a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

# Mathematics
# Probability: Counting

Science and Mathematics
Education Research Group

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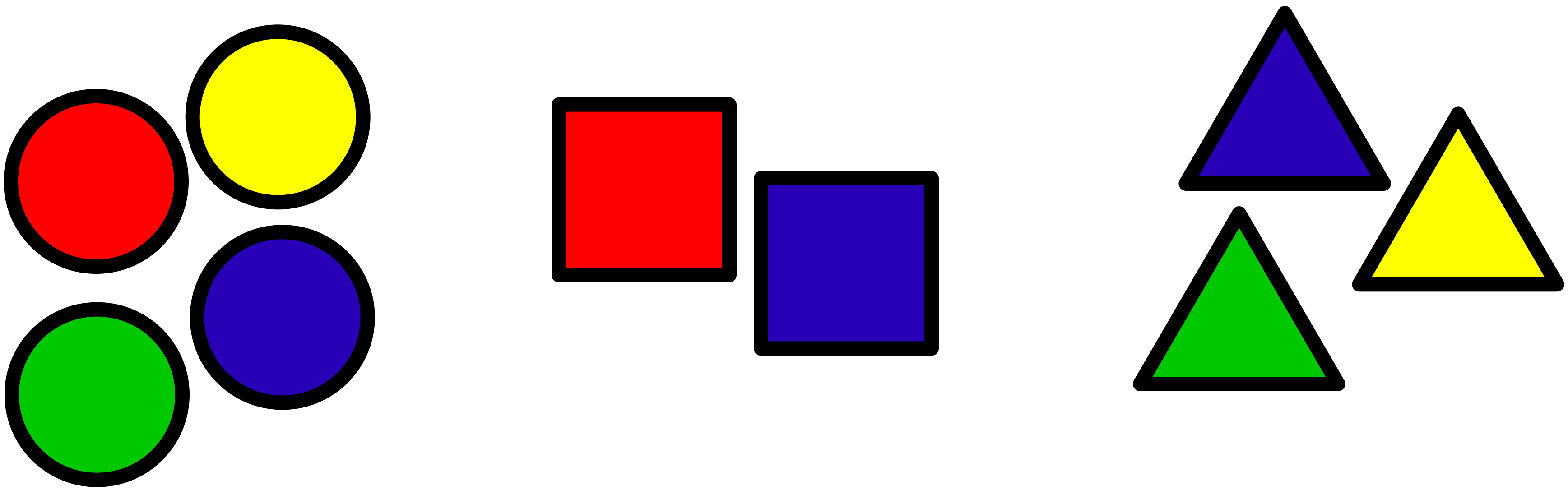
# Counting Techniques



$\underline{\;4\;} \times \underline{\;2\;} \times \underline{\;3\;}$

# Counting Techniques

Suppose two random numbers between 0 to 9  are chosen to form a 2 digit number.  (The two numbers may be the same.)

For example, if the numbers 4 and 7 are chosen, then the number 47 is formed.

How many numbers can be formed in this way?

A. 10
B. 20
C. 81
D. 99
E. 100

# Solution

**Answer:** E

**Justification:** There are 10 different ways to choose the first digit of the unknown number. For each of these 10 first digits, there are 10 different ways for the second digit to be chosen. There are 10 ways to choose the second digit because there can be repeated numbers. Therefore, there are a total of 10 x 10 = 100 different numbers that can be formed.

Intuitively, we know the numbers that can be formed are:

00, 01, 02, ... 97, 98, 99

# Counting Techniques II

Jeremy is at a fast food restaurant with the following menu:

| **Pizza**: | **Side:** | **Drinks:** |
|---|---|---|
| *Cheese* | *Fries* | *Orange Juice* |
| *Pepperoni* | *Chicken Wings* | *Apple Juice* |
| *Ham* | | *Lemonade* |
| *Veggie* | | |

A combo comes with a slice of pizza, 1 side, and 1 drink.  How many different combos can Jeremy choose from?

A. 9

B. 23

C. 24

D. 4096

E. None of the above

# Solution

**Answer:** C

**Justification:** Jeremy has 4 different choices for the type of pizza, 2 different choices for the side, and 3 different choices for the drink. These choices are done independently of each other. In order words, the choice of pizza has no effect on the choice of the side or drink.

