a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

# Mathematics
## Parabolas

Science and Mathematics
Education Research Group

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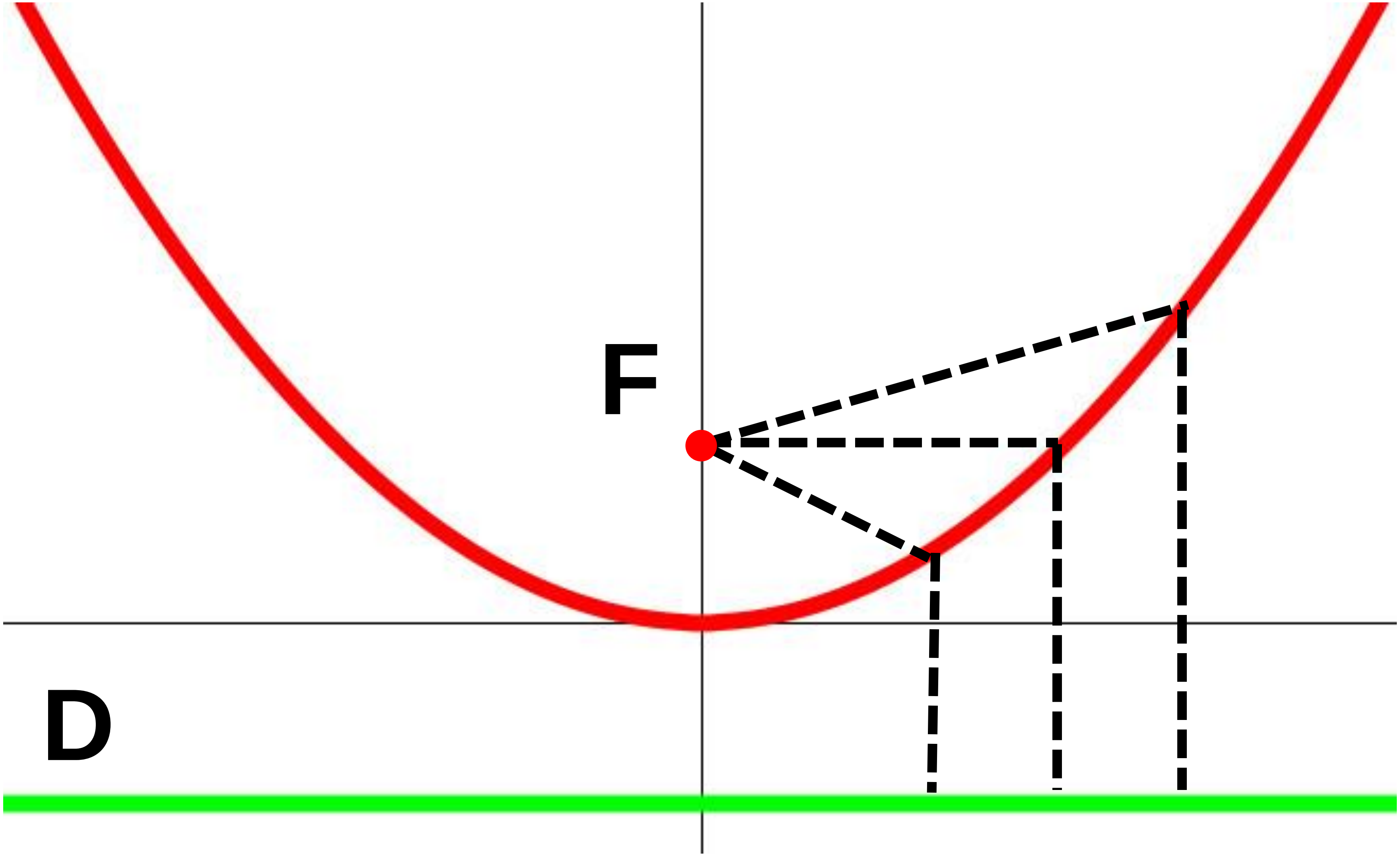
# Parabolas



# Parabolas I

The graph of $y = x^2$ is shown in red. The graph is then shifted 2 units right and 3 units down.

