a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Mathematics
Linear Relations

Science and Mathematics
Education Research Group

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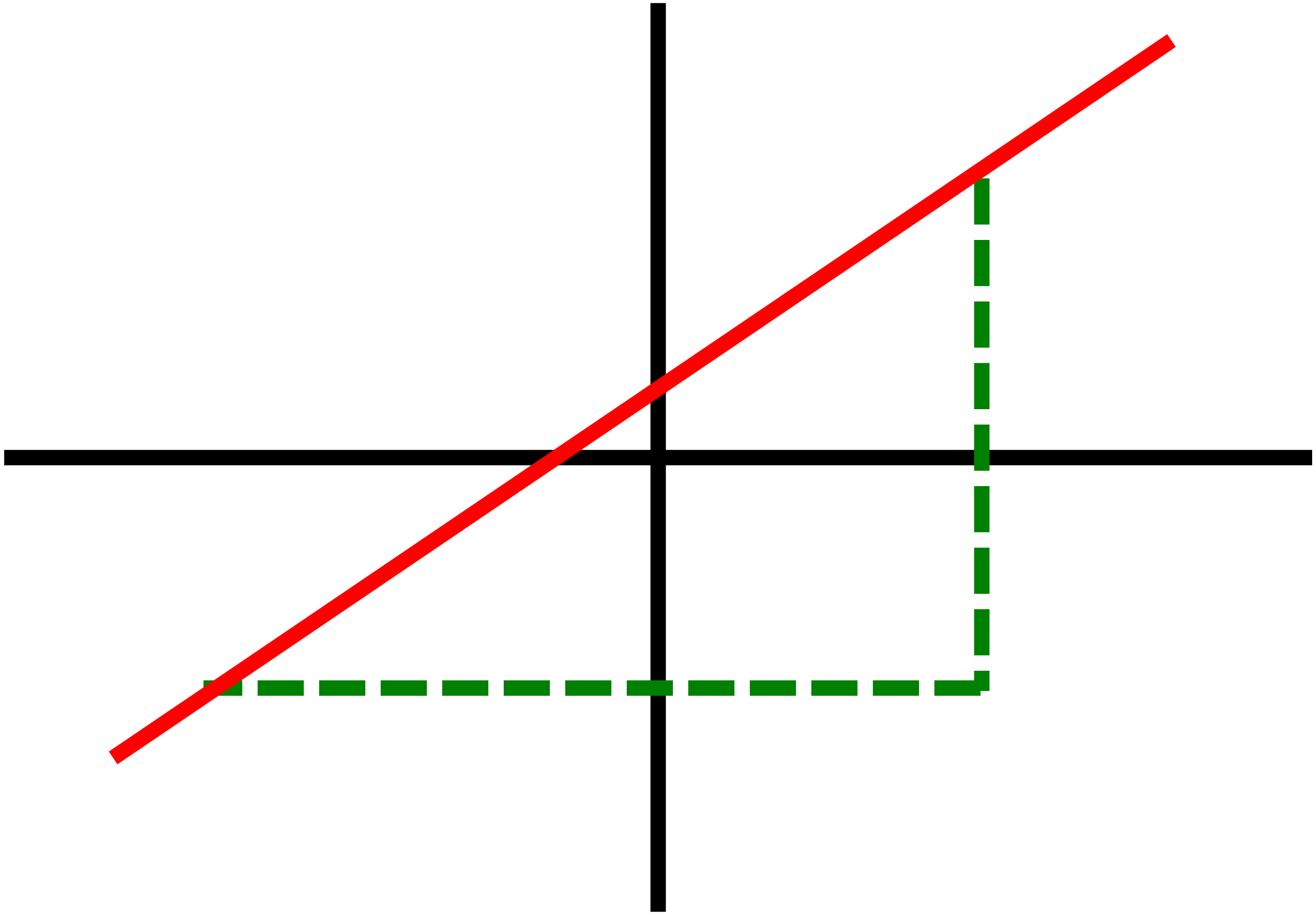
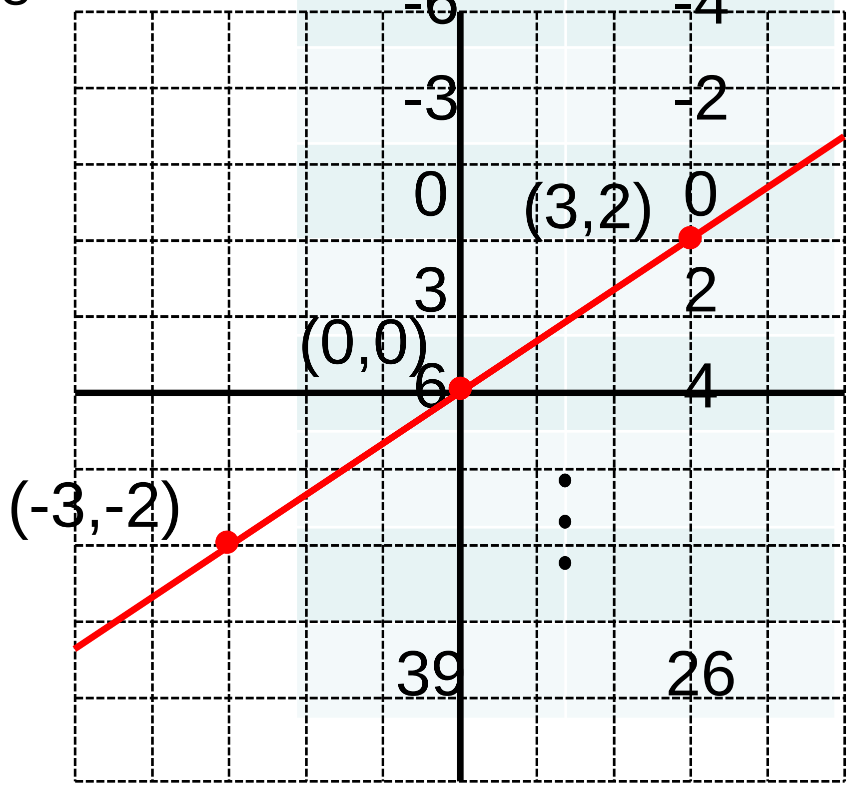
Linear Relations



Linear Relations I

The points in the table lie in a line when plotted on a graph. Which of the following points also belongs in the table?

x	y
-6	-4
-3	-2
0	0
3	2
6	4
⋮	
39	26



Press to show table graph

A. x = 45, y = 27

B. x = 45, y = 28

C. x = 45, y = 29

D. x = 45, y = 30

E. x = 45, y = 31

Solution

Answer: D

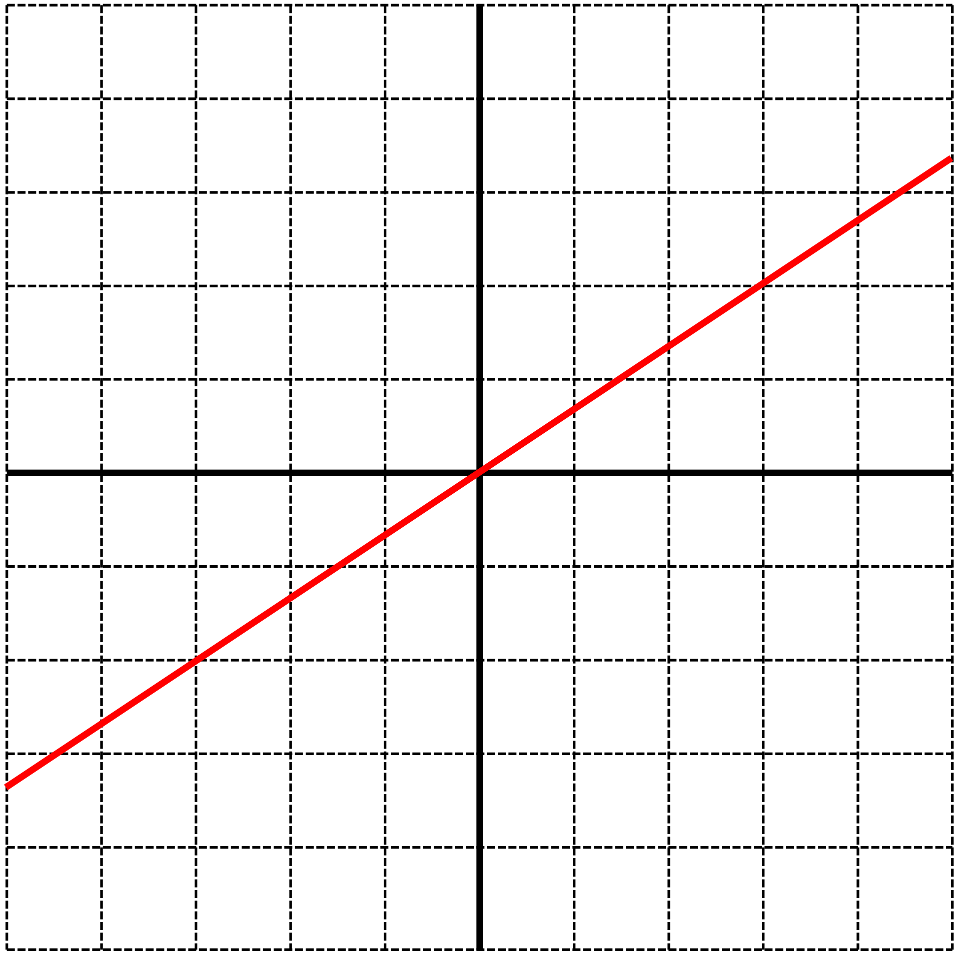
Justification: Notice that values of x in the table increase by 3, while the values of y increase by 2. Every change in x by 3 causes a change in y by 2.

