a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Mathematics

Numbers: Absolute Values

Science and Mathematics
Education Research Group

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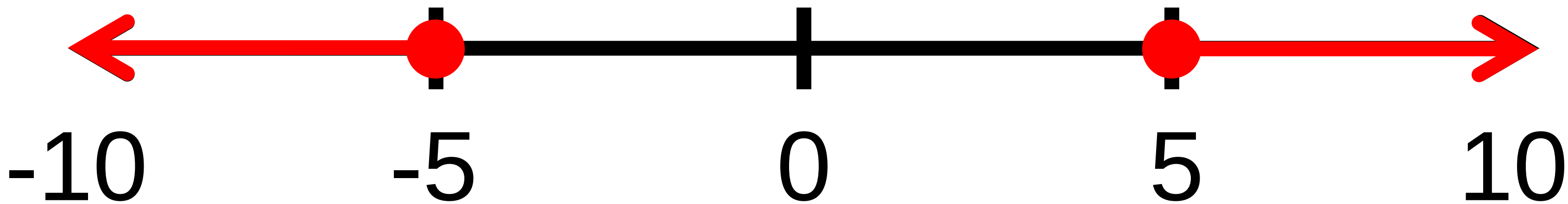
Absolute Values



Absolute Values

The two vertical bars $|\ |$ refer to the absolute value of the number.

What is the value of $|-24|$?

A. 24

B. -24

C. ± 24

D. 24^2

E. 0

Solution

Answer: A

Justification: The absolute value of a real number is its value without regarding its sign. It is often referred to as the *magnitude* of the number.

$$\left|-24\right| = 24$$

In this example, the value inside the absolute value is negative. After computing the absolute value, we are returned the positive value 24.

If the number inside the absolute value were positive, the number would not change.

$$\left|24\right| = 24$$

Absolute Values II

What is the value of the following expression?

$$\left| -1 \times 7^2 \right|$$

A. 49

B. -49

C. ±49

D. ±7

E. 0

Solution

Answer: A

Justification: The absolute value of a real number is its value without regarding its sign.

Compute all the calculations inside absolute value signs as you would with brackets.

$$\left|-1\times 7^2\right| = \left|-49\right| = 49$$

The absolute value of -49 is 49.

Absolute Values III

What is the value of the following expression?

$$-2\left|(-2)(2)\right|$$

A. 8

B. -8

C. ±8

D. 2

E. -2

Solution

Answer: B

Justification: Compute all the calculations inside absolute value signs as you would with brackets.

$$-2\left|(-2)(2)\right| = -2\left|-4\right|$$

After applying the absolute value, the -4 becomes positive 4. We then multiply by the -2 outside the absolute value.

$$-2\left|-4\right| = -2(4)$$

$$= -8$$

Even though this expression includes an absolute value, the value of the expression is still negative because first -2 is outside of the absolute value sign.

Absolute Values IV

What is the value of the following expression?

$$2|3-4|\times(-3)|5-2|$$

A. 9

B. -9

C. 18

D. -18

E. -6

Solution

Answer: D

Justification: Compute all the calculations inside absolute value signs as you would with brackets.

$$2|3-4|\times(-3)|5-2| = 2|-1|\times(-3)|3|$$

Evaluate the absolute values and simplify:

$$\begin{aligned} 2|3-4|\times(-3)|5-2| &= 2|-1|\times(-3)|3| \\ &= 2(1)\times(-3)(3) \\ &= 2\times(-9) \\ &= -18 \end{aligned}$$

Absolute Values V

Let x be a real number. Which one of the following is the correct definition of the absolute value of x?

A. $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

B. $|x| = \begin{cases} x, & x \leq 0 \\ -x, & x > 0 \end{cases}$

C. $|x| = \begin{cases} x, & x \geq 0 \\ 0, & x < 0 \end{cases}$

D. $|x| = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$

E. $|x| = \begin{cases} x, & x < 0 \\ -x, & x > 0 \end{cases}$

Solution

Answer: A

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Justification: The definition A states that if the value of x is zero or positive, we leave the value unchanged. If the value of x is negative, we must multiply x by -1 in order to make its value positive. No matter what the value of x, we are returned the positive value for x by the piecewise function.