Which one of the following equations represents the translated green graph?

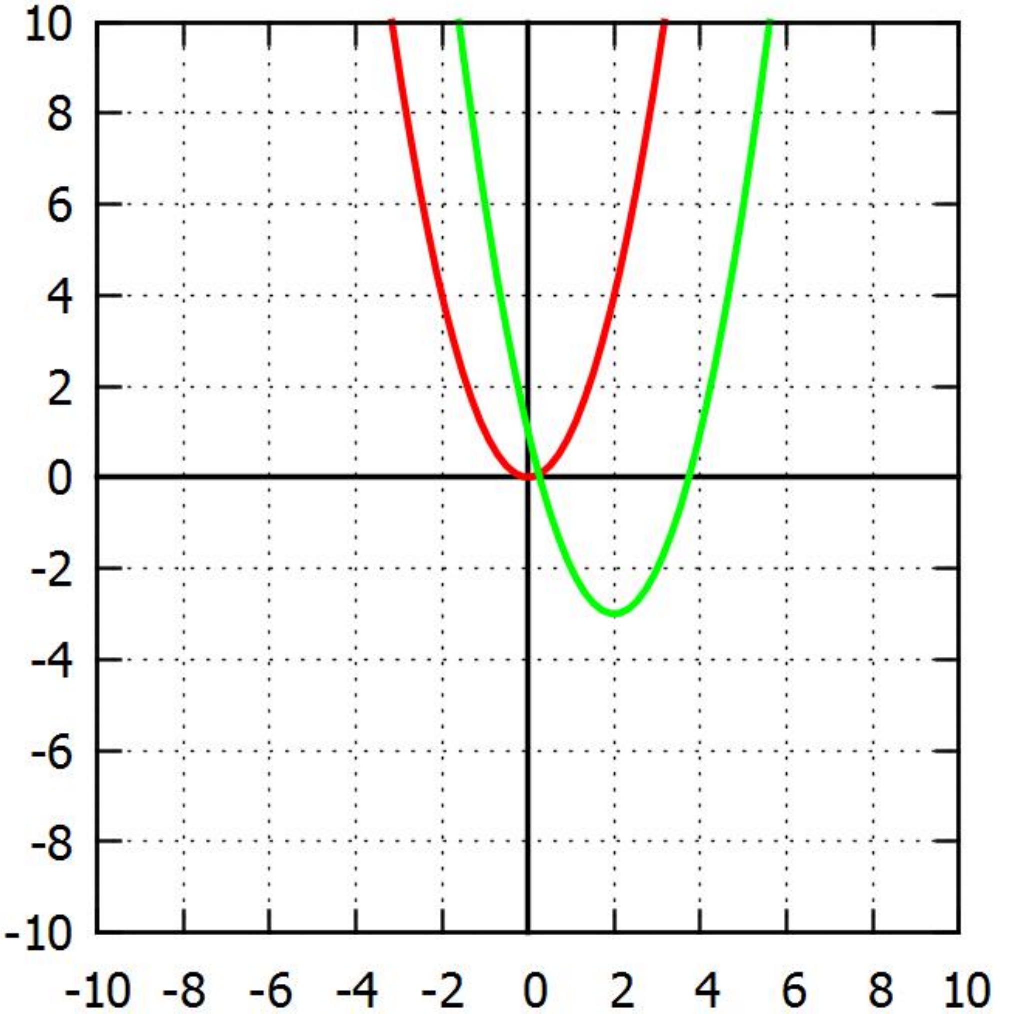
A. $y = (x+2)^2 + 3$

B. $y = (x+2)^2 - 3$

C. $y = (x-2)^2 + 3$

D. $y = (x-2)^2 - 3$

E. $y = x^2 - 1$



# Solution

**Answer:** D

**Justification:** The graph of y = $x^2$ is shifted to the right 2 units by replacing x with (x – 2). It is shifted down 3 units by replacing y with (y + 3). $y = x^2$

$$y + 3 = (x - 2)^2 \quad \textit{shifts graph } 2 \longrightarrow, 3 \downarrow$$

$$y = (x - 2)^2 - 3$$

The vertex of y = a$x^2$ is (0, 0). After shifting the graph, the vertex becomes (2, 3). In general, the vertex of the equation

$y = a(x - p)^2 + q$ is ($p$, $q$).