Since x increases by 6 when changing from 39 to 45, we can expect a change in y by 4. Therefore when x = 45, our linear relationship gives y = 26 + 4 = 30.

x	y
-6	-4
-3	-2
0	0
3	2
6	4
⋮	
39	26
42	28
45	30

Linear Relations II

Consider any point on the red line. If the x-coordinate of this point increases by 3, what happens to the y-coordinate?



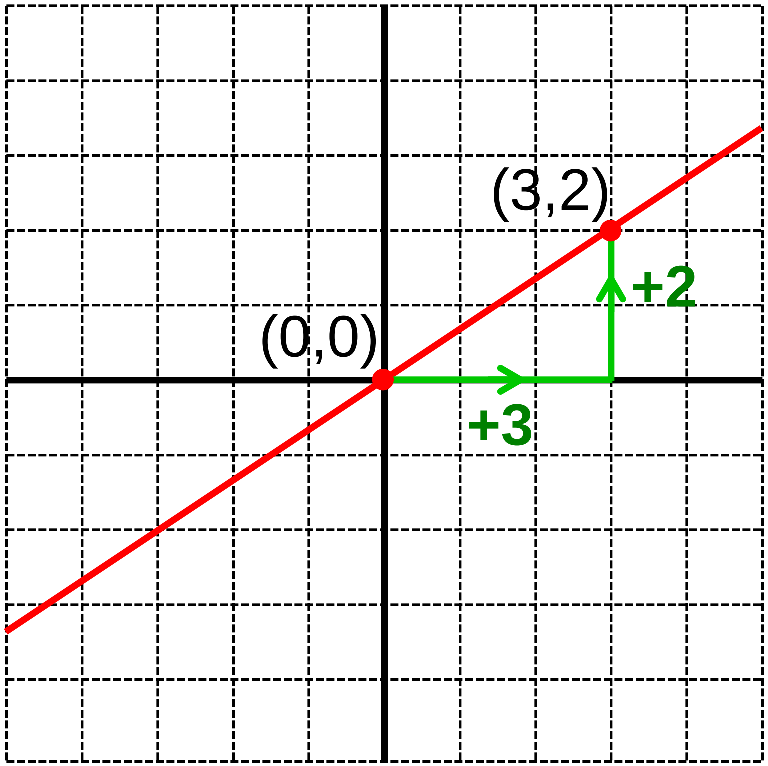
A. Increases by 3
B. Increases by 2
C. Increases by 1
D. Increases by 1/3
E. Depends on the point chosen

Solution

Answer: B

Justification: Consider the point on the red line (0,0). If x is increased from 0 to 3, the y-coordinate on the line increases from 0 to 2. This corresponds to (3,2) on the line.

Try starting at a different point, such as (-3,-2) and verify that the y-coordinate always increases by 2.



Linear Relations III

In which of the following equations does y increase by 2 if x is increased by 3?

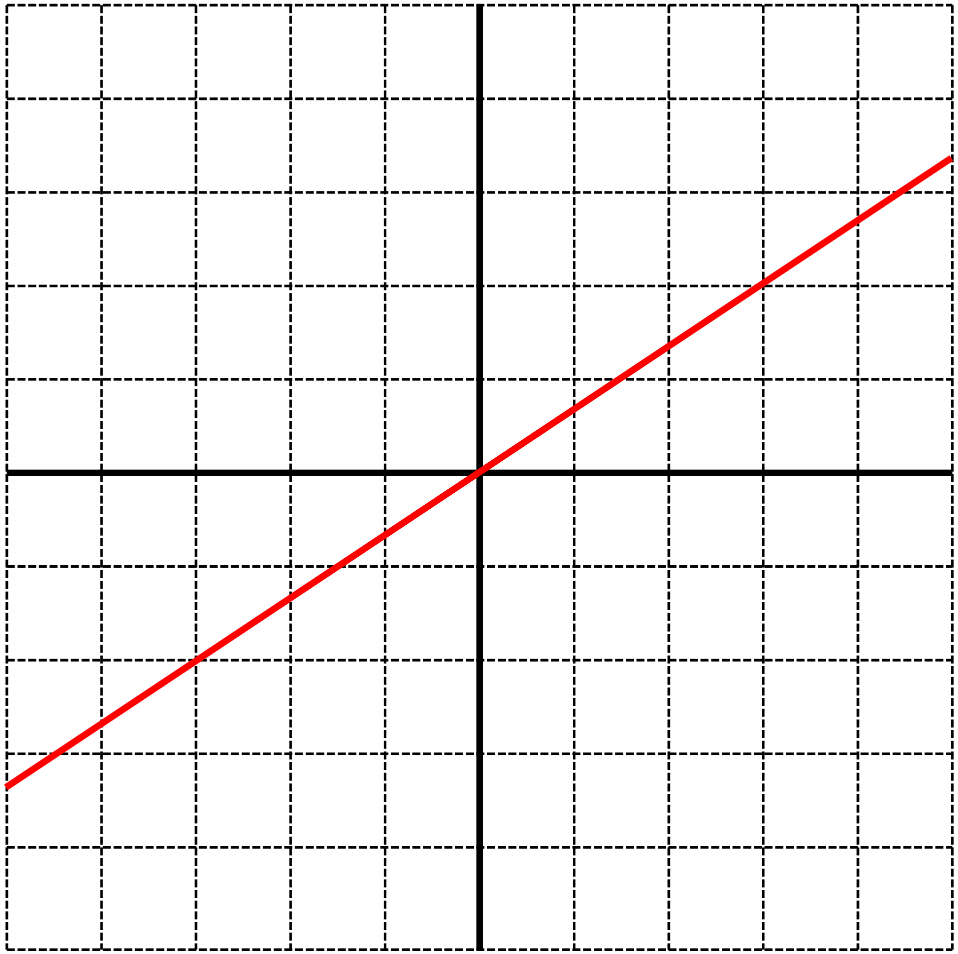
A. $y = 3x$

B. $y = 2x$

C. $y = \frac{2}{3}x$

D. $y = \frac{3}{2}x$

E. $y = x + 2$



Solution

Answer: C

Justification: Consider what happens to each equation when x is replaced with x + 3. Let y_1 be the original y-value, and y_2 be the new y-value after x increases by 3.

A. $y_1 = 3x \quad \Rightarrow \quad y_2 = 3(x+3) = 3x+9 = y_1 + 9$

B. $y_1 = 2x \quad \Rightarrow \quad y_2 = 2(x+3) = 2x+6 = y_1 + 6$

C. $y_1 = \frac{2}{3}x \quad \Rightarrow \quad y_2 = \frac{2}{3}(x+3) = \frac{2}{3}x + 2 = y_1 + 2$ (y rises by 2 from the original)

D. $y_1 = \frac{3}{2}x \quad \Rightarrow \quad y_2 = \frac{3}{2}(x+3) = \frac{3}{2}x + \frac{9}{2} = y_1 + \frac{9}{2}$

E. $y_1 = x+2 \quad \Rightarrow \quad y_2 = (x+3)+2 = x+5 = y_1 + 3$

Linear Relations IV

Consider the two points labelled in the graph. What change in the x- and y-values (Δx and Δy) is needed to move from P_1 to P_2?

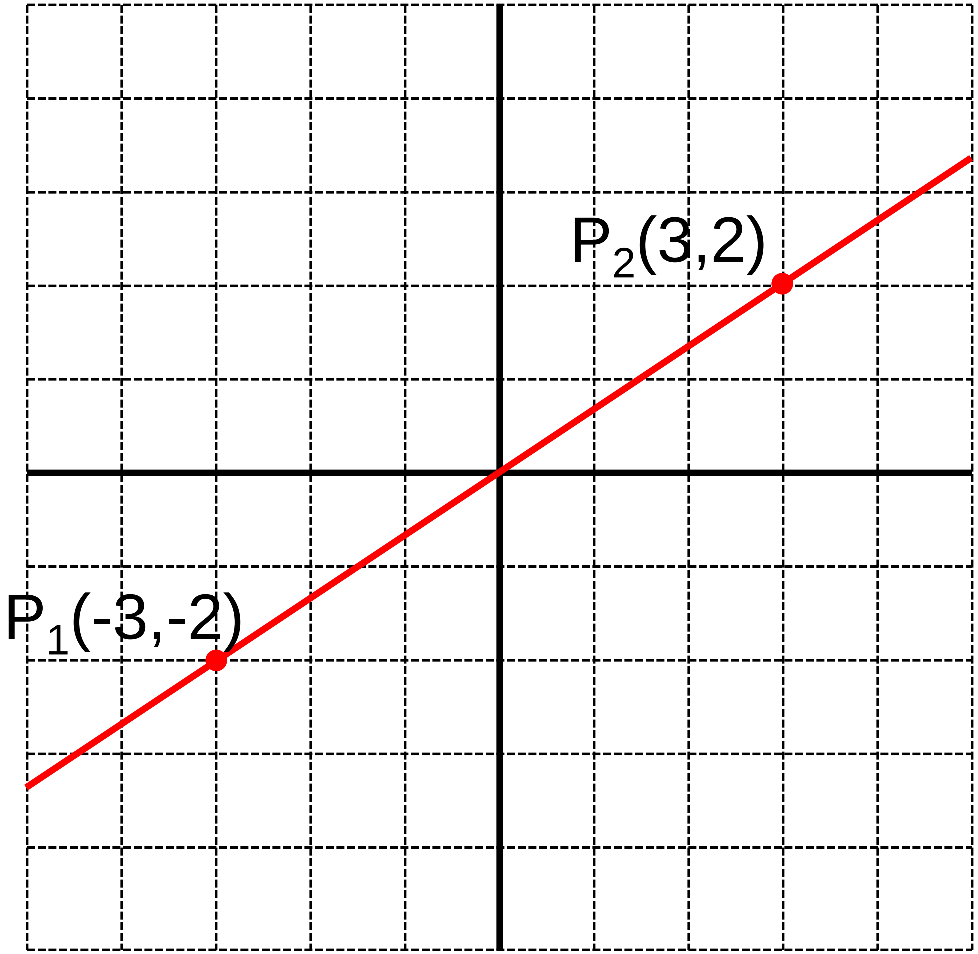
A. $\Delta x = 4$, $\Delta y = 6$

B. $\Delta x = -4$, $\Delta y = -6$

C. $\Delta x = 6$, $\Delta y = 4$

D. $\Delta x = -6$, $\Delta y = -4$

E. None of the above



Solution

Answer: C

Justification: The x-values of P_1 and P_2 are -3 and +3 respectively. x_1 must be *increased* by 6 to reach x_2.