Definition D is very close to the answer, although it does not define a value for x = 0. The value of x when x = 0 can be either x or -x.

Absolute Values VI

What is the correct piecewise definition for $|-2x+9|$?

A. $|-2x+9| = \begin{cases} -2x+9, & x \geq \frac{9}{2} \\ 2x+9, & x < \frac{9}{2} \end{cases}$

B. $|-2x+9| = \begin{cases} -2x+9, & x \geq \frac{9}{2} \\ 2x-9, & x < -\frac{9}{2} \end{cases}$

C. $|-2x+9| = \begin{cases} -2x+9, & x \leq \frac{9}{2} \\ 2x-9, & x > \frac{9}{2} \end{cases}$

D. $|-2x+9| = \begin{cases} -2x+9, & -2x+9 \geq 0 \\ 2x+9, & -2x+9 < 0 \end{cases}$

E. $|-2x+9| = \begin{cases} -2x+9, & 2x+9 \leq 0 \\ 2x-9, & 2x+9 > 0 \end{cases}$

Solution

Answer: C

Justification: Definition of absolute value:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Substituting $-2x+9$ into the definition for x gives:

$$|-2x+9| = \begin{cases} -2x+9, & -2x+9 \geq 0 \\ -(-2x+9), & -2x+9 < 0 \end{cases}$$

Re-write the subdomains with x isolated on one side of the inequalities:

$$|-2x+9| = \begin{cases} -2x+9, & x \leq \frac{9}{2} \\ 2x-9, & x > \frac{9}{2} \end{cases}$$

Remember to change the sign of the inequality when dividing by -2

Absolute Values VII

Solve the following equation:

$$4|x+1|=100$$

A. $x=\pm 24$

B. $x=\pm 26$

C. $x=24, 26$

D. $x=24, -26$

E. $x=-24, 26$

Solution

Answer: D

Justification: First simplify the equation so that the absolute value is isolated:

$$4|x+1| = 100$$
$$|x+1| = 25$$

Use the definition of the absolute value to break the equation into two separate equations. The LHS will equal 25 if the expression inside the absolute value is either 25 or -25:

$$x+1 = 25 \quad or \quad x+1 = -25$$
$$x = 24 \qquad\qquad x = -26$$

Absolute Values VIII

Which one of the following is equivalent to $|(x+3)(x-1)|$ in piecewise notation?

$$|(x+3)(x-1)| =$$

A.
$$\begin{cases} (x+3)(x-1), & x < -1 \\ -(x+3)(x-1), & -1 \le x \le 3 \\ (x+3)(x-1), & x > 3 \end{cases}$$

B.
$$\begin{cases} -(x+3)(x-1), & x < -1 \\ (x+3)(x-1), & -1 \le x \le 3 \\ -(x+3)(x-1), & x > 3 \end{cases}$$

C.
$$\begin{cases} (x+3)(x-1), & x < -3 \\ -(x+3)(x-1), & -3 \le x \le 1 \\ (x+3)(x-1), & x > 1 \end{cases}$$

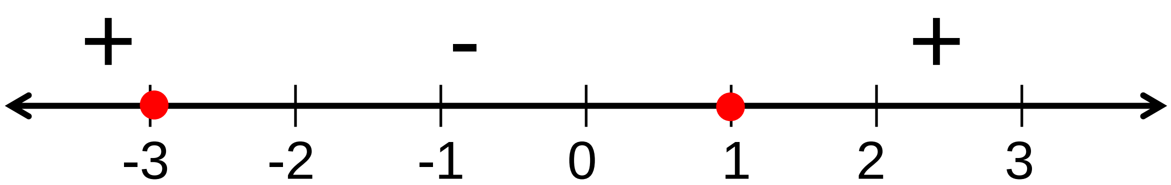
D.
$$\begin{cases} -(x+3)(x-1), & x < -3 \\ (x+3)(x-1), & -3 \le x \le 1 \\ -(x+3)(x-1), & x > 1 \end{cases}$$