The total number of ways to form a combo is 4(2)(3) = 24.

When we multiply possibilities together to find the total number of combinations, we are using the Fundamental Counting Principle.

# Counting Techniques III

How many numbers between 1 and 999 are divisible by 2?

A. 499

B. 500

C. 501

D. 502

E. 999/2

# Solution

**Answer:** A

**Justification:** $1^{st}$ digit ____ $2^{nd}$ digit ____ $3^{rd}$ digit ____

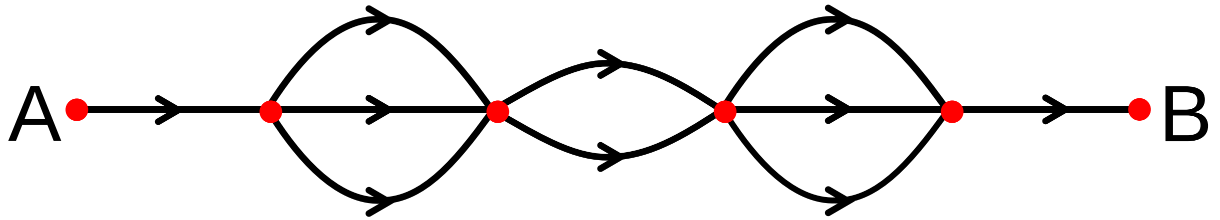
There are 10 ways to choose the first digit: the numbers 1 to 9 or blank.

There are also 10 ways to choose the second digit, the numbers 0 to 9 (or blank instead of 0 if the first digit is blank).

A number is divisible by 2 if the last digit ($3^{rd}$ digit) is either 0, 2, 4, 6 or 8. The only exception is when both the first and $2^{nd}$ digit are blank where we cannot choose 0. After subtracting the zero case, there are 10(10)(5)-1 = 499 different possible numbers.

# Counting Techniques IV

The diagram below shows the possible one-way paths to each red dot. How many unique paths are there from A to B?