# Parabolas II

What is the vertex of the graph of

$$y = -2\left(x + \frac{8}{3}\right)^2 + \frac{15}{2}?$$

Is the vertex a maximum or minimum point?

A. $\left(\frac{8}{3}, \frac{15}{2}\right)$, maximum

B. $\left(\frac{8}{3}, \frac{15}{2}\right)$, minimum

C. $\left(-\frac{8}{3}, \frac{15}{2}\right)$, maximum

D. $\left(-\frac{8}{3}, \frac{15}{2}\right)$, minimum

E. $\left(-\frac{16}{3}, -15\right)$, maximum

# Solution

**Answer:** C

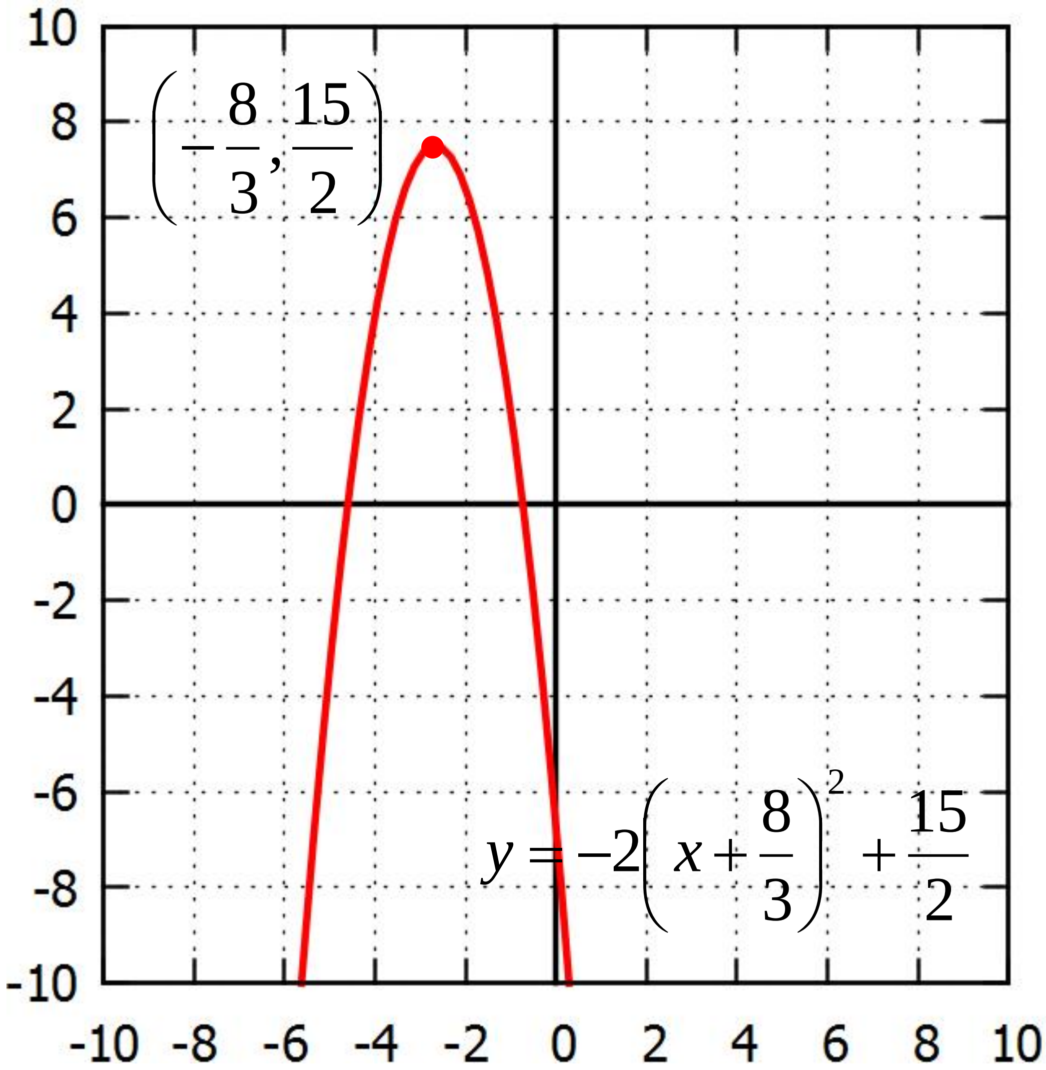
**Justification:** Recall that the vertex of $y = a(x-p)^2 + q$ is ($p$, $q$). Therefore:

$$\text{Vertex} = \left(-\frac{8}{3}, \frac{15}{2}\right)$$

The vertex is a maximum because $a = -2 < 0$ in

$$y = a(x-p)^2 + q$$

Since $a(x-p)^2$ is always negative in this case, the graph will open downwards.



# Parabolas III

The red line shows the graph of $y = x^2$. All the other lines are in the form: $y = ax^2$

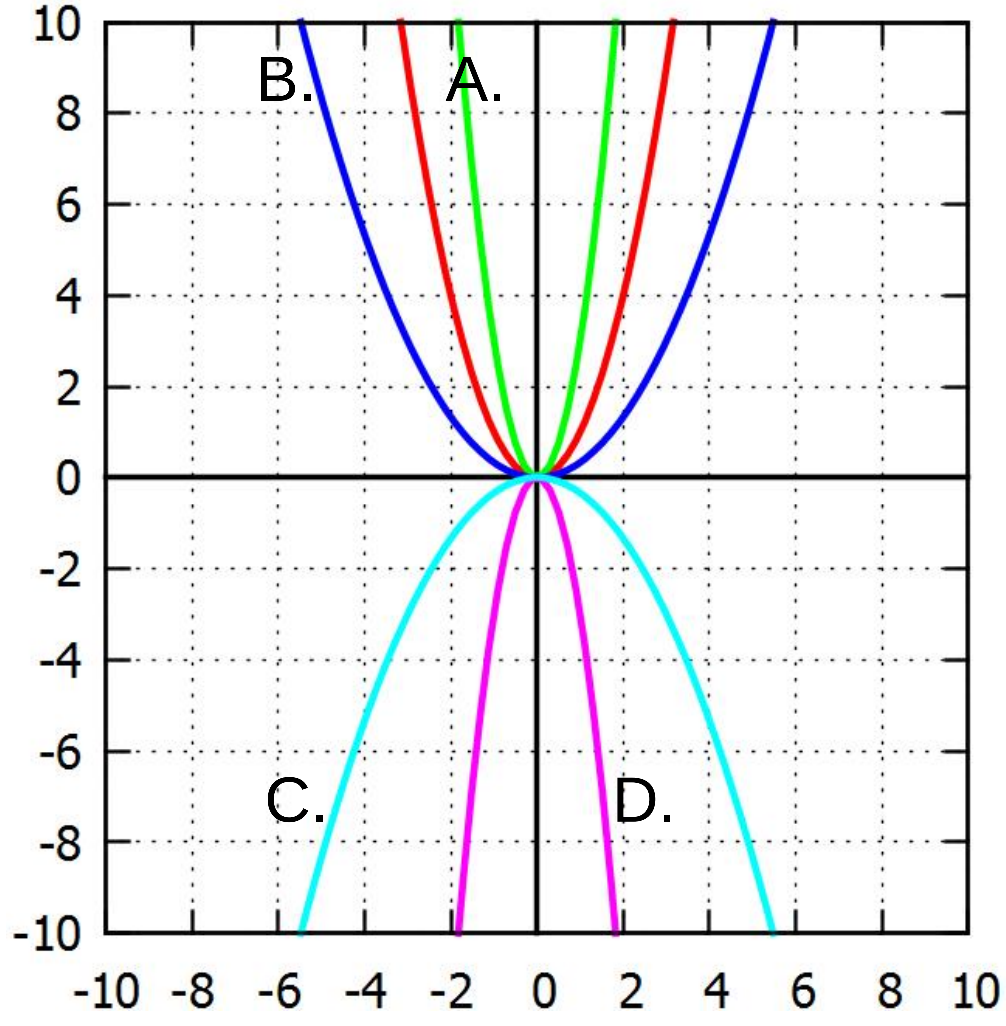
In which one of the graphs is $0 < a < 1$ ?