E. None of the above

Solution

Answer: C

Justification: The expression $\left|(x+3)(x-1)\right|$ is zero at the points -3 and 1. Pick test values between these intervals to determine where the expression inside the absolute value is positive or negative:



For example, choosing -1 as the test point for the interval -3 < x < 1 shows that the expression is negative since

$$\left((-1)+3\right)\left((-1)-1\right)<0$$

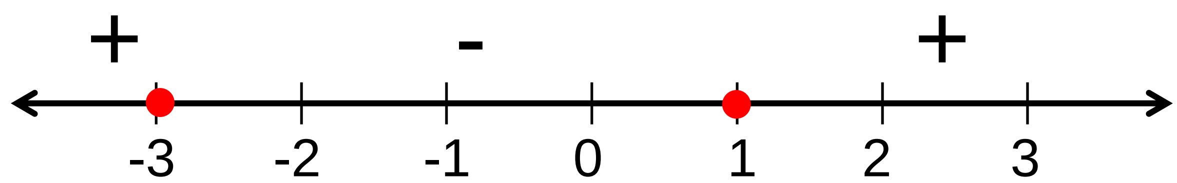
This means that (x+3)(x-1) is negative for -3 < x < 1.

Answer continues on the next slide

Solution Cont'd

Answer: C

Justification:



Where the expression inside the absolute value is negative, we must multiply by -1. Where the expression is positive, we leave the same:

$$|(x+3)(x-1)| = \begin{cases} (x+3)(x-1), & x < -3 \\ -(x+3)(x-1), & -3 \leq x \leq 1 \\ (x+3)(x-1), & x > 1 \end{cases}$$

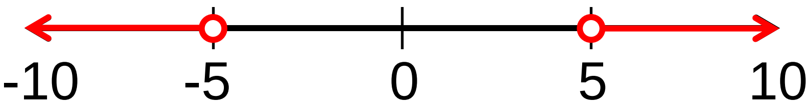
(Note: The above expression is a quadratic polynomial that opens upwards, which means the middle region in between the roots must be negative, while the outer regions are positive)

Absolute Values IX

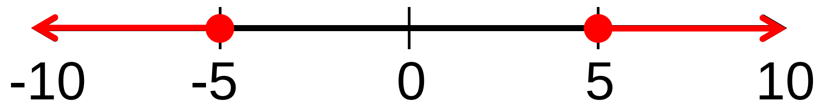
Which number line best represents the following inequality?

$$|x| < 5$$

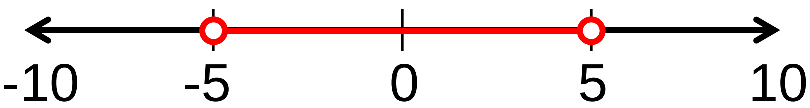
A.



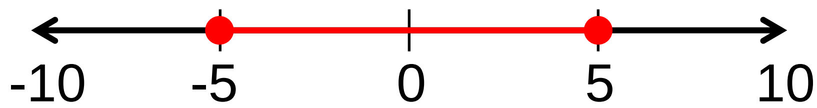
B.



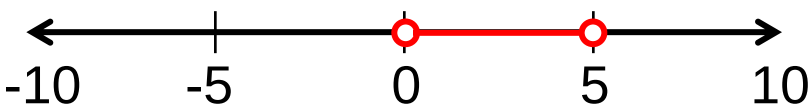
C.



D.