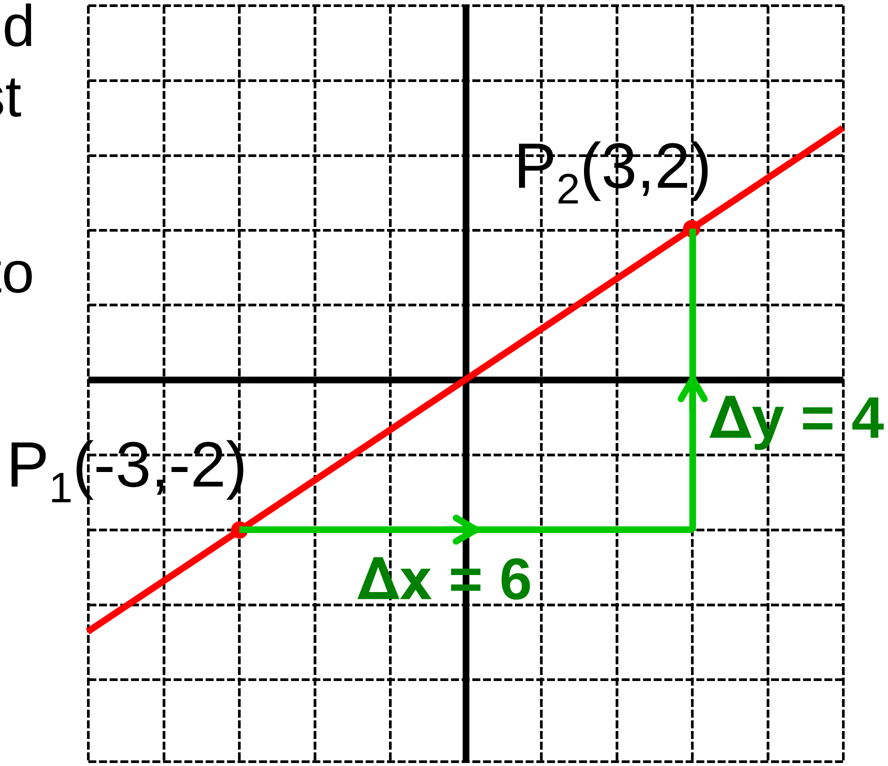
Similarly, y_1 must be *increased* by 4 to reach y_2. This can be calculated as follows:

$$\Delta x = x_2 - x_1 = (3) - (-3) = 6$$

$$\Delta y = y_2 - y_1 = (2) - (-2) = 4$$

The slope of a line (denoted with the letter m) is defined as the ratio:

$$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



Slope = $\frac{4}{6}$ = $\frac{2}{3}$

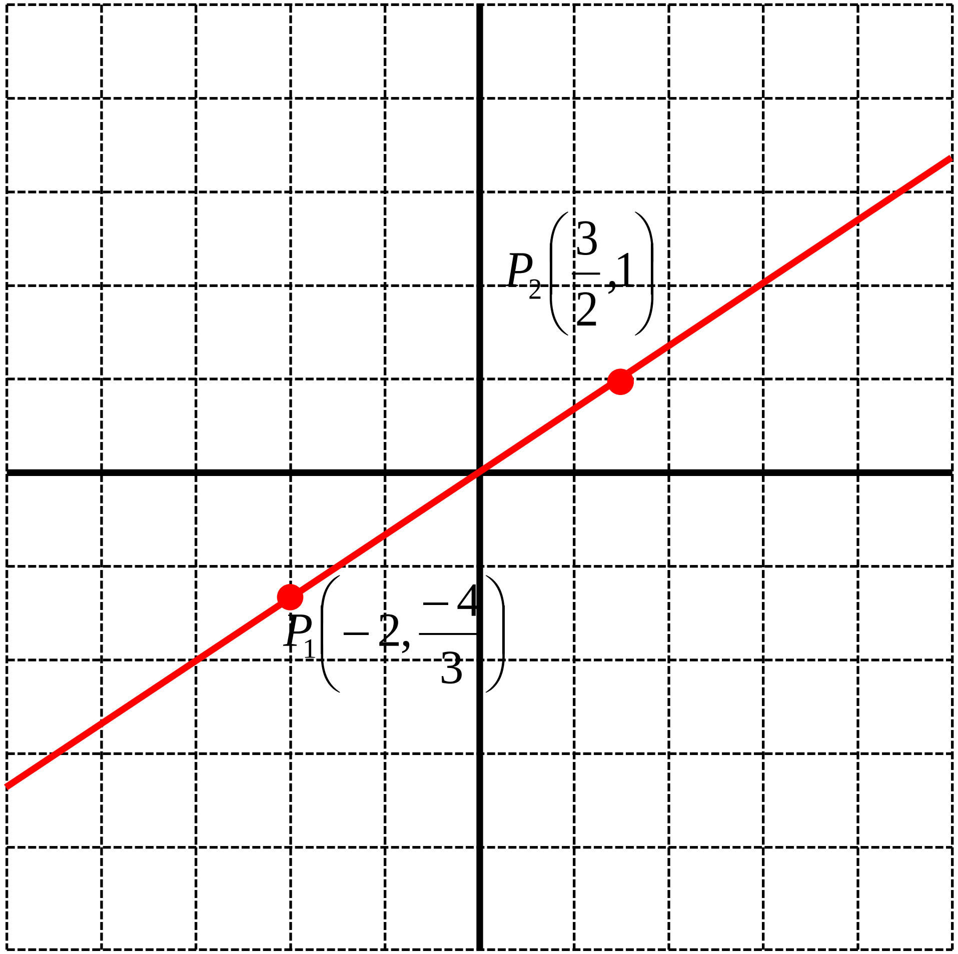
Linear Relations V

Consider the 2 points P_1 and P_2 shown on graph. The line is the same line as in the previous question. What is the slope using these two points?

$$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

A. $\frac{4}{7}$ B. $\frac{3}{7}$ C. $\frac{3}{2}$

D. $\frac{2.5}{3.5}$ E. $\frac{2}{3}$



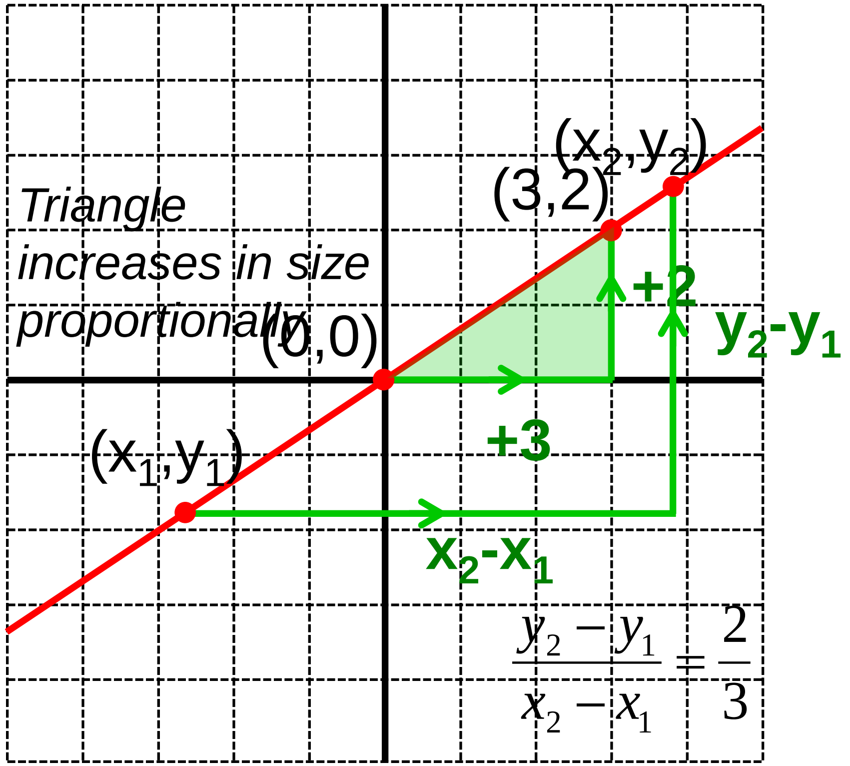
Solution

Answer: E

Justification: Recall that if x increases by 3, y increases by 2. The ratio between the change in y and change in x remains constant between any 2 points on the same line. (see animation)