A. Green graph

B. Blue graph

C. Cyan graph

D. Purple graph



# Solution

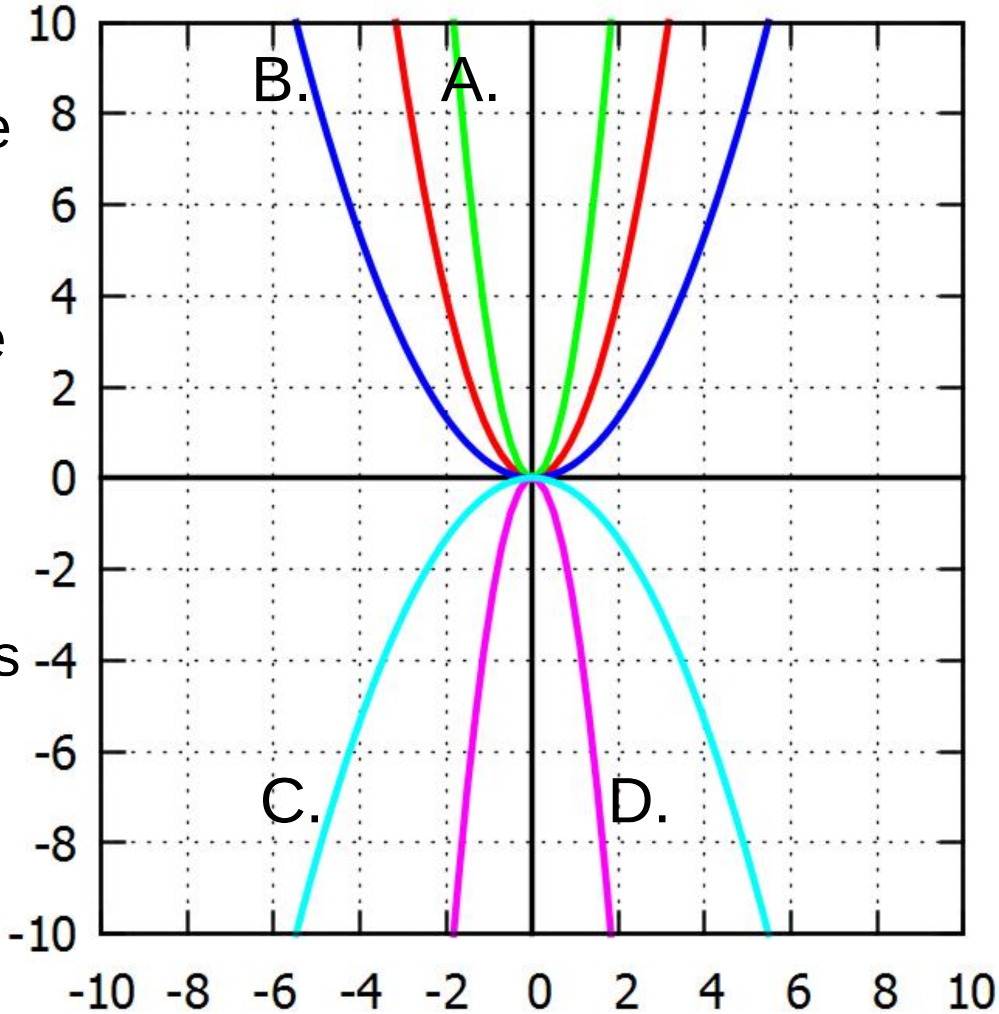
**Answer:** B

**Justification:** When $0 < a < 1$, the graph of $y = ax^2$ is always positive so it must lie above (or on) the x-axis. Therefore graph C and D are incorrect.

Since $a < 1$, the graph we are looking for must lie below the red graph because its y-values are less than $y = x^2$.

