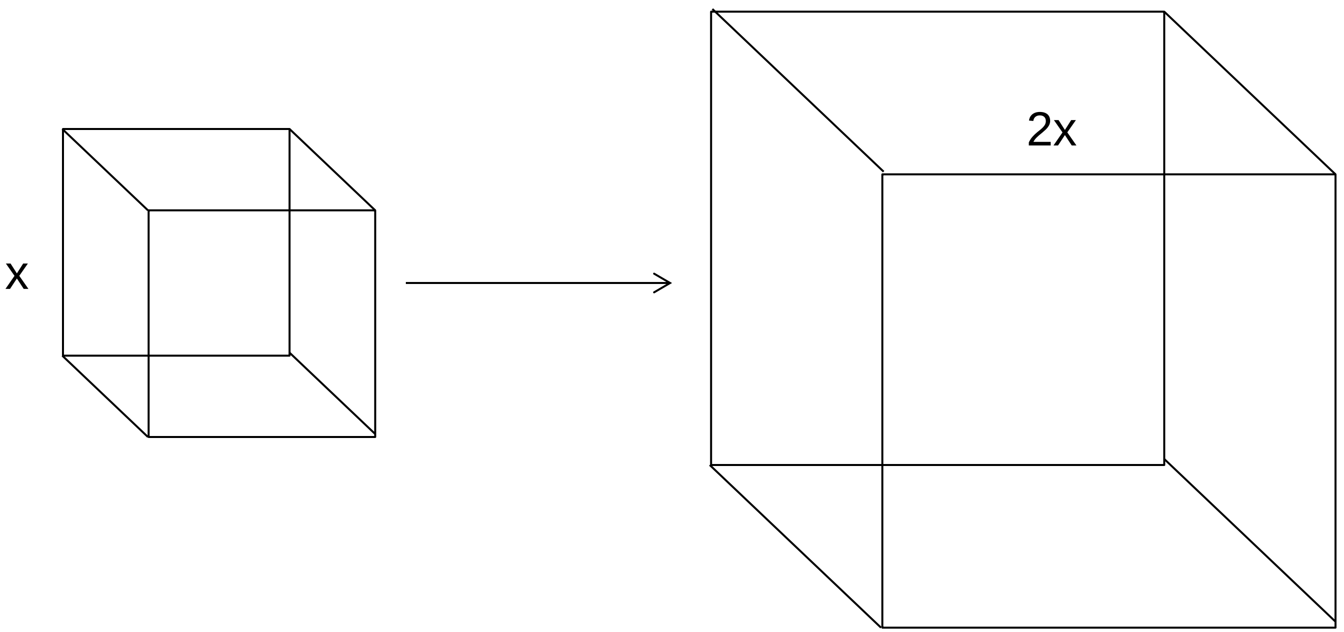
A. $48x^2$

B. $36x^2$

C. $28x^2$

D. $24x^2$

E. $24x$



Solution

Answer: D

Justification: A cube has 6 faces, each with a surface area of x^2. This gives a total surface area of $6x^2$. If each side length is doubled, then the surface area of each face becomes $6(2x)^2 = 6(4x^2) = 24x^2$.

Alternatively, the surface area of each face increases by a factor of 4, $4x^2$, and there are 6 faces. This gives

$$6 \times 4x^2 = 24x^2$$

Dimensions VI

The volume of a cube with side length x is x^3. Each of the edge lengths in the cube are then doubled. What is the volume of the new cube?