We can conclude that the slope is the same as in question 2. It is not necessary to recalculate the slope with these new points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2}{3}$$



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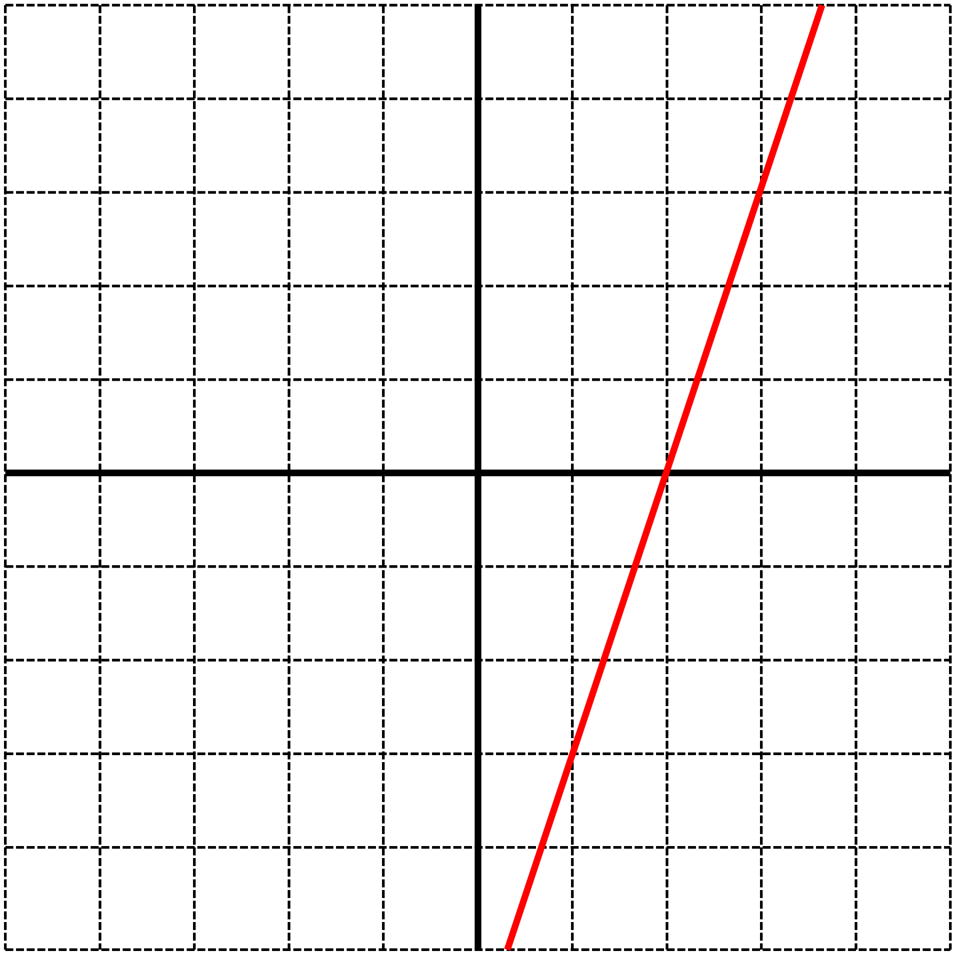
Linear Relations VI

What is the slope of the line?



A. 6

B. 3

C. $\frac{1}{3}$

D. $\frac{2}{6}$

E. Both A and B are correct

Solution

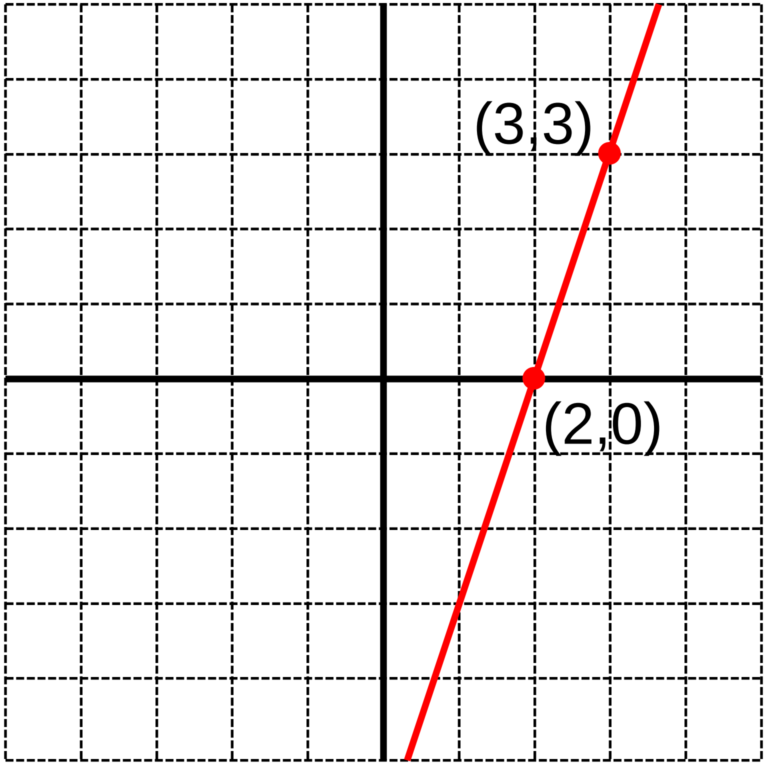
Answer: B

Justification: Look for 2 points on the line that intersect the grid, for example (2,0) and (3,3). Using the formula to calculate the slope:

$$(x_1, y_1) = (2,0), \quad (x_2, y_2) = (3,3)$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3-0}{3-2} = 3$$

Note: If (3,3) was used as point 1 and (2,0) as point 2, the same result would be achieved.



Linear Relations VII

What is the slope of the following line?