The correct answer is graph B:

$$y = \frac{1}{3}x^2$$



# Parabolas IV

Which one of the following is true about *a* and *q* if the equation of the given parabola is written in the form

$$y = a(x-p)^2 + q \text{ ?}$$

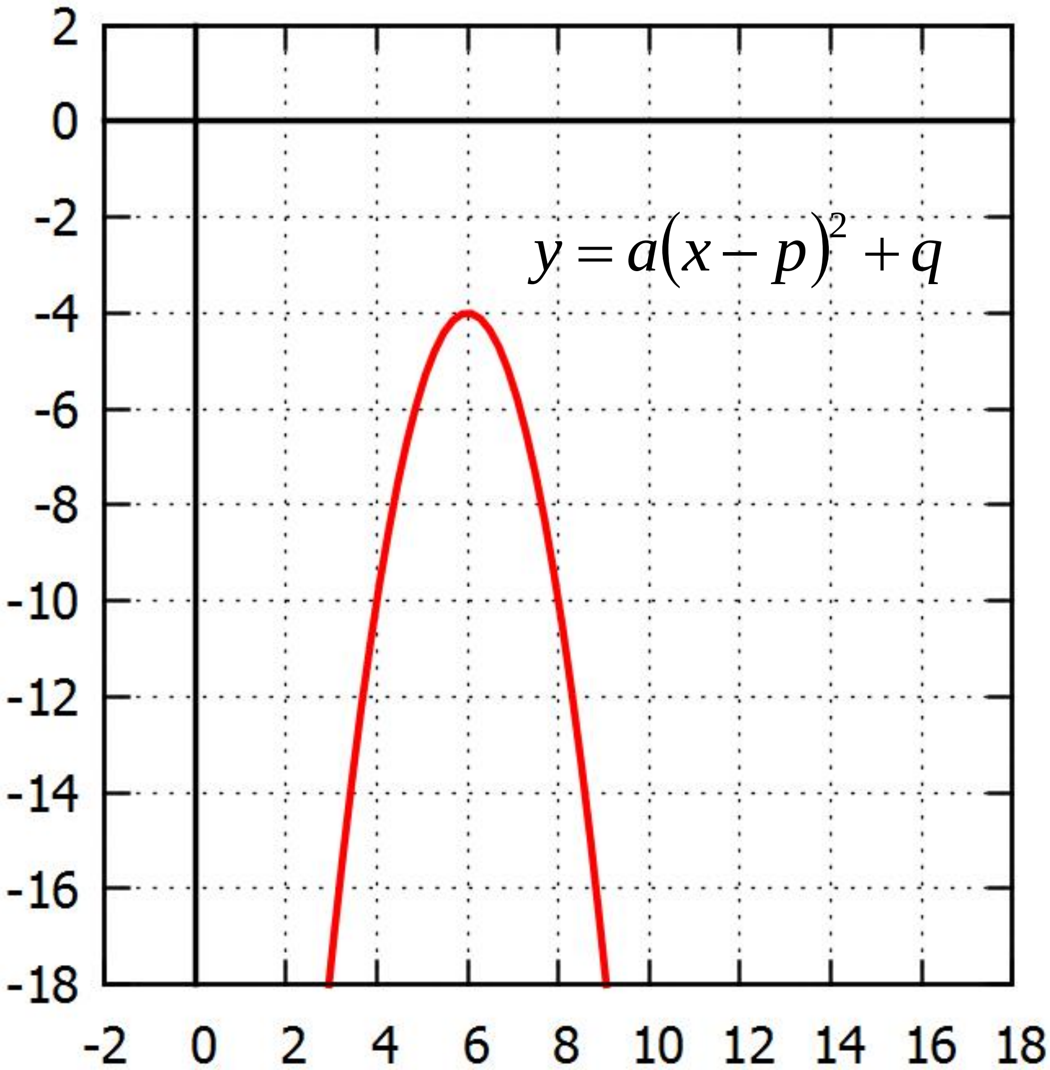
A. $a > 0, \quad q > 0$

B. $a < 0, \quad q > 0$

C. $a > 0, \quad q < 0$

D. $a < 0, \quad q < 0$

E. $a < 0, \quad q = 0$