A. 8

B. 18

C. 36

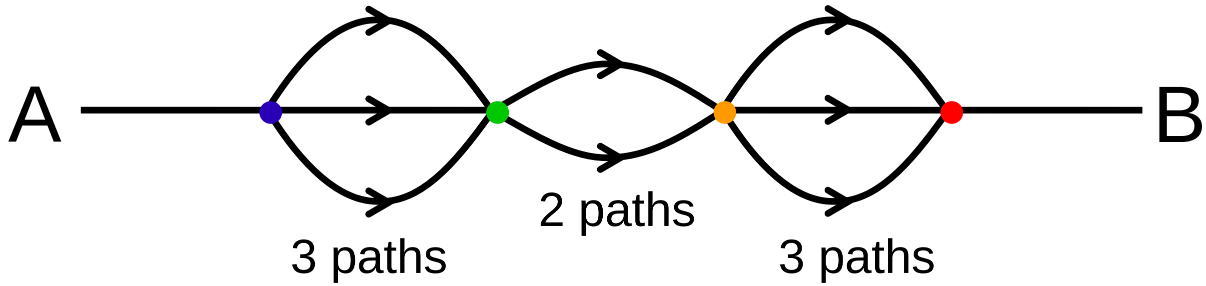
D. 72

E. 120

# Solution

**Answer:** B

**Justification:**

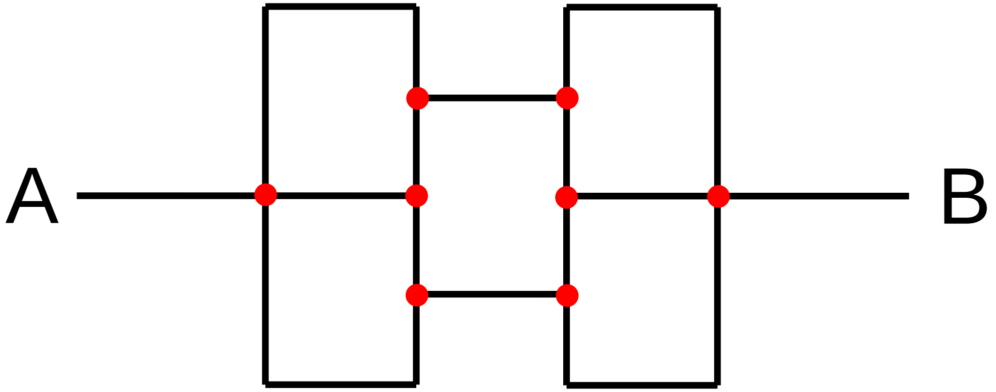



After reaching the first junction (blue dot), there are different paths to reach the green dot. For each of these paths, there are 2 paths to the orange dot, and 3 more paths to the red dot. Selecting a path does not alter the choice of the next path, so by the Fundamental Counting Principle, the number of unique paths is:

$$3(2)(3) = 18$$

# Counting Techniques V

In the maze below, you are only allowed to move up, down, or right, and you can only go over a path once. How many unique paths are there from A to B?




A. 8

B. 9

C. 18

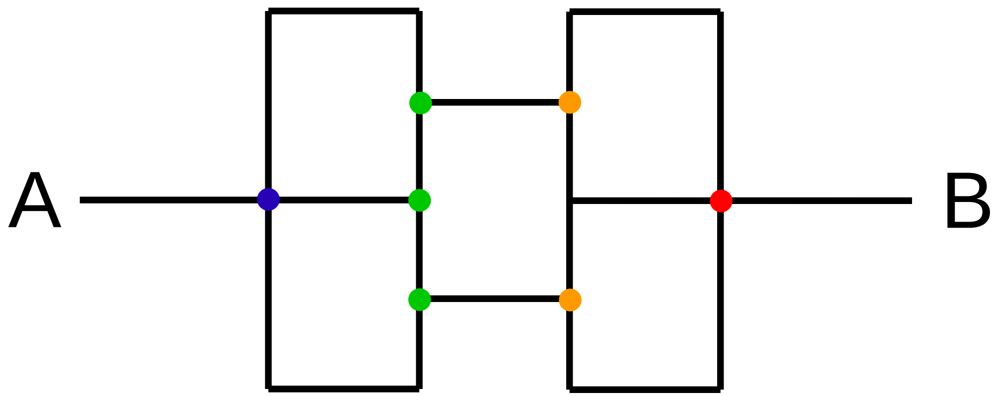
D. 24

E. 36

# Solution

**Answer:** C

**Justification:**




This question is exactly the same as the previous question.

After reaching the first junction (blue dot), there are different paths (up, down, left or right). From any of the 3 green dots, there are only 2 unique paths. You either reach the top orange dot, or the bottom orange dot. From either orange dot, there 3 unique paths to the red dot. By the Fundamental Counting Principle, the number of unique paths is:

$$3(2)(3) = 18$$

# Counting Techniques VI

A locker combination consists of 3 numbers, where each number ranges from 0 to 59.  How many different locker combinations are there on the lock?

