



a place of mind

FACULTY OF EDUCATION

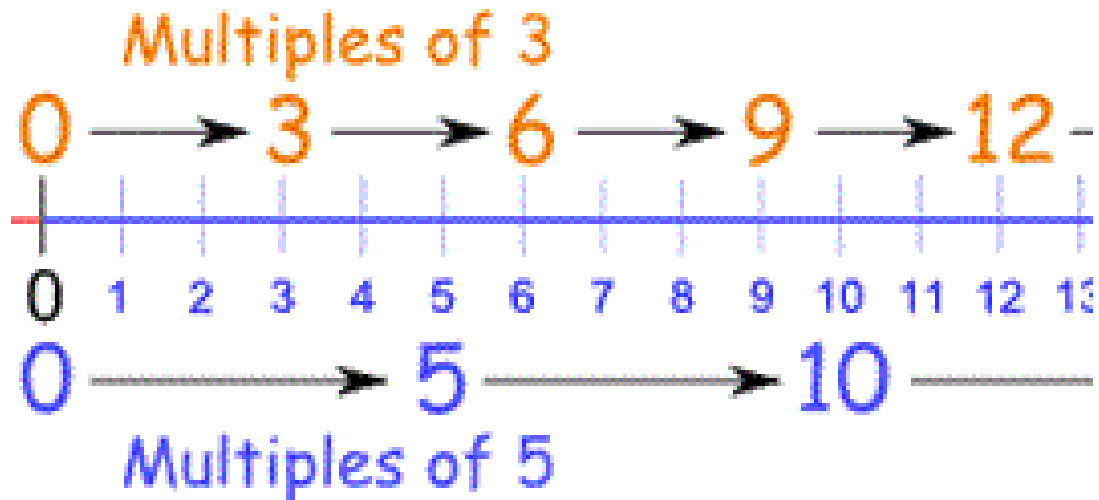
Department of
Curriculum and Pedagogy

Mathematics

Numbers: Multiples

Science and Mathematics
Education Research Group

Multiples



Multiples

What are the multiples of 6?

- A. 1, 2, 3, 6
- B. 1, 3, 6, 9, 12
- C. 6, 12, 18, 24...
- D. 1, 2, 3, 6, 9, 12, 15, 18...
- E. 1, 2, 3, 6, 12, 18, 24...

Solution

Answer: C

Justification: Multiples of a certain number are natural numbers (1, 2, 3 ...) that are multiplied by another natural numbers.

For example, for multiples of 2, we are multiplying 2 starting with 1, 2, 3, etc. Thus, multiples of 2 are

$$2 \times 1, 2 \times 2, 2 \times 3, 2 \times 4, 2 \times 5, \dots$$

2 4 6 8 10 ...