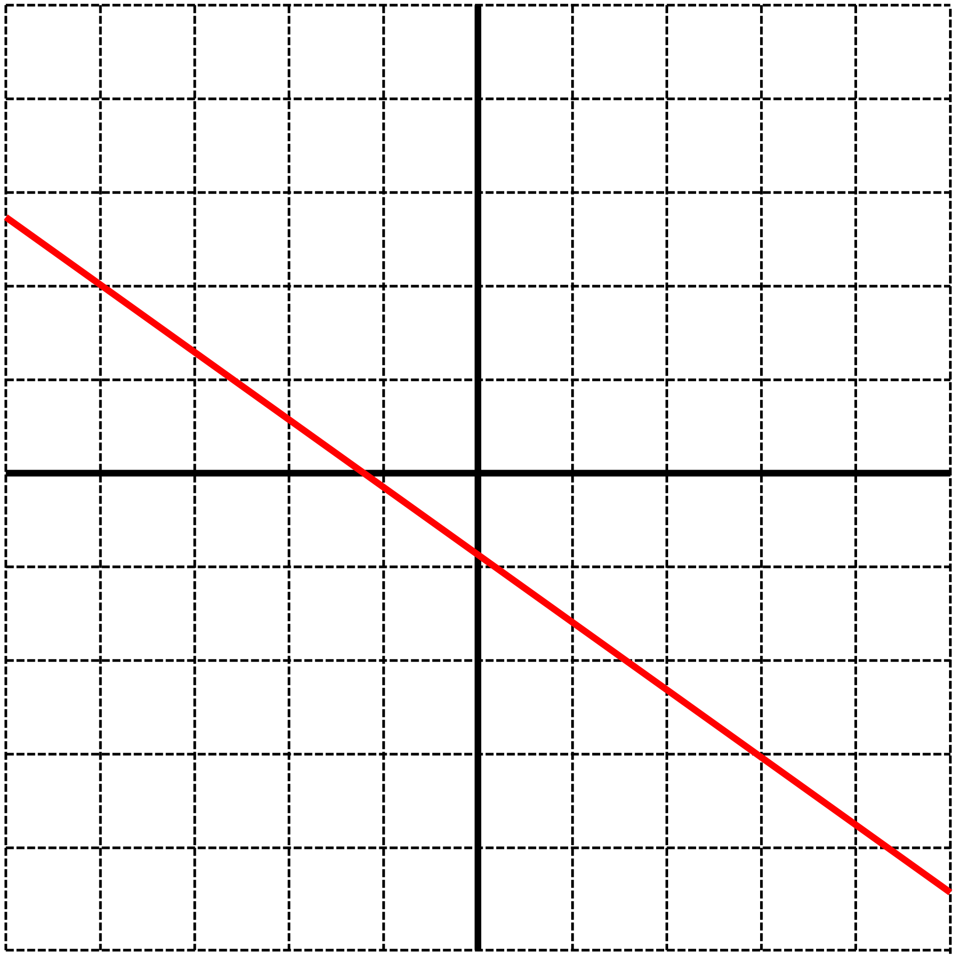
A. $\frac{7}{5}$

B. $-\frac{7}{5}$

C. $\frac{5}{7}$

D. $-\frac{5}{7}$

E. None of the above



Solution

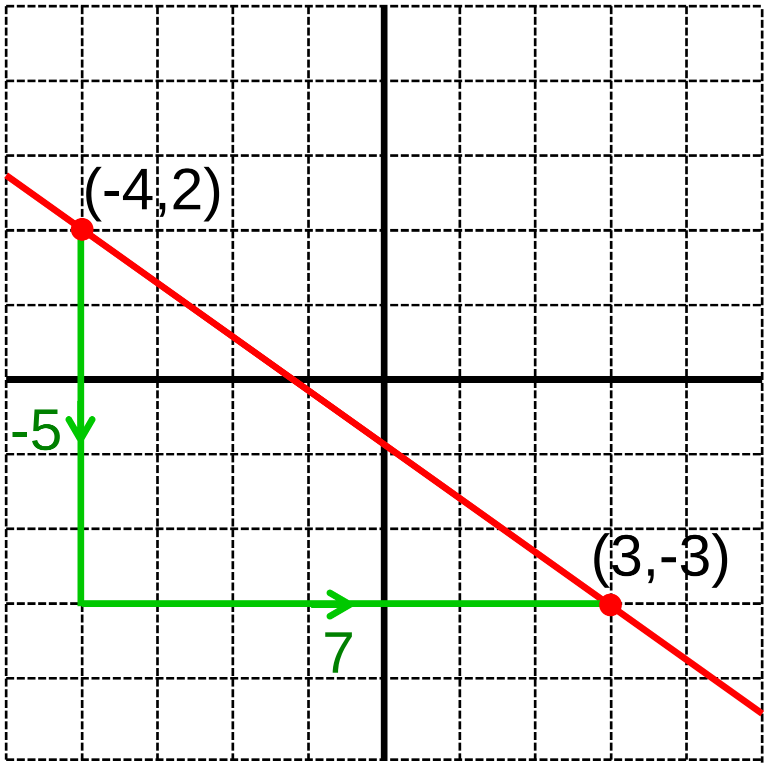
Answer: D

Justification: Look for 2 points on the line that intersect the grid, for example (-4,2) and (3,-3). Using the formula to calculate the slope:

$$(x_1, y_1) = (-4,2), \quad (x_2, y_2) = (3,-3)$$

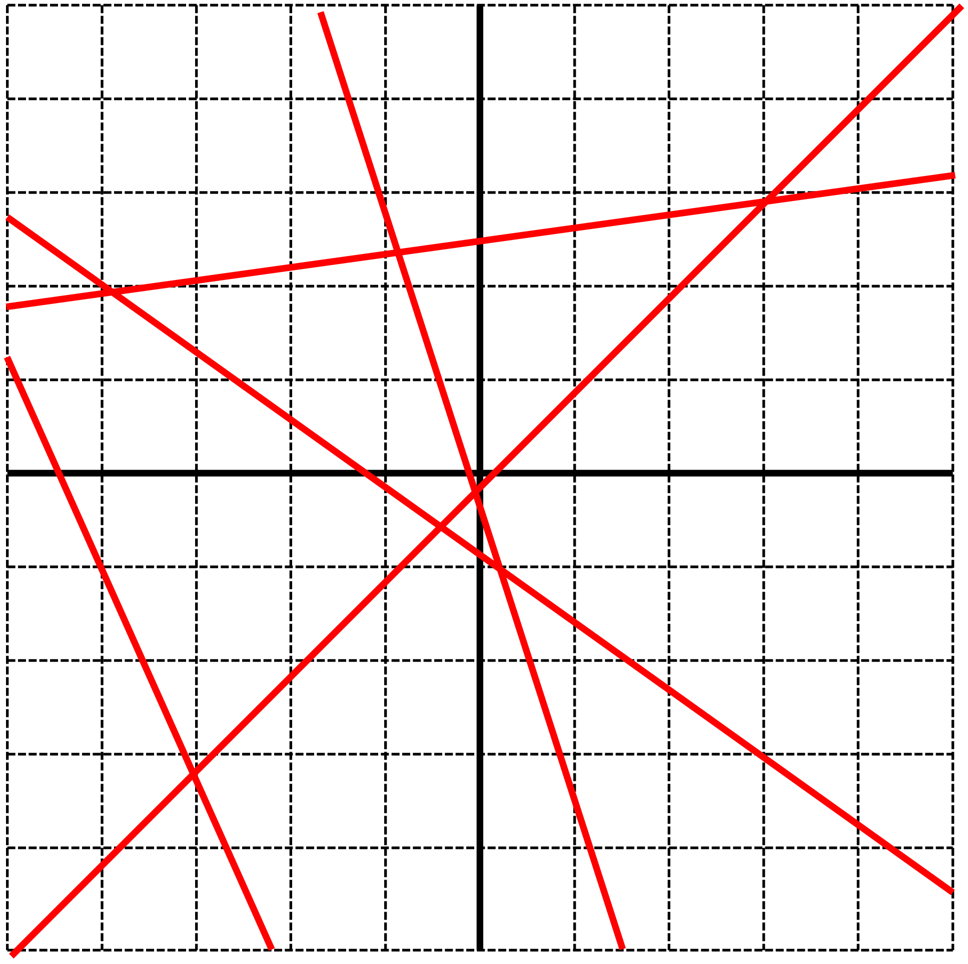
$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-2}{3-(-4)} = \frac{-5}{7}$$

A negative slope implies that point 2 is either down and to the right, or up and to the left of point 1.



Linear Relations VIII

How many of the following lines have a negative slope?



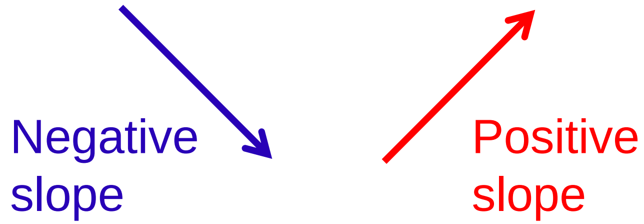
A. 1
B. 2
C. 3
D. 4
E. 5

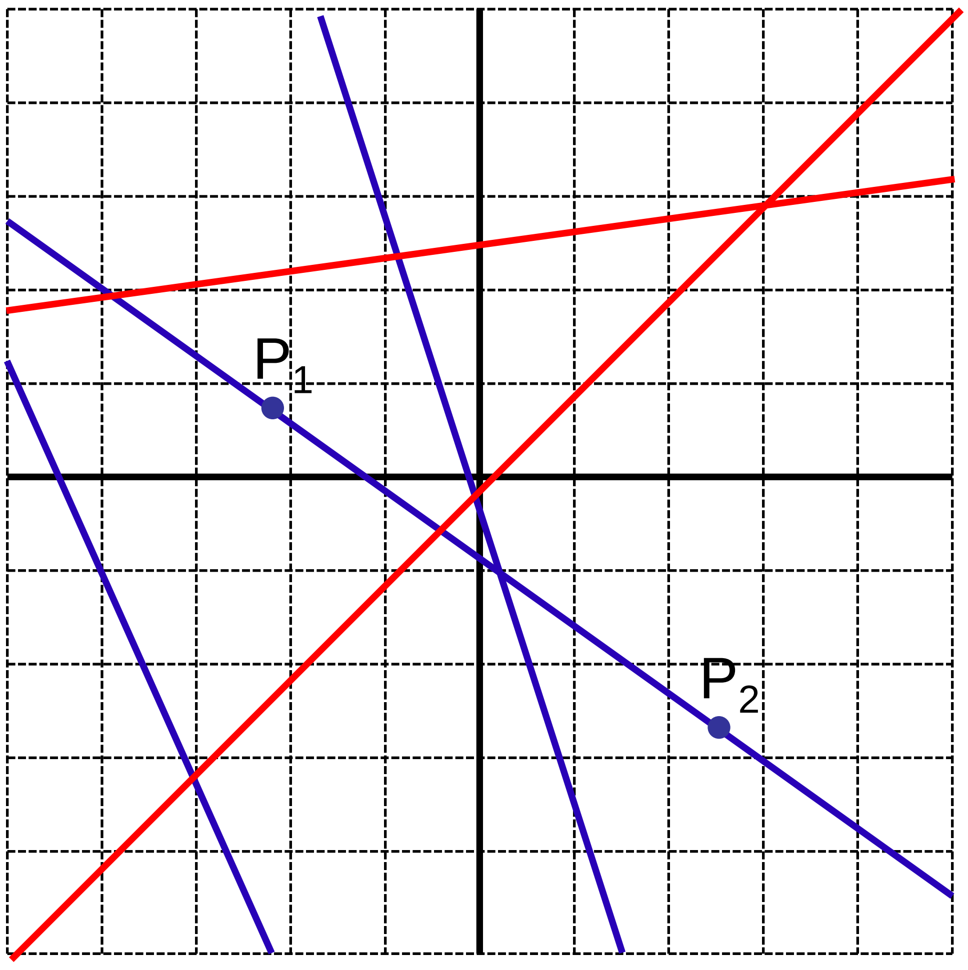
Solution

Answer: C

Justification: The blue lines have a negative slope.