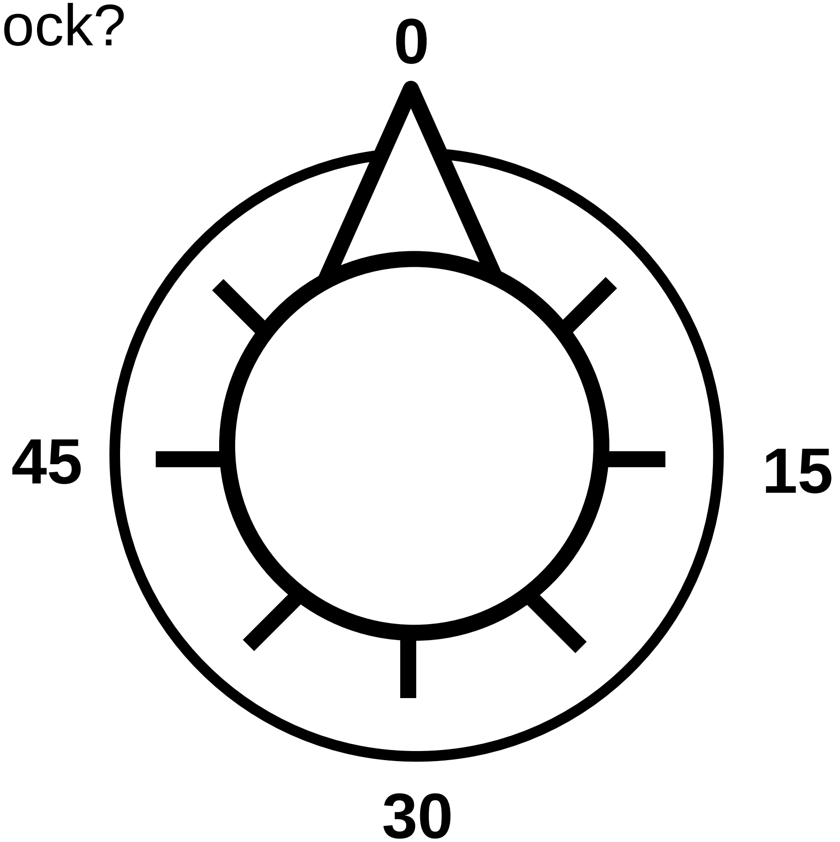
A. $59^3$

B. $60^3$

C. $61^3$

D. 60(59)(58)

E. Too many to count



# Solution

**Answer:** B

**Justification:** Each of the numbers in the locker combination can be anything from 0 to 59. This is a total of 60 choices for each of the numbers. Since there are 3 numbers in a locker combination, there are a total of:

$$60(60)(60) = 60^3 = 216000 \text{ different combinations.}$$

Reminder: To find the how many numbers there are between *x* and *y* (inclusive), we must calculate $y-x+1$.

# Counting Techniques VII

Mary is worried that $60^3$ different combinations on her lock is not safe enough. She decides to put 3 separate locks onto her locker. How many different ways can Mary choose the numbers for her 3 locks?

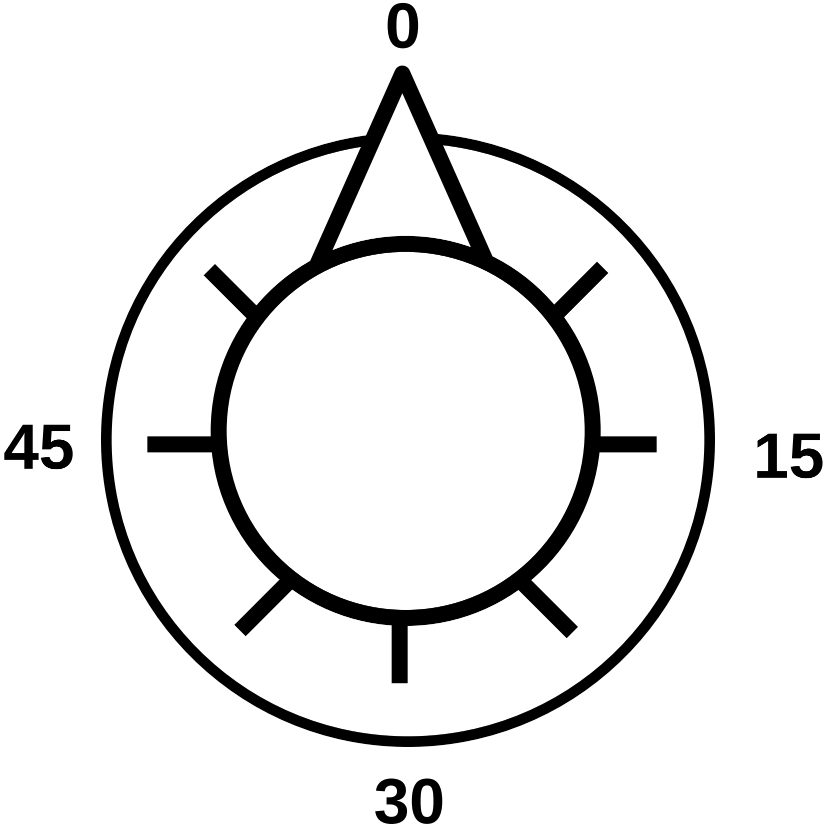
A. $3(60^3)$

B. $3(60^3) - 3$

C. $60^9$

D. $60^9 - 3$

E. $60^3(59^3)(58^3)$



# Solution

**Answer:** C

**Justification:** Mary must choose a number between 0 to 59 nine times in order to get combinations for her 3 locks. This is a total of $60^9 \approx 10^{16}$ different combinations.