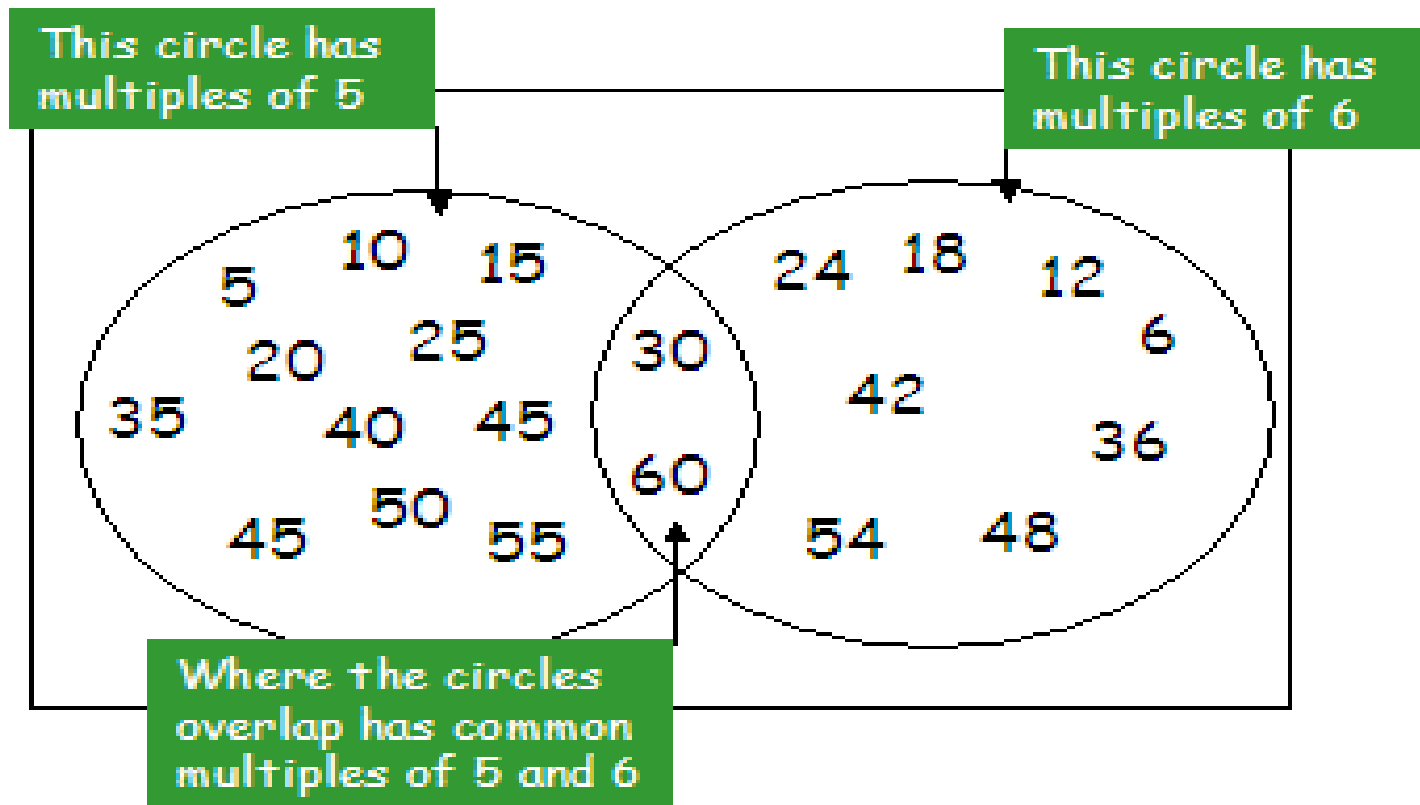
Similarly, multiples of 6 are:

$$6 \times 1, 6 \times 2, 6 \times 3, 6 \times 4, 6 \times 5, \dots$$

6 12 18 24 30 ...

Thus, our answer is C.

Common Multiples



Common Multiples

What are the common multiples of 3 and 4?

- A. 1, 3, 4
- B. 3, 4, 6, 8, 9, 12...
- C. 1, 2, 3, 4, 6, 12
- D. 3, 4, 9, 16...
- E. 12, 24, 36, 48...

Solution

Answer: E

Justification: We can start listing the multiples of 3 and 4 as we did in the previous question.

For multiples of 3, we are multiplying 3 starting with 1, 2, 3, etc.

Thus, multiples of 3 are

$3 \times 1, 3 \times 2, 3 \times 3, 3 \times 4, 3 \times 5, 3 \times 6, 3 \times 7, 3 \times 8 \dots$

3 6 9 12 15 18 21 24

On the other hand, multiples of 4 are:

$4 \times 1, 4 \times 2, 4 \times 3, 4 \times 4, 4 \times 5, 4 \times 6 \dots$

4 8 12 16 20 24

Thus, the first two common multiples are 12 and 24. Then, since 24 is a multiple of 12, we know that the common multiples of 3 and 4 will be the multiples of 12. The answer is E.

Common Multiples II

What are the common multiples of 6 and 9?

- A. 1, 2, 3, 6, 9
- B. 6, 9, 12, 18, 24...
- C. 6, 9, 36, 81...
- D. 18, 36, 54, 108...
- E. 54, 108, 162, 216...

Solution

Answer: D

Justification: We can start listing the multiples of 6 and 9 as we did in the previous question.