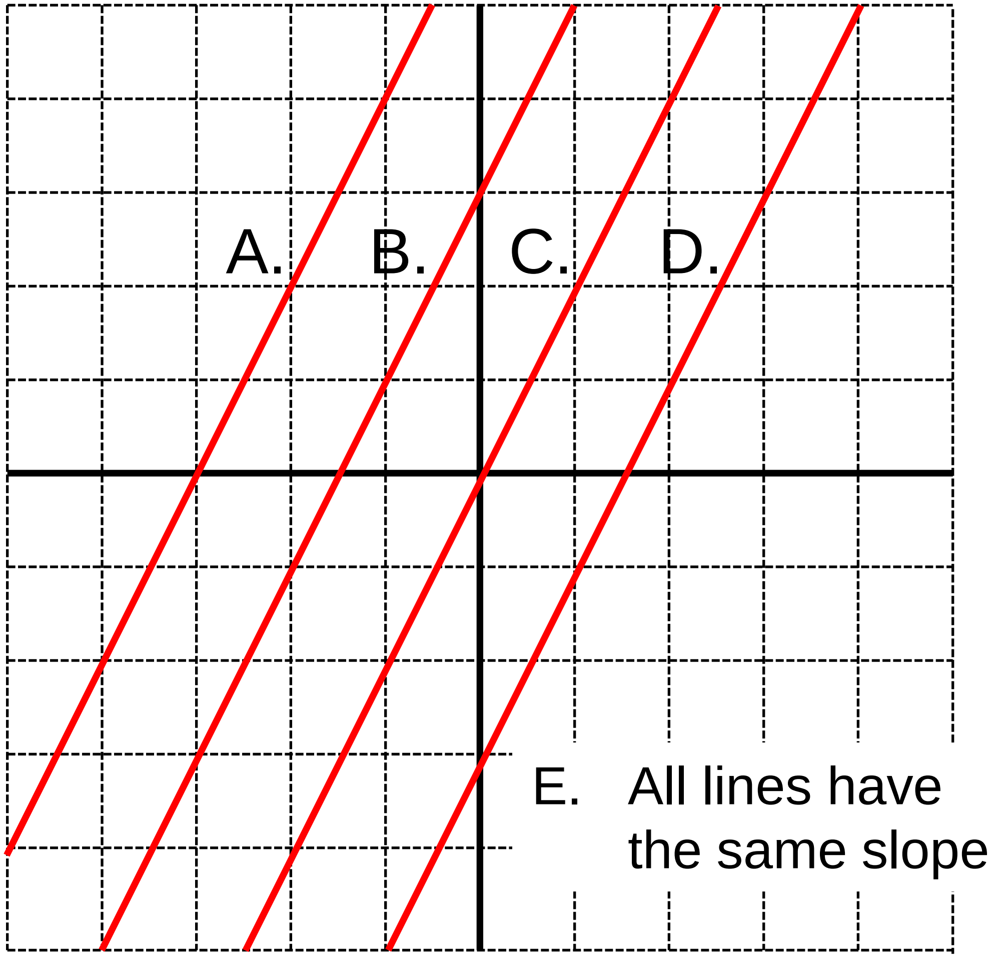
Consider any 2 points on a line, with P_2 the point farther right. If P_2 is lower than P_1, the slope will be negative. This is because Δx is positive but Δy is negative.





Linear Relations IX

Which of the following lines has the largest slope?



A. See graph

B. See graph

C. See graph

D. See graph

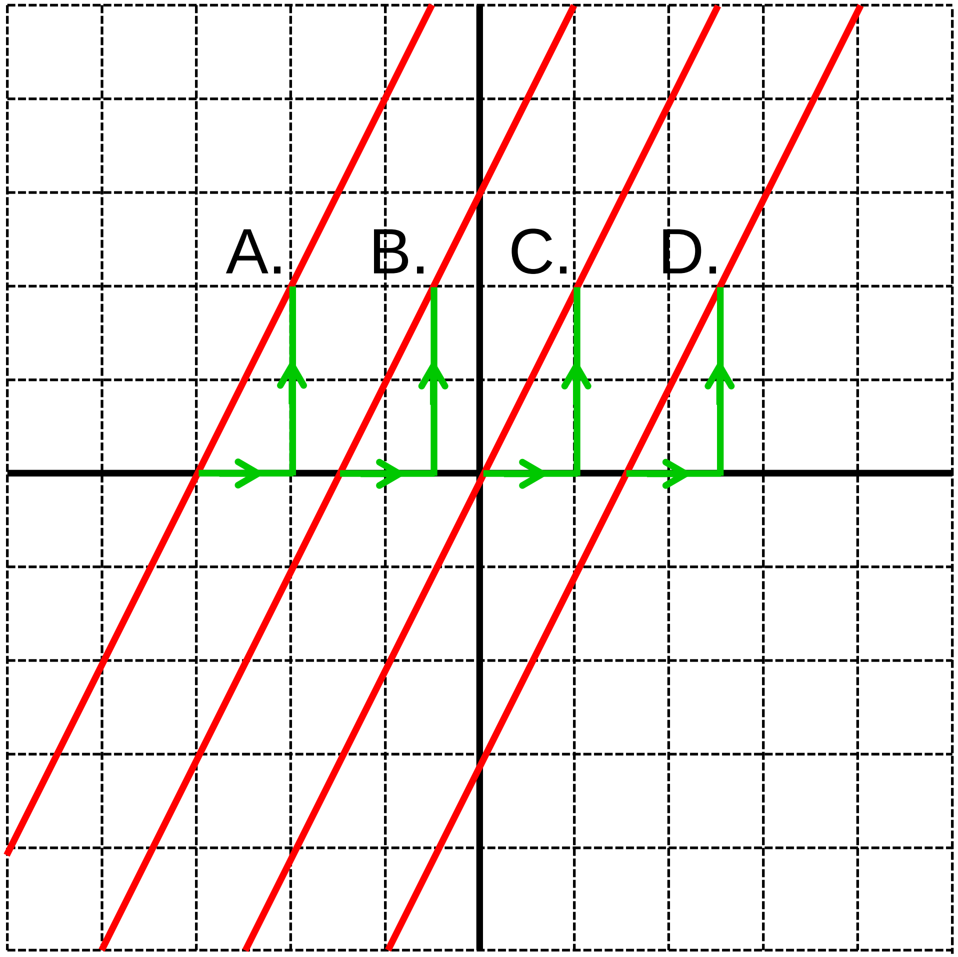
E. All lines have the same slope

Solution

Answer: E

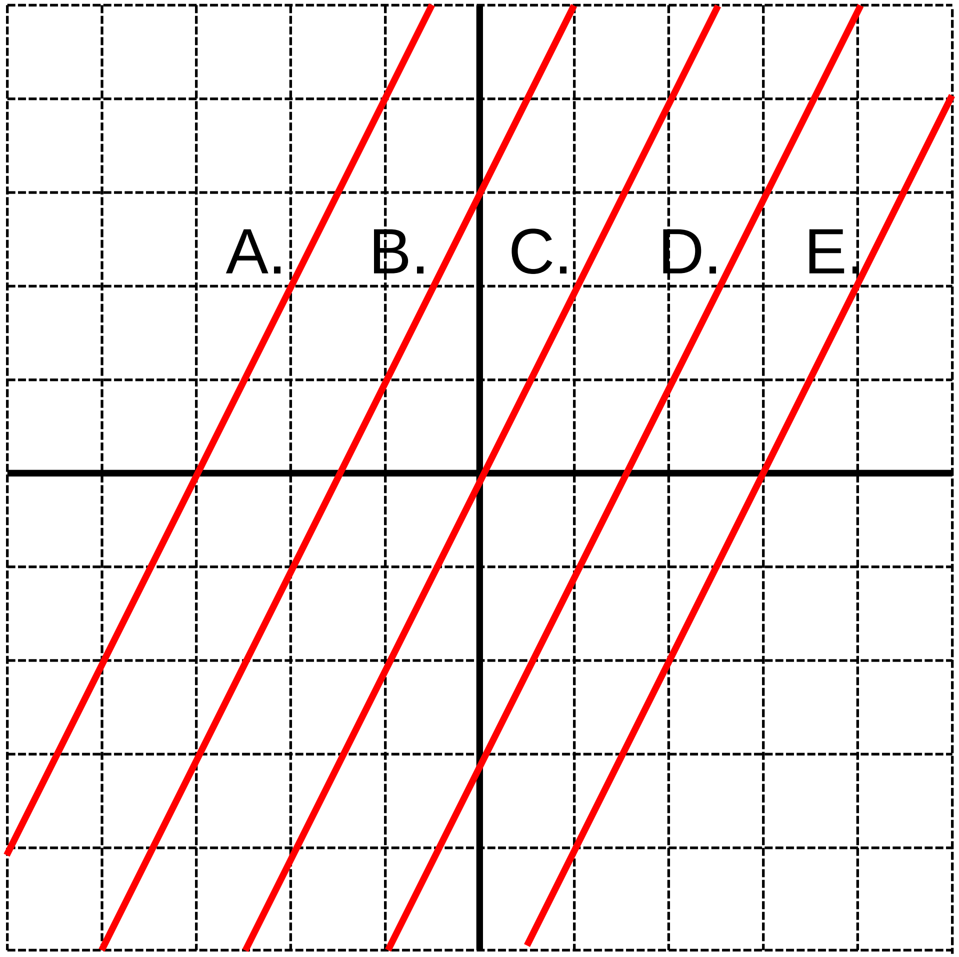
Justification: Every line has a rise of 2 and run of 1. This shows that all lines have the same slope of 2. The position of each line on the graph has no affect on the slope.

From this question we can conclude that knowing the slope of a line cannot distinguish it from every other line.



Linear Relations X

All of the following lines have a slope of 2. Which line corresponds to the equation y = 2x + 3?



Press for hint

Look at where each line intersects the y-axis. At this point, the x-coordinate is 0.

Solution

Answer: B

Justification: Consider where each line intersects the y-axis. At this point, the x-coordinate is 0.