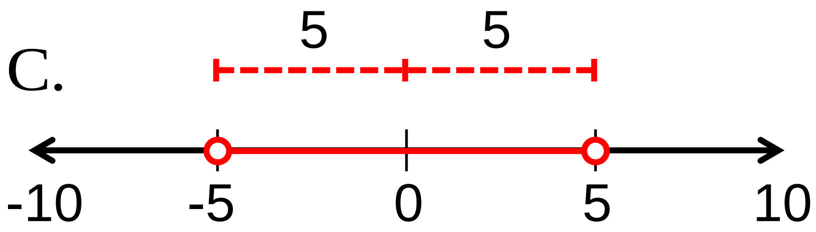
E.



Solution

Answer: C

Justification: Inequalities involving absolute values can be thought of as the distances from the origin. $|x| < 5$, and can be interpreted as all distances less than 5 from the origin:



Open circles denote that the points 5 and -5 are not included

Inequalities involving absolute values can be rewritten without absolute values as shown:

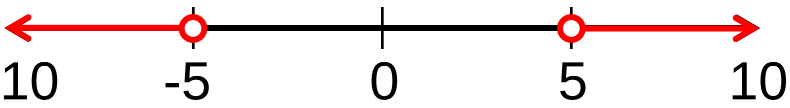
$$|x| < a = -a < x < a$$

Absolute Values X

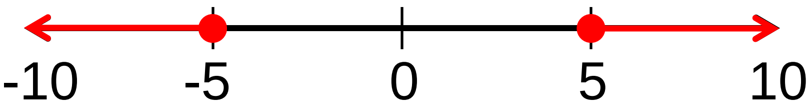
Which number line best represents the following inequality?

$$|x| \geq 5$$

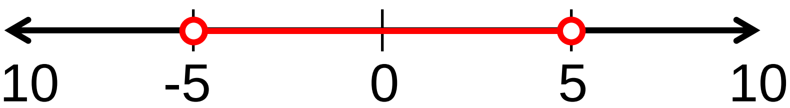
A.



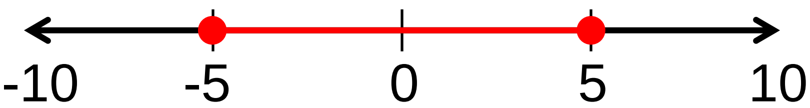
B.



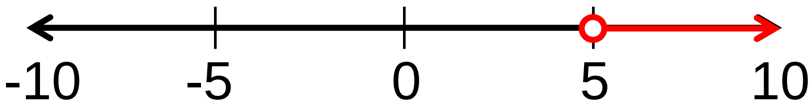
C.



D.



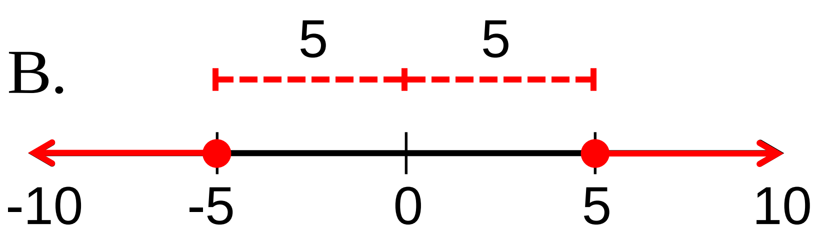
E.



Solution

Answer: B

Justification: Unlike the previous question, the inequality now involves a greater than or equal to sign. We want distances from the origin that are 5 units or greater.



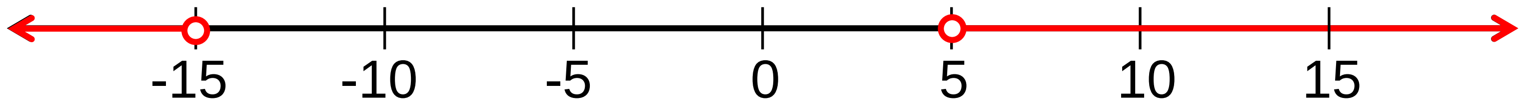
Closed / filled circles include the points 5 and -5

Inequalities involving absolute values can be rewritten without absolute values as shown:

$$|x| \geq a = x \leq -a \quad or \quad x \geq a$$

Absolute Values XI

Which of the following inequalities best represents the following number line?