For the multiples of 6, we are multiplying 3 starting with 1, 2, 3, etc. Thus, multiples of 3 are

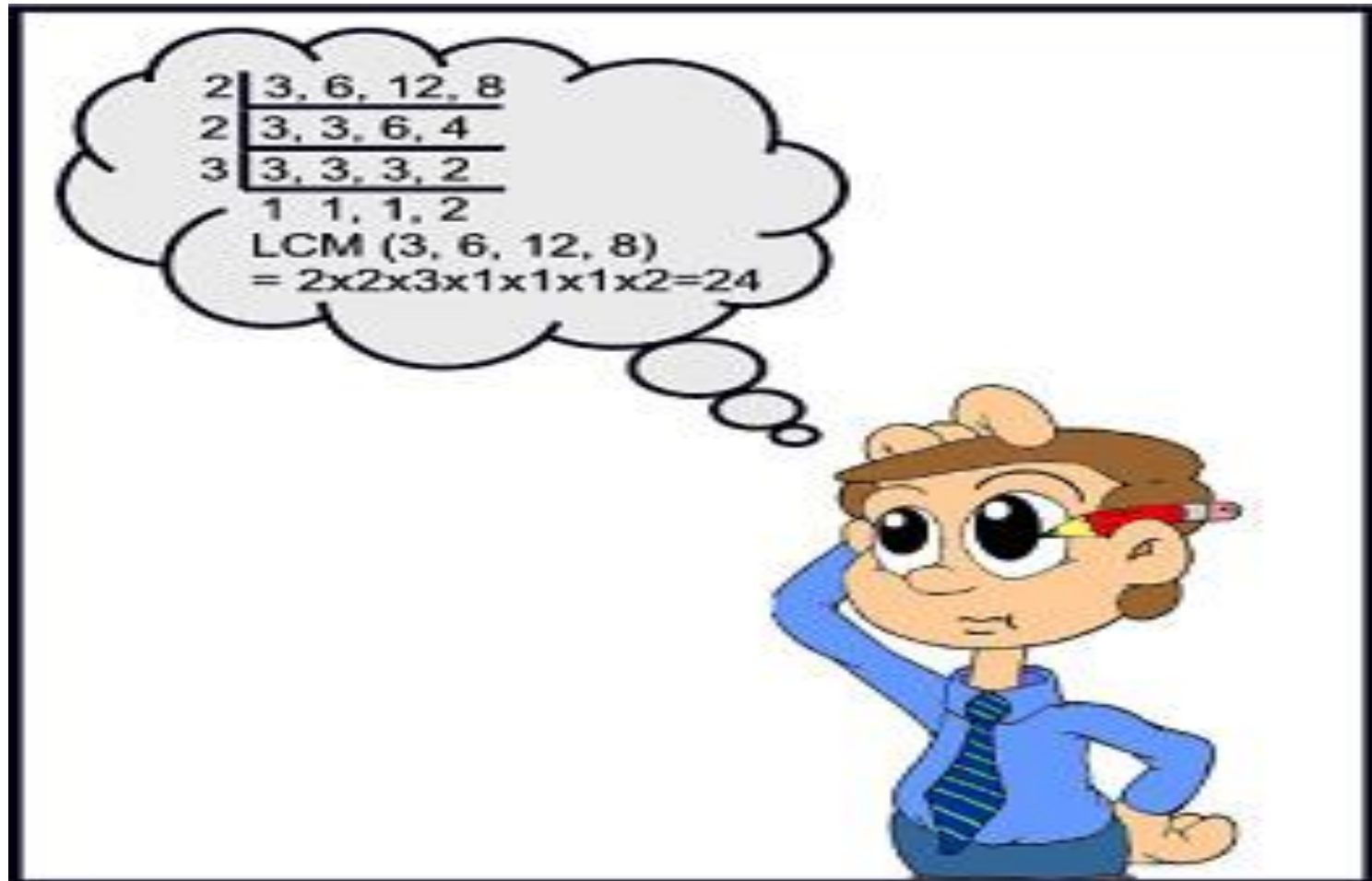
$6 \times 1, 6 \times 2, 6 \times 3, 6 \times 4, 6 \times 5, 6 \times 6, 6 \times 7, 6 \times 8 \dots$
6 12 18 24 30 36 42 48

On the other hand, multiples of 9 are:

$9 \times 1, 9 \times 2, 9 \times 3, 9 \times 4, 9 \times 5, 9 \times 6 \dots$
9 18 27 36 45 54

From this, the first two common multiples are 18 and 36. Since 36 is a multiple of 18, we know that the common multiples of 6 and 9 will be multiples of 18. The answer is D.

