a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Mathematics
Probability: Odds

Science and Mathematics
Education Research Group

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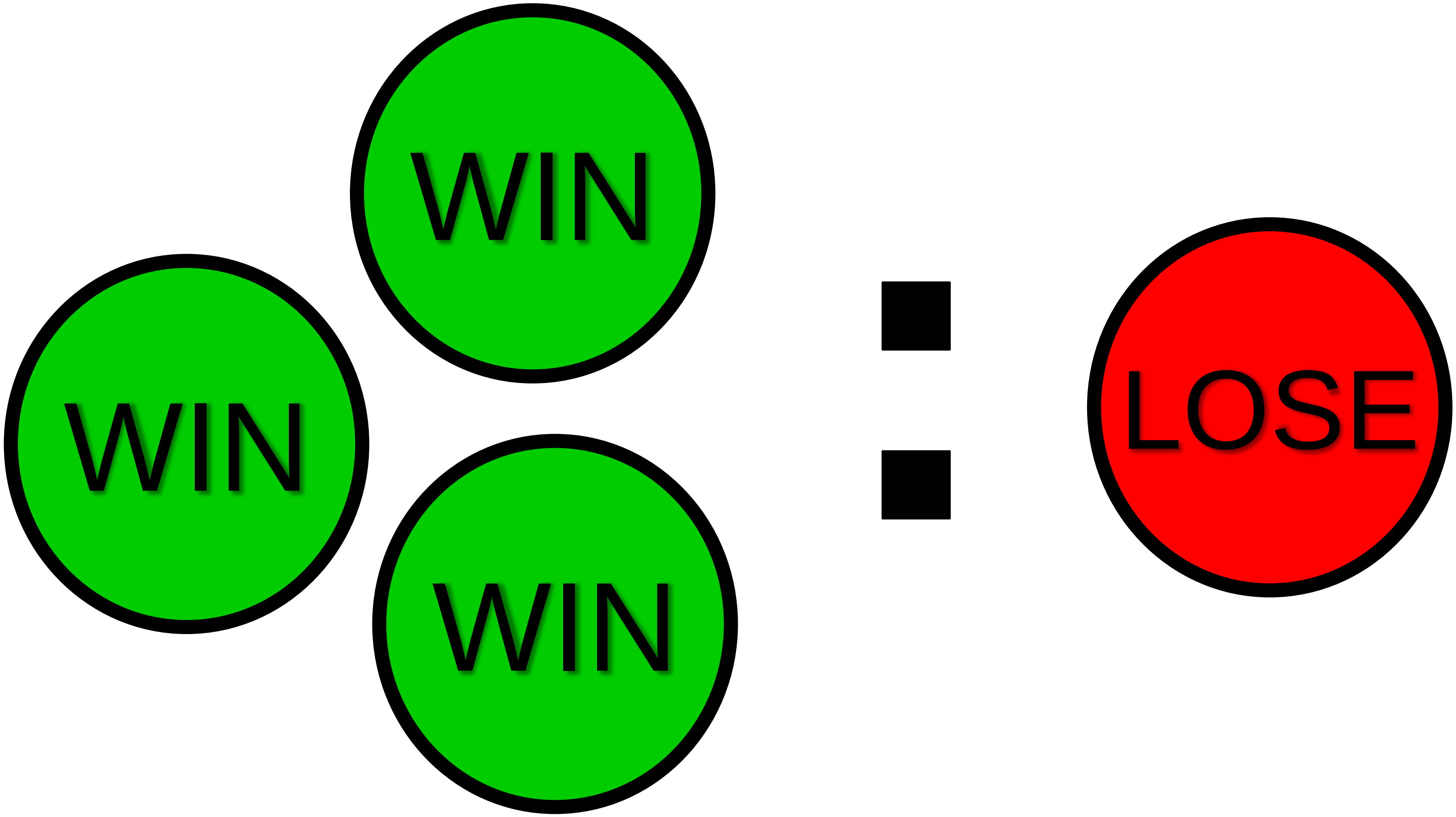
Odds



Odds

The message on a lottery ticket says “Odds of winning are 1:3.” Which one of the following statements is true?

A. There is 1 winning ticket for every 4 tickets
B. There is 1 winning ticket for every 3 tickets
C. There are 3 winning tickets for every 4 tickets
D. Buying 3 tickets guarantees that you will win
E. None of the above statements are true

Solution

Answer: A

Justification:

Odds express the ratio of the number of times you win to the number of times you lose. The odds of winning are 1 to 3, which means that there is 1 winning ticket for every 3 losing tickets. There is 1 winning ticket for every 4 total tickets.

Note that even though there is 1 winning ticket for every 4 tickets, buying 4 tickets does not guarantee that you will win if you buy 4 tickets.