A. $|x+5|>15$

B. $|x-5|>15$

C. $|x+5|>10$

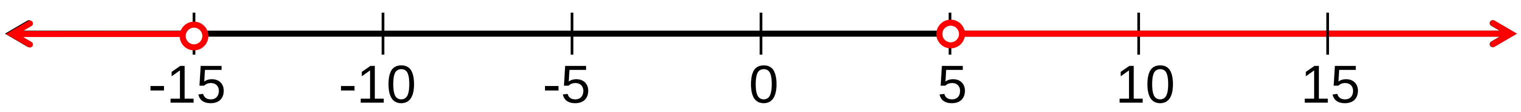
D. $|x-5|>10$

E. $|x+5|>5$

Solution

Answer: C

Justification: The line represents the inequality: $x < -15 \quad or \quad x > 5$



One way to determine which inequality matches this number line is to convert all the multiple choice options into inequalities without absolute values.

C. $|x+5| > 10$

$x+5 > 10 \quad or \quad x+5 < -10$

$x > 5 \quad or \quad x < -15$

Both the positive case and negative case match the number line

Note that if the positive case does not match the number line, it is not necessary to check the negative case of the inequality.

Absolute Values XII

Which one of the following properties shown below is true?

A. $-|p| < p < |p|$

B. $-|p| < p \leq |p|$

C. $-|p| \leq p < |p|$

D. $-|p| \leq p \leq |p|$

E. $-|p| = p = |p|$

Press for hint

Consider cases when *p* is positive and when *p* is negative. Try substituting some values for *p*.

Solution

Answer: D $-|p| \le p \le |p|$

Justification: Case 1: $p > 0$

Let *p* = 5. In this case,

$$-|5| \le 5 \le |5|$$

$$-5 < 5 = 5$$

This shows it is possible for *p* to be equal to $|p|$, but greater than $-|p|$.

Case 2: $p < 0$

Let *p* = -2. In this case,

$$-|-2| \le -2 \le |-2|$$

$$-2 = -2 < 2$$

This shows it is possible *p* to be equal to $-|p|$, but greater than $|p|$.

Combine the results from both cases to show that $-|p| \le p \le |p|$ for any value of *p*. Note, if $p = 0$, then $-|p| = p = |p|$.

Absolute Values XIII

Which one of the following properties shown below is true?

A. $|p+q| = |p| + |q|$

B. $|p+q| > |p| + |q|$

C. $|p+q| \geq |p| + |q|$

D. $|p+q| < |p| + |q|$

E. $|p+q| \leq |p| + |q|$

Press for hint

Try substituting a positive value for *p* and a negative value for *q*. Compare the right-hand side with the left-hand side.

Solution

Answer: E $\quad |p+q| \leq |p|+|q|$

Justification: Case 1: Imagine that both *p* and *q* are positive. The absolute value will have no effect on either the RHS or LHS.

$$|2+3| = |2|+|3|$$

$$|5| = 2+3$$

$$5 = 5$$

Since the property must cover the equality case, options B and D are false.

Case 2: Imagine that $p > 0$ and $q < 0$. Notice the RHS will always be greater than *p* or *q*, while the LHS is reduced.

$$|-4+3| < |-4|+|3|$$

$$1 < 7$$

Answer continues on the next slide

Solution Cont'd

Answer: E $\left|p+q\right| \leq \left|p\right| + \left|q\right|$

Justification: Case 3: Both *p* and *q* are negative. This case is similar to case 1 where *p* and *q* are positive:

$$\left|-2-3\right| = \left|-2\right| + \left|-3\right|$$

$$\left|-5\right| = 2+3$$

$$5 = 5$$

Combining the results of the 3 cases can be generalized to:

$$\left|p+q\right| \leq \left|p\right| + \left|q\right|$$

Bonus: Try verifying that:

$$\left|p-q\right| \geq \left|p\right| - \left|q\right|$$