Least Common Multiple (LCM)



A cartoon boy with a pencil in his ear is thinking. A thought bubble above him shows the following calculation:

$$\begin{array}{r|l} 2 & 3, 6, 12, 8 \\ \hline 2 & 3, 3, 6, 4 \\ \hline 3 & 3, 3, 3, 2 \\ \hline & 1, 1, 1, 2 \end{array}$$

LCM (3, 6, 12, 8)
= $2 \times 2 \times 3 \times 1 \times 1 \times 1 \times 2 = 24$

Least Common Multiple

What is the least common multiple of 8 and 12?

- A. 4
- B. 8
- C. 12
- D. 24
- E. 36

Solution

Answer: D

Justification: We can start listing the multiples of 8 and 12.

For multiples of 8, we are multiplying 8 starting with 1, 2, 3, etc.
Thus, multiples of 8 are

$$8 \times 1, 8 \times 2, 8 \times 3, 8 \times 4, 8 \times 5 \dots$$

8 16 24 32 40

On the other hand, multiples of 12 are:

$$12 \times 1, 12 \times 2, 12 \times 3, 12 \times 4, 12 \times 5 \dots$$

12 24 36 48 60

The least common multiple would be the first common multiple appearing from the multiples of 8 and 12, which is 24. The answer is D.

Solution II

Answer: D

Justification: For 8 and 12, we can start dividing them by the smallest possible prime number. In our case, we start with 2.

2	8	12
2	4	6
2	2	3

Once we can no longer divide by same prime number, we can stop. In order to find the LCM, we have to multiply the numbers we have been dividing both 8 and 12, and all of the remaining prime numbers at the bottom of the operation (the numbers in red circles). By multiplying $2 \times 2 \times 2 \times 3$, we get 24. Thus, our LCM is 24. The answer is D.

Least Common Multiple II

What is the least common multiple of 6, 12 and 15?

- A. 2
- B. 3
- C. 6
- D. 60
- E. 120

Solution I

Answer: D

Justification: We can start listing the multiples of 6, 12 and 15.

For multiples of 6, we are multiplying 6 starting with 1, 2, 3, etc.

Thus, multiples of 6 are

$6 \times 1, 6 \times 2, 6 \times 3, 6 \times 4, 6 \times 5, 6 \times 6, 6 \times 7, 6 \times 8, 6 \times 9, 6 \times 10, 6 \times 11 \dots$
6 12 18 24 30 36 42 48 54 60 66

Similarly, multiples of 12 are:

$12 \times 1, 12 \times 2, 12 \times 3, 12 \times 4, 12 \times 5, 12 \times 6 \dots$
12 24 36 48 60 72

Lastly, multiples of 15 are:

$15 \times 1, 15 \times 2, 15 \times 3, 15 \times 4, 15 \times 5 \dots$
15 30 45 60 75

The first common multiple we can see from all three numbers is 60. Our answer is D.

Solution II

Answer: D

Justification: For 6, 12 and 15, we can start dividing them by the smallest possible prime number. In our case, we start with 3.

$$\begin{array}{r|rrr} 3 & 6 & 12 & 15 \\ \hline & 2 & 4 & 5 \end{array}$$

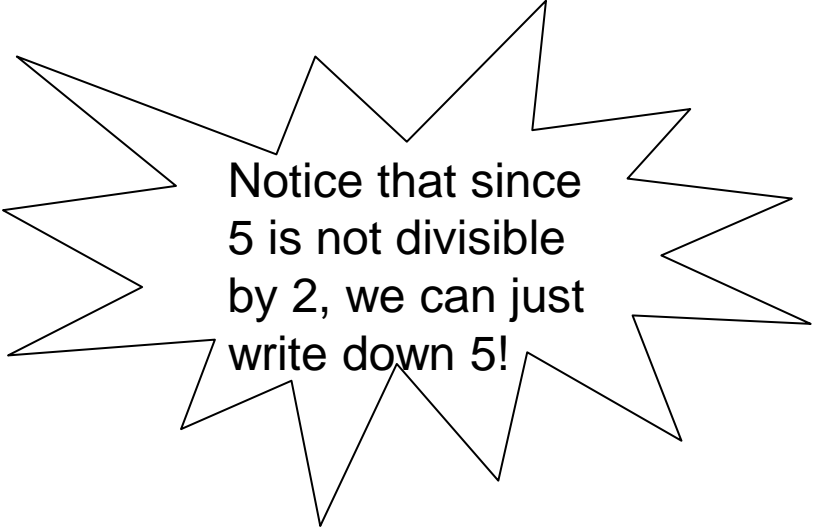
When we divide 6, 12, and 15 with 3, we get 2, 4, and 5. Now, there is no prime number that we can divide all three of them.