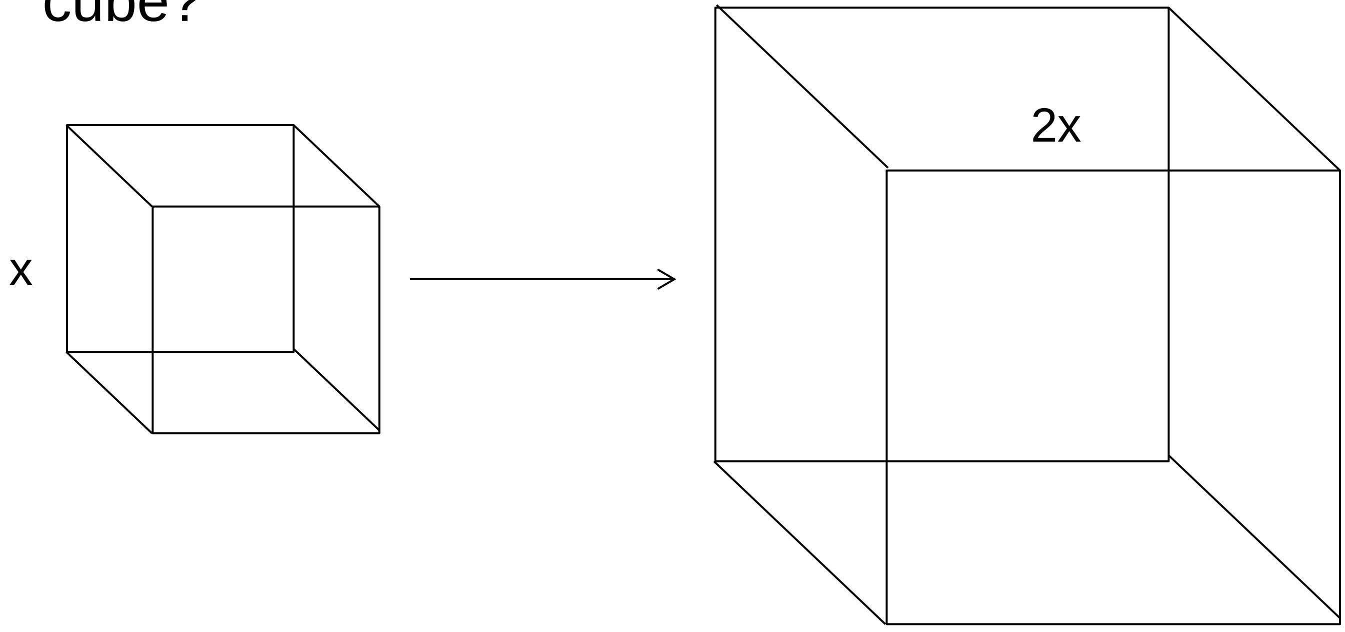
A. $9x^3$

B. $8x^3$

C. $6x^3$

D. $8x^2$

E. $2x^3$

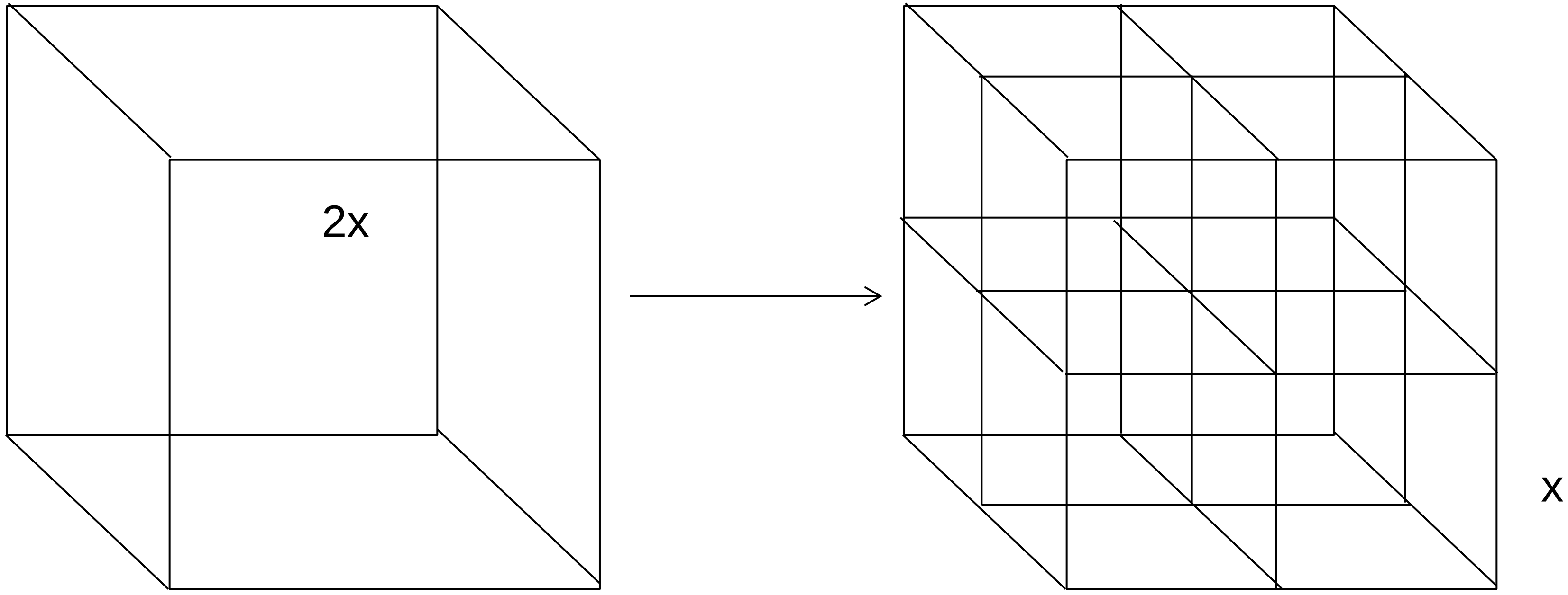


Solution

Answer: D

Justification: The volume of a cube is the edge length cubed. This gives: $(2x)^3 = (2x)\times(2x)\times(2x) = 8x^3$

As seen in the diagram below, when the side length doubles, 8 of the original cubes can fit inside the new, larger cube.



Dimensions VII

The edge length of a cube is increased by a factor of F (the new length is Fx), where F is called the scaling factor. By how many times will the surface area increase?