Odds II

Pick a number between 1 to 6 and roll a six-sided die. What are the odds that the number you pick is the same as the die roll?

A. 1 to 6

B. 1 to 5

C. 6 to 1

D. 5 to 1

E. 1 : 36

Solution

Answer: B

Justification:

The probability of guessing the outcome of rolling a six-sided die is:

$$P(\text{Correct die guess}) = \frac{1}{6}$$

This means that after rolling a die 6 times, you should expect to guess correct once and incorrect 5 times. The odds of guessing correct are therefore *1 to 5*.

Note that if you say that the odds of winning are *5 to 1*, this is interpreted that you will win 5 times every time you lose once. Be careful with the order of the ratios and see if the question is asking for the odds of winning or losing.

Odds III

Suppose that the odds of winning a game are *m : n*. What is the probability that you will win the game?

A. $P(\text{win}) = \dfrac{m}{n}$

B. $P(\text{win}) = \dfrac{n}{m}$

C. $P(\text{win}) = \dfrac{m}{n+1}$

D. $P(\text{win}) = \dfrac{n}{m+1}$

E. $P(\text{win}) = \dfrac{m}{m+n}$

Solution

Answer: E

Justification: When the odds of winning are *m to n*, you expect to win *m* times and lose *n* times after *m* + *n* total games.

The probability of winning is therefore:

$$P(\text{win}) = \frac{\text{number of wins}}{\text{number of total games}} = \frac{m}{m+n}$$

Recall that the odds of predicting a die roll were 1:5, and the probability of guessing the correct roll was 1/6. This matches with the formula above:

$$P(\text{guess die roll}) = \frac{1}{1+5} = \frac{1}{6}$$

Odds IV

The probability that tomorrow will be sunny happens to be 55%. What are the odds that tomorrow will be sunny?

A. 55 to 100
B. 55 to 155
C. 11 to 31
D. 11 to 20
E. 11 to 9

Solution

Answer: E

Justification: We are given the probability that it will be sunny tomorrow:

$$P(\text{Sunny}) = 55\% = \frac{55}{100}$$

If this probability remains the same after 100 days, we should expect that 55 of the days were sunny, while 45 of the days were not. This translates to the odds 55 to 45.

Similar to fractions and ratios, these odds can be reduced to 11 : 9.

Odds V

Your friend challenges you to a game where the odds of you winning are 1 : 3. If you win, your friend gives you $30. However, if you lose, you must pay your friend $10.

Are the stakes for winning and losing fair?

A. Yes

B. No

Solution

Answer: A

Justification: Since the odds of you winning are 1 : 3 you can expect to win once for every three times you lose. The probability of winning and losing can be calculated as shown:

$$P(\text{Win}) = \frac{m}{m+n} = \frac{1}{4} \qquad P(\text{Lose}) = \frac{n}{m+n} = \frac{3}{4}$$

Intuitively we can see that the stakes are fair. Even though you expect to lose the game 3 times as much, you also gain 3 times as much money for winning (you gain $30 for winning but only give away $10 when you lose). On average, winning 1 game and losing 3 will result in no money being transferred between you and your friend.

See the next slide for a calculated answer

Solution

Answer: A

Justification: We can also calculate how much we can expect to win (or lose) as follows:

Let the amount you win after the game be X. X can be one of two values, X_{win} = \$30 or X_{lose} = - \$10.

The expected value of X can be found by multiplying the values of X by the probability that X occurs:

$$E(\mathrm{X}) = P(\mathrm{win})X_{win} + P(\mathrm{lose})X_{lose}$$

Notice that P(win) + P(lose) = 1

$$= \frac{1}{4}(\$30) - \frac{3}{4}(\$10) = \$0$$

When the expected value is \$0, we can expect that we will win as much money as we lose (if we play the game enough times).

Odds VI

Suppose that the odds of winning a lottery is 1 : 14 000 000. Each lottery ticket only costs $2. How large should the grand prize be so that your expected return is $0? (Assume that there is only a grand prize, and if you win, the money is not split with another winner).

A. Approximately $7 000 000
B. Approximately $14 000 000
C. Approximately $28 000 000
D. Approximately $196 000 000
E. Larger than $196 000 000

Solution

Answer: C

Justification: Since you expect to lose 14 million times before you win and every losing ticket costs \$2, the grand prize should be about \$28 000 000 in order to break even.

$$E(\mathrm{X}) = P(\mathrm{win})X_{win} + P(\mathrm{lose})X_{lose}$$

$$\$0 = \frac{1}{14000001}X_{win} - \frac{14000000}{14000001}(\$2)$$

$$X_{win} = \frac{14000000}{14000001}(\$2)(14000001)$$

$$X_{win} = \$28000000$$

Remember this calculation is an approximation because the rules of the lottery were simplified. Why is it not wise to try and buy every single ticket combination to guarantee that you win?