However, contrast to obtaining the GCF, when we are calculating the LCM, we can divide these numbers by a prime number if it can divide any two of them.

Solution II Cont'd

Thus, since 2 and 4 are divisible by 2, we can continue with our operation.

3	6	12	15
2	2	4	5
	1	2	5



Notice that since 5 is not divisible by 2, we can just write down 5!

Since none of the pair of 1, 2, and 5 cannot be divided by a prime number, we can stop. In order to find the LCM, we have to multiply the numbers we have been dividing both 8 and 12, and all remaining prime numbers on the bottom of the operation (the numbers in red circles). By multiplying $3 \times 2 \times 1 \times 2 \times 5$, we get the LCM of 60. The answer is D.

Least Common Multiple III

In the UBC bus loop, 99 B-Line leaves the loop every 3 minutes, 84 VCC-Clark leaves every 8 minutes, and 44 Downtown leaves every 12 minutes. If all three buses have left the loop at 2:00pm, when is the next time that all three buses leave together?

- A. 2:12 *pm*
- B. 2:24 *pm*
- C. 2:36 *pm*
- D. 2:48 *pm*
- E. 3:00 *pm*

Solution I

Answer: B

Justification: Since this problem wants to find the smallest time that coincide between 3, 8, and 12 minutes, this problem is about obtaining the LCM between these 3 numbers.

We can start this question by listing the multiples of these 3 numbers

Bus #99: 3 6 9 12 15 18 21 (24) 30 ... 60

Bus #84: 8 16 (24) 32 40 48 54

Bus #44: 12 (24) 36 48 60

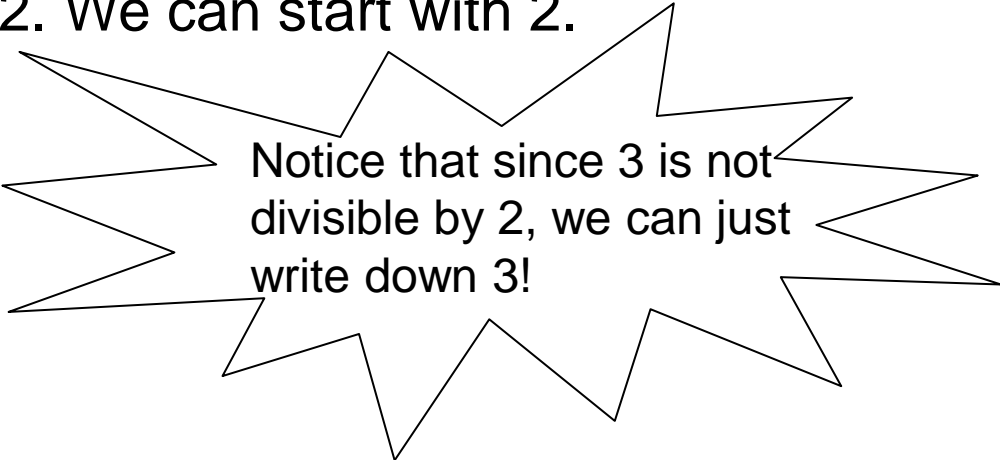
Since the question is asking for the very next time after 2:00pm, our desired time is 2:00pm + 24 minutes = 2:24 pm.
The answer is B.

Solution II

Answer: B

Justification: For 3, 8 and 12, there is no prime number that we can divide all three of them. However, since this question is about getting the LCM, we only have find a number that we can divide any two of 3, 8, and 12. We can start with 2.

2	3	8	12
2	3	4	6
3	3	2	3
1	2	1	1



Notice that since 3 is not divisible by 2, we can just write down 3!

Thus, after the operation, we can start multiplying numbers in the red circle. Then, we get 24 minutes.

Thus, 2:00pm + 24 minutes = 2:24pm. The answer is B.