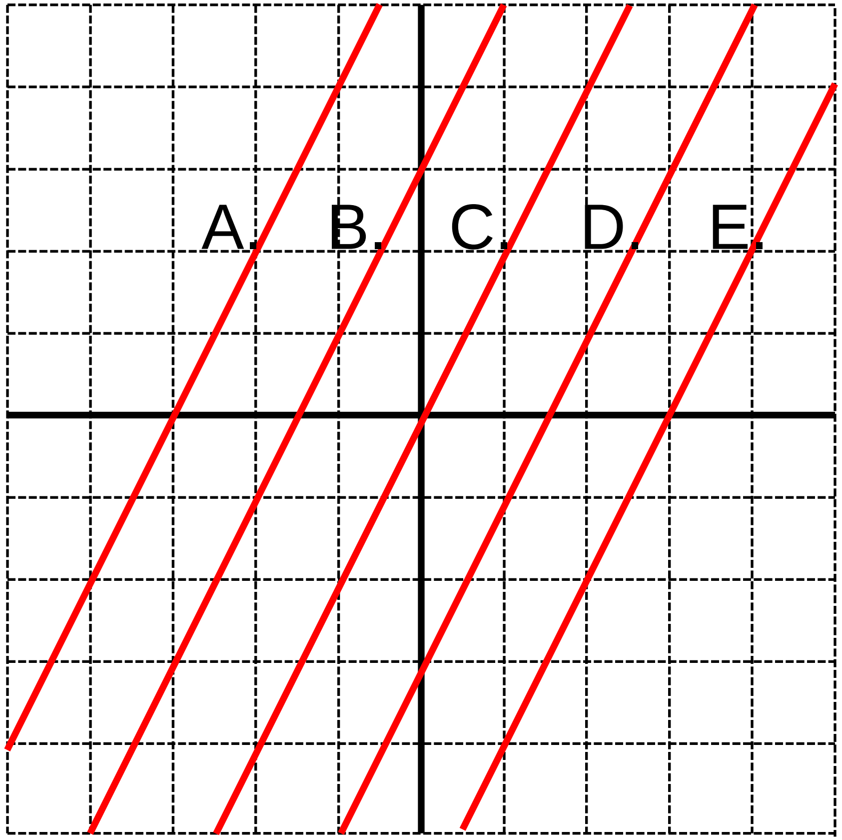
If we let x = 0 in the equation:

$$y = 2x + 3$$

$$y = 2(0) + 3$$

$$y = 3$$

This means that the line representing this equation must have a y-intercept of 3. This corresponds to line B.



Linear Relations XI

Which of the following equations represents the line shown in the graph?

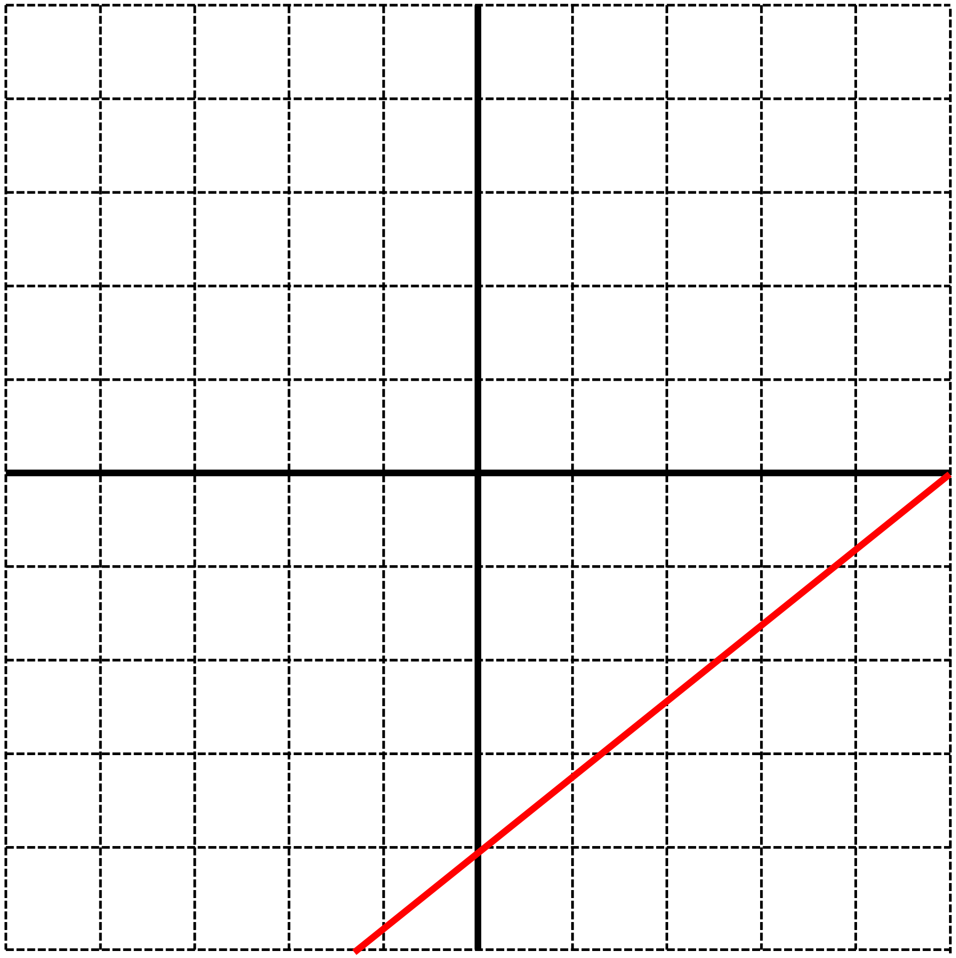
A. $y = \frac{4}{5}x$

B. $y = \frac{4}{5}x + 4$

C. $y = \frac{4}{5}x - 4$

D. $y = \frac{5}{4}x + 4$

E. $y = \frac{5}{4}x - 4$



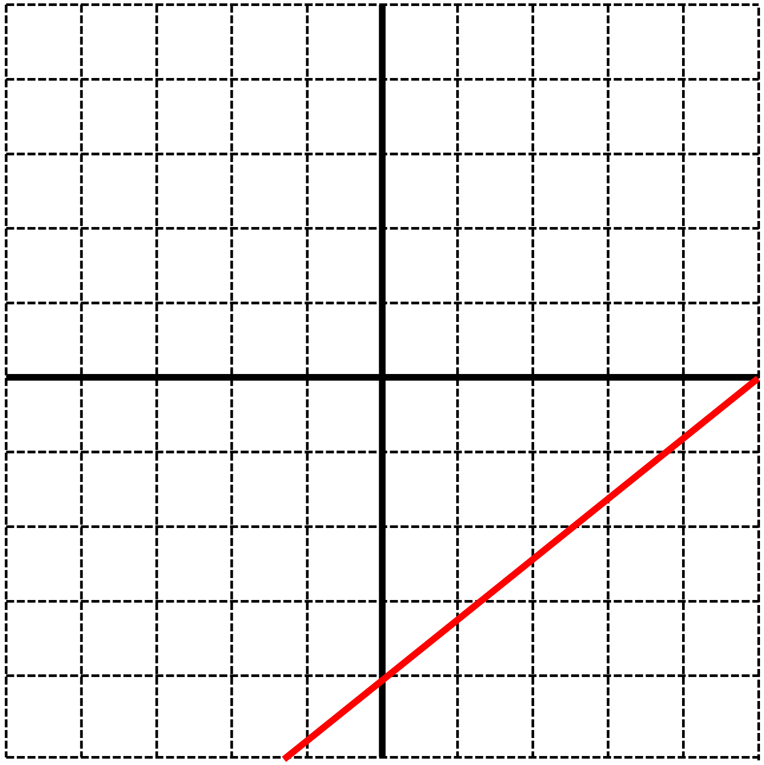
Solution

Answer: C

Justification: Consider the 2 points on the line (0,-4) and (5,0). Every change in x of 4 results in a change in y of 5. The slope is therefore: $\frac{4}{5}$.

The line also intersects the y-axis at -4. This means that if x=0 is plugged into the equation, the results must be y=-4. The correct equation is therefore:

$$y = \frac{4}{5}x - 4$$



Linear Relations XII

Which of the following equations represents the line shown in the graph?