# Solution

**Answer:** D

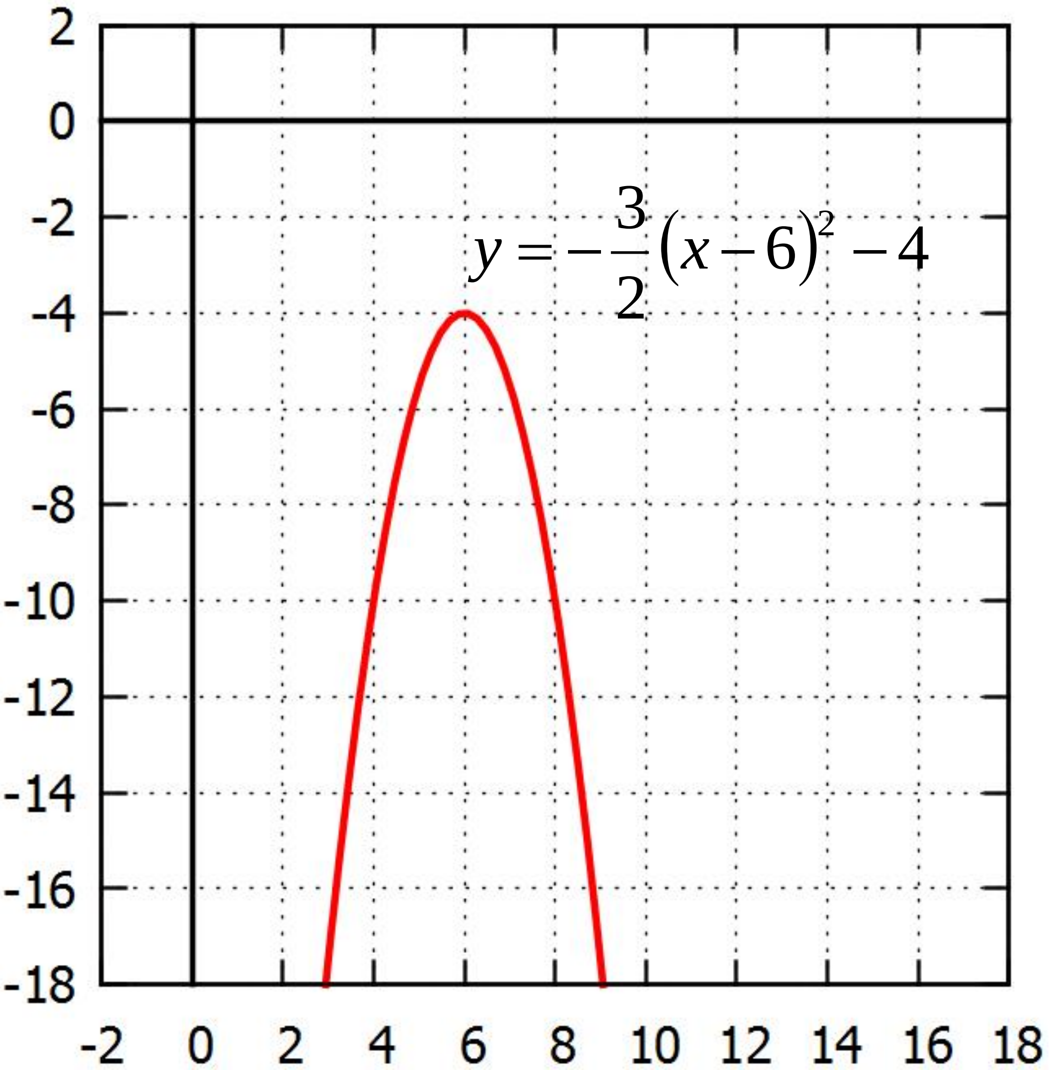
**Justification:** The vertex of the graph is at (6, -4). This shows that

$$q = -4 < 0$$

$$p = 6$$

This point is a maximum point because the parabolas grows to negative infinity (it opens downwards). Therefore

$$a < 0$$



# Parabolas V

How many solutions are there to the equation

$$0 = -\frac{3}{2}(x-6)^2 - 4 \text{ ?}$$