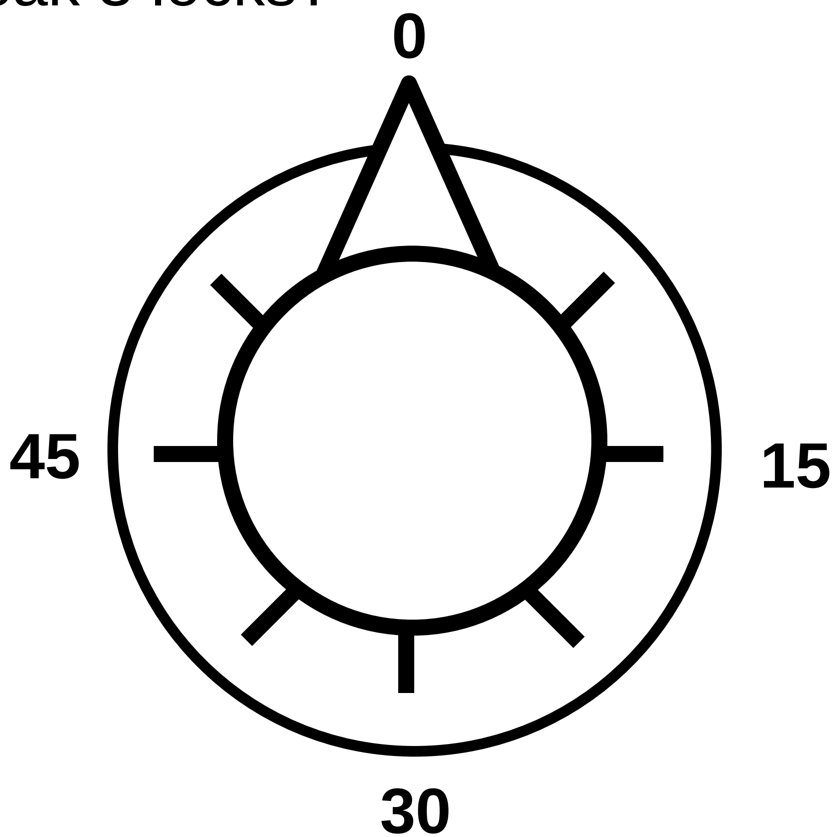
Alternative Solution:

Mary has $60^3$ ways to choose the combination of her first lock. For each of these combinations, there are another $60^3$ ways to choose the second lock. For each of these combinations, there are another $60^3$ ways to choose the third lock. Thus the total number of different combinations is:

$$60^3(60^3)(60^3) = 60^9 \approx 10^{16}$$

# Counting Techniques VIII

Suppose it takes a robber T minutes to try $60^3$ combinations. What is the maximum amount of time required for the robber to break 3 locks?




A. $60^6$(T)
B. $60^9$(T)
C. T
D. 3T
E. 60T

# Solution

**Answer:** D

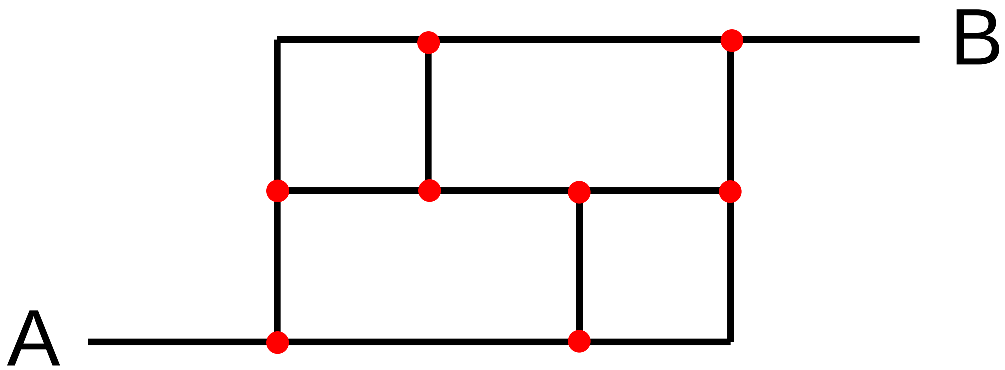
**Justification:** Even though 3 locks have $60^9$ different combinations, the robber only has to spend T minutes to break the first lock, T minutes to break the second lock, and T minutes to break the third lock. The total time is 3T minutes.