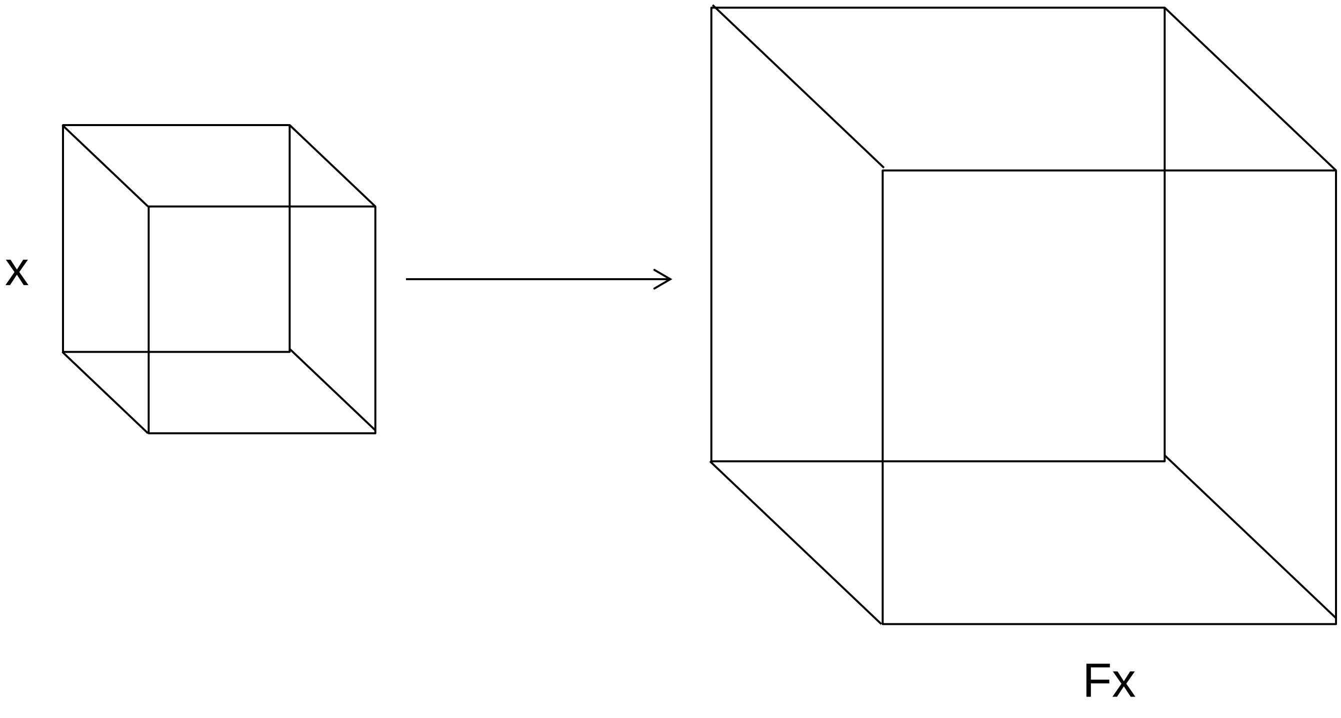
A. F^3

B. F^2

C. $2F^2$

D. $4F^2$

E. F



Solution

Answer: B

Justification: Think about the previous questions.

The surface area of a cube of length x is $6x^2$. If the side length increases by a factor of F, the side length becomes Fx. The surface area then becomes $6(Fx)^2$, which is F^2 times larger than the original surface area:

$$A_{original} = 6x^2; A_{new} = 6(Fx)^2 = 6F^2x^2 = F^2 6x^2 = F^2 A_{original}$$

You can check this by thinking about the question in which we doubled the side length. The surface area of the original cube was $6x^2$. When the side lengths were doubled, the surface area became $6(2x)^2 = 24x^2$

Dimensions VIII

The edge length of a cube is increased by a factor of F. By how many times will the volume increase?