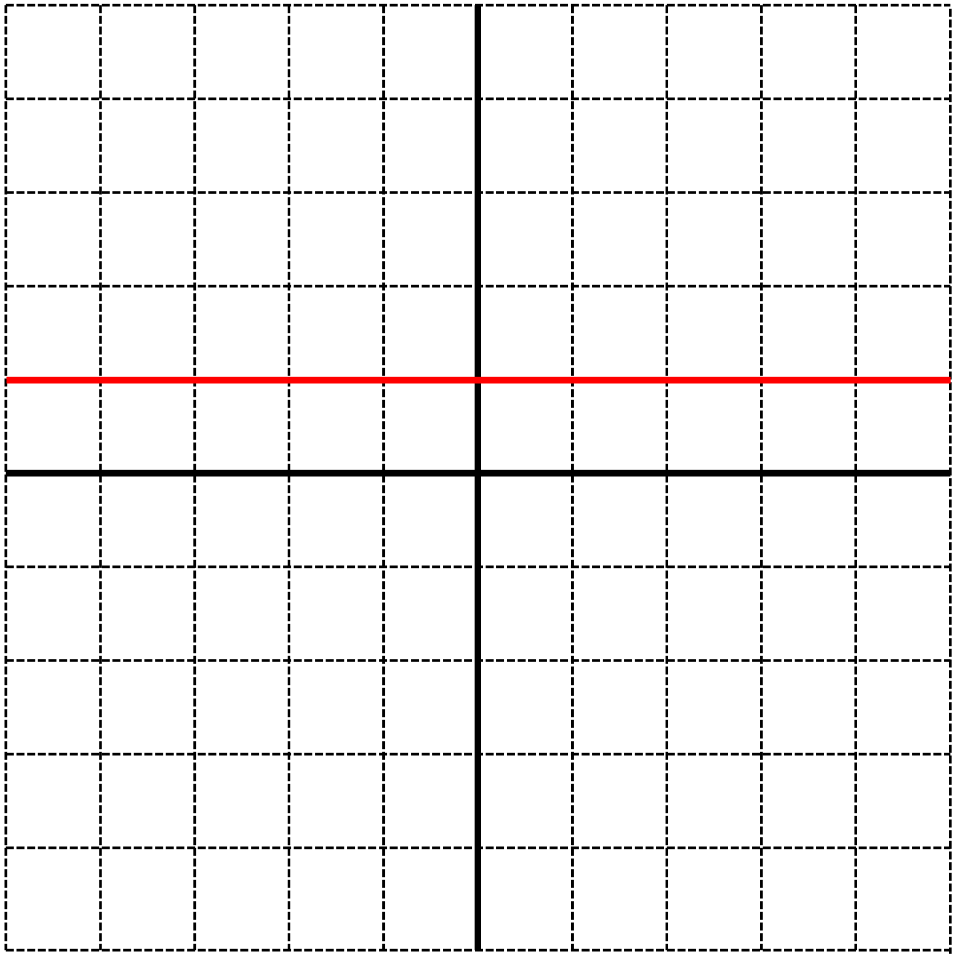
A. $y = x + 1$

B. $y = -x + 1$

C. $y = 1$

D. $x = 1$

E. None of the above



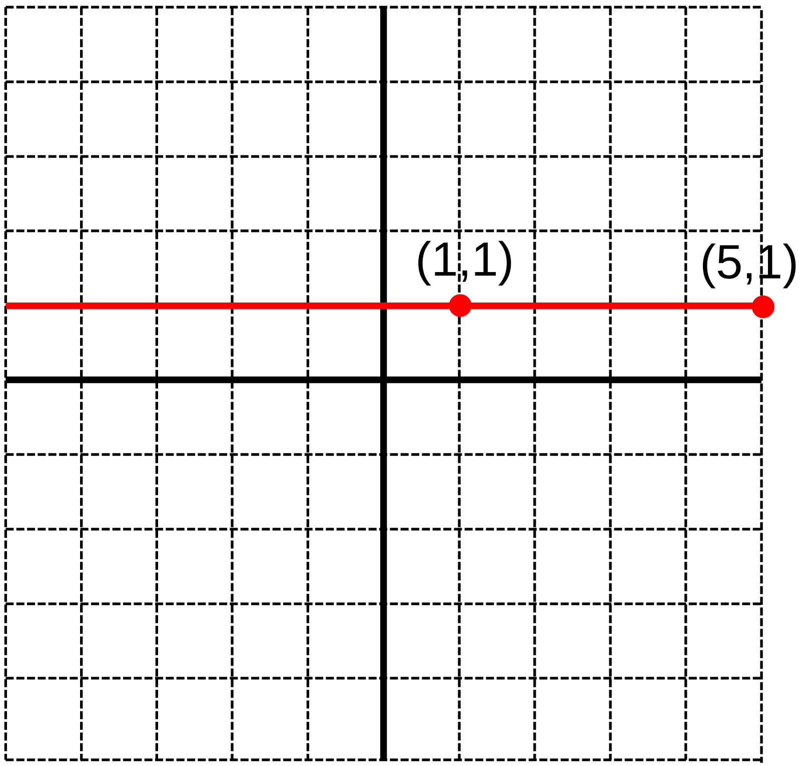
Solution

Answer: C

Justification: For every value x, y = 1. Since the equation does not depend on x, the equation is y = 1.

The slope of this line is 0. This is because there is no change in y (there is no rise), no matter what the change in x is.

Consider the points (1,1) and (5,1):

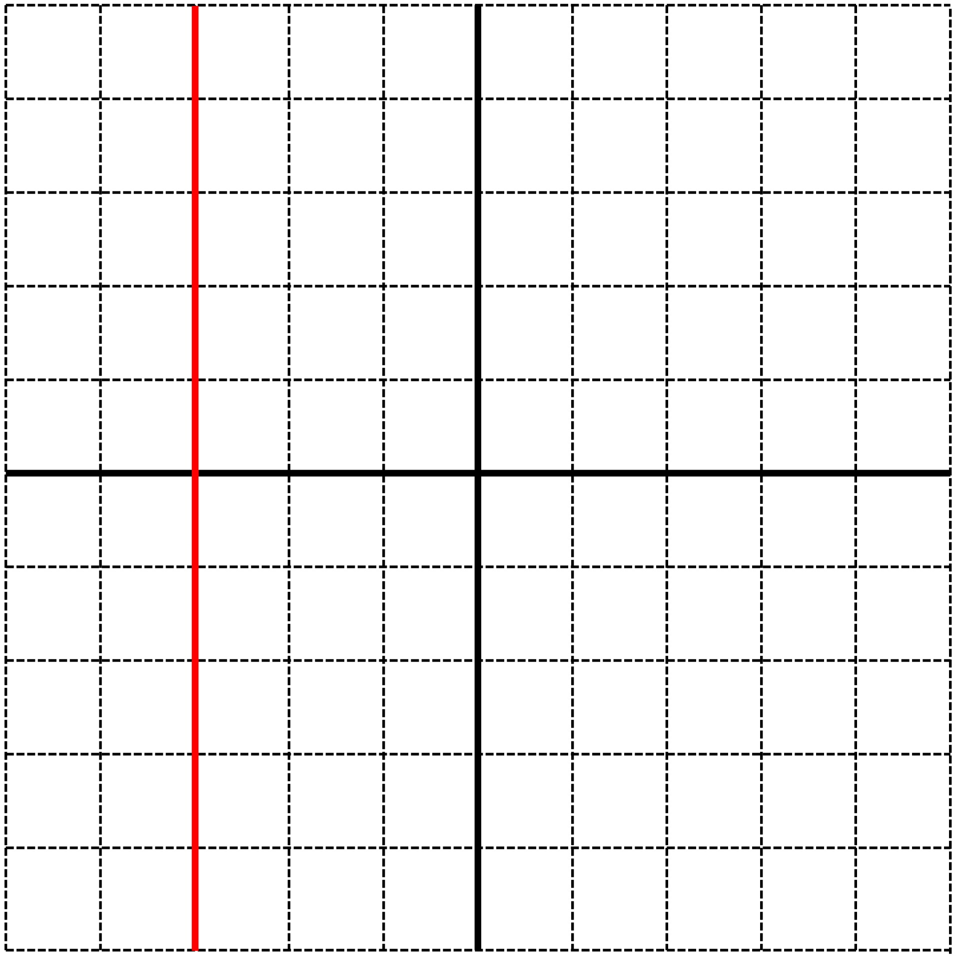


$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-1}{5-1} = 0$$

Linear Relations XIII

Which of the following equations represents the line shown in the graph?

A. $y = x - 3$
B. $x = y - 3$
C. $y = -3$
D. $x = -3$
E. None of the above



Solution

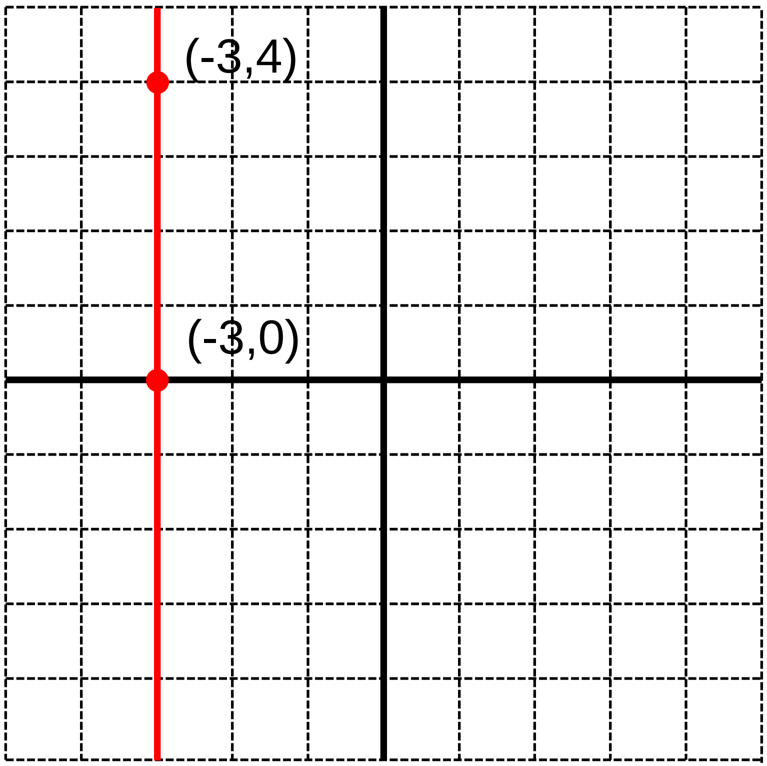
Answer: D

Justification: The x value of every point on the line is -3. Since x does not depend on y, the equation of the line is x = -3.

The slope of this line is undefined. Since the run (Δx) is always 0, the formula for the slope has 0 in the denominator.

Consider the points (-3,4) and (-3,0):

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{-3-(-3)} = \frac{4}{0}$$



Linear Relations Summary

Slope of a line:

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

General equation of a line (slope-intercept form):

$$y = mx + b$$

where b is the y-intercept of the line