Note: If instead a single lock consists of 9 numbers ranging from 0 to 59, there will also be $60^9$ different combinations. However, it will take the robber:

$$60^9 \cdot \frac{T}{60^3} = 60^6 T$$

# Counting Techniques IX

In the maze below, you are only allowed to move up, down, or right, and you can only go on a path once.  How many unique paths are there from A to B?




A. 8

B. 9

C. 10

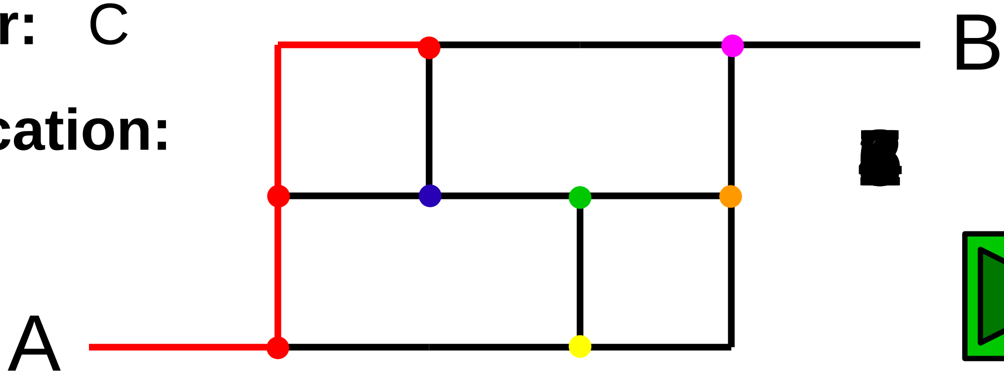
D. 12

E. 18

# Solution

**Answer:** C

**Justification:**




*Press to show solutions*

Consider reaching one of the red dots by following the red path. The number of ways to reach B from each of these 3 red dots is:

- Top red dot: 3 ways to reach B
- Middle red dot: 3 ways to reach B
- Bottom red dot: 2 ways to reach B

Adding each of these 3 cases give a total of 8 ways to reach B. Notice that the 3 cases are independent of each other, so the number of ways are not multiplied together.

# Counting Techniques X

Consider a 10 character email address (before the @) where the domain is one of *email.com, coolmail.com,* or *math.com.* Characters include any digit (0 to 9), any lowercase (a-z) and any uppercase (A-Z). How many unique 10 character email addresses can be made?

A. $3(26+26+10)^{10}$
B. $3(26+26+9)^{10}$
C. $3^{10}(26+26+10)^{10}$
D. $(3+26+26+9)^{10}$
E. $3^{10}(26^{10})(26^{10})(10^{10})$

Examples:

character8@email.com

cHaRaCtEr8@email.com

character8@coolmail.com

(All the above are unique)

# Solution

**Answer:** A

**Justification:** There are 26+26+10 = 62 different choices for each character (a lowercase, uppercase, or digit). This choice is made independently 10 times to form the 10 character address before the @. For every one of these $62^{10}$ unique names, there are 3 choices for the domain. We therefore multiply our total number of email addresses by 3; one for each domain.

$$3(26+26+10)^{10} = 3(62^{10}) = 2.52 \times 10^{18}$$

# Counting Techniques XI

Consider a 6 character (a-z, A-Z, 0-9) password. Let the number of passwords that can be made using 6 lowercase characters be X. How many passwords (in terms of X) can be made using 5 lowercase and 1 uppercase ?

A. X
B. 5X
C. 6X – 1
D. 6X
E. $X^6$

# Solution

**Answer:** D

**Justification:** Consider any 6 lowercase password xxxxxx (where x represents any lowercase a-z). If we now have 5 lowercase and 1 uppercase, we can represent the password xxxxxx in 6 different ways:

Xxxxxx, xXxxxx, xxXxxx, xxxXxx, xxxxXx, xxxxxX

This means that for every one of the 6 lowercase passwords, we can now change one of the characters to an uppercase, giving us 6 times as many passwords.