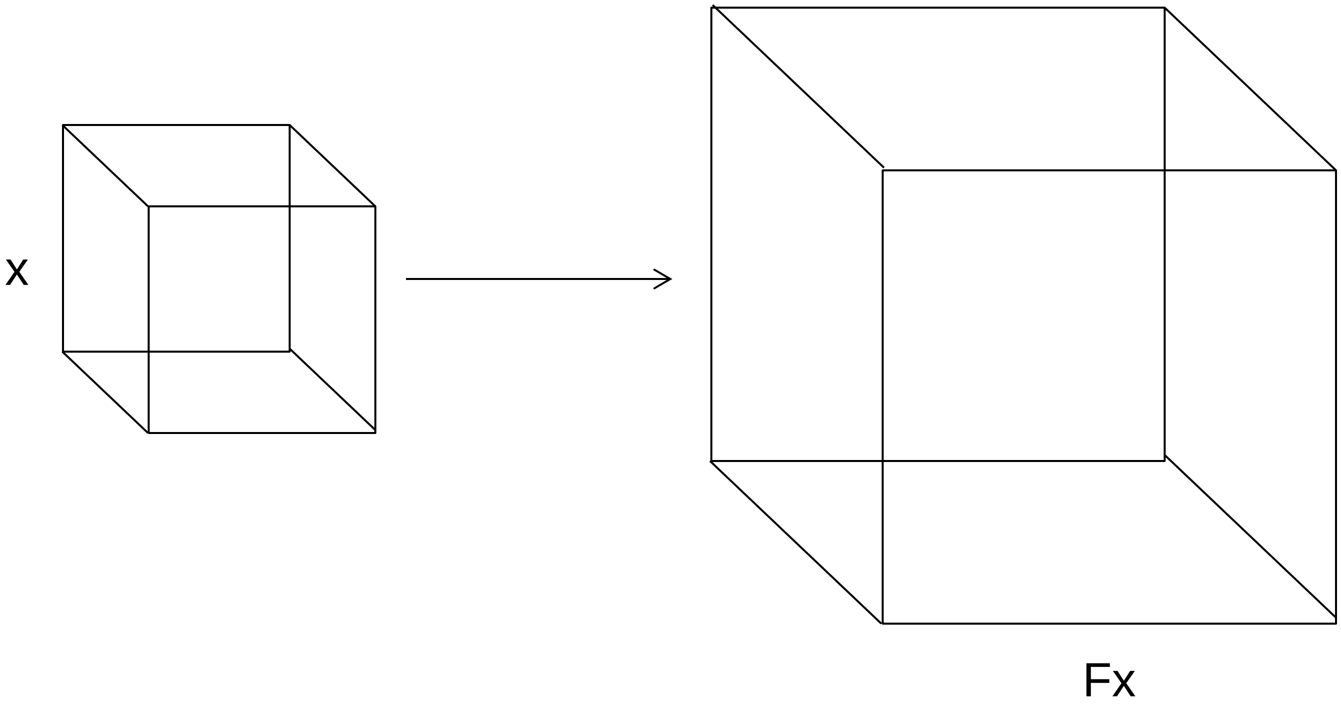
A. F^3

B. F^2

C. $2F^2$

D. $4F^2$

E. F



Solution

Answer: A

Justification: The volume of a cube of length x is x^3. If the side length increases by a factor of F, the side length becomes Fx. The volume then becomes $(Fx)^3$:

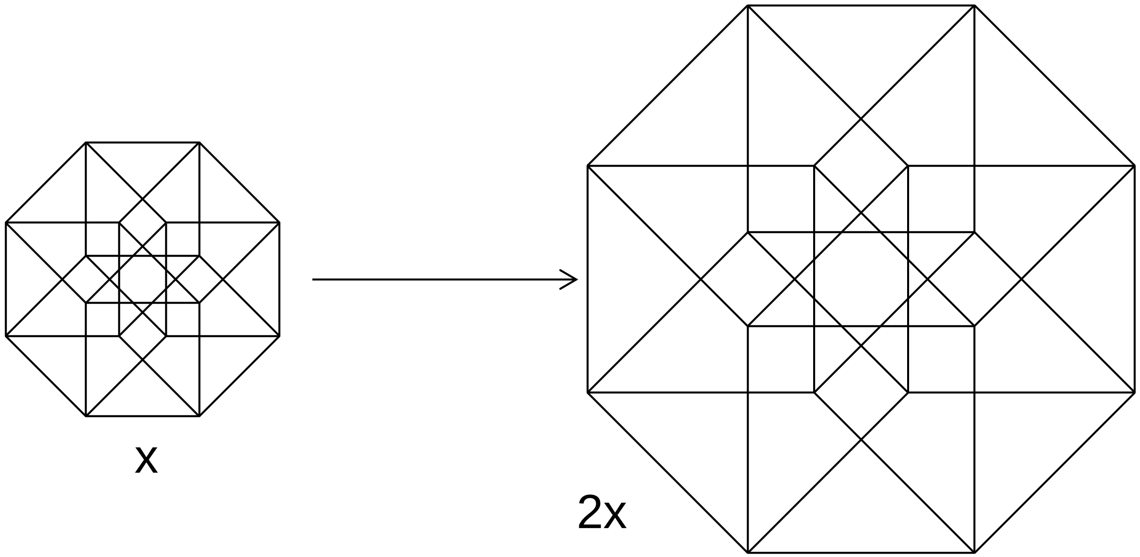
$$V_{original} = x^3; V_{new} = (Fx)^3 = F^3x^3 = F^3V_{original}$$

You can check this by thinking about the question in which we doubled the side length. The volume of the original cube was x^3. When the side lengths were doubled, the surface area became $(2x)^3 = 8x^3$.

Dimensions IX

The edge length of a tesseract (cube of 4 spatial dimensions) is doubled. What is the factor that the 3D volume increases by?

A. 16
B. 8
C. 4
D. 2
E. No idea



Solution

Answer: B

Justification: As we have deduced before, the number of dimensions of the object does not affect the scaling of its constituent parts, as in, squares and cubes both scale by a factor of F^2 in terms of their surface area. Both cubes and the 3D volume on the tesseract scale the same (F^3) when we talk about 3D volume, and therefore the answer is $2^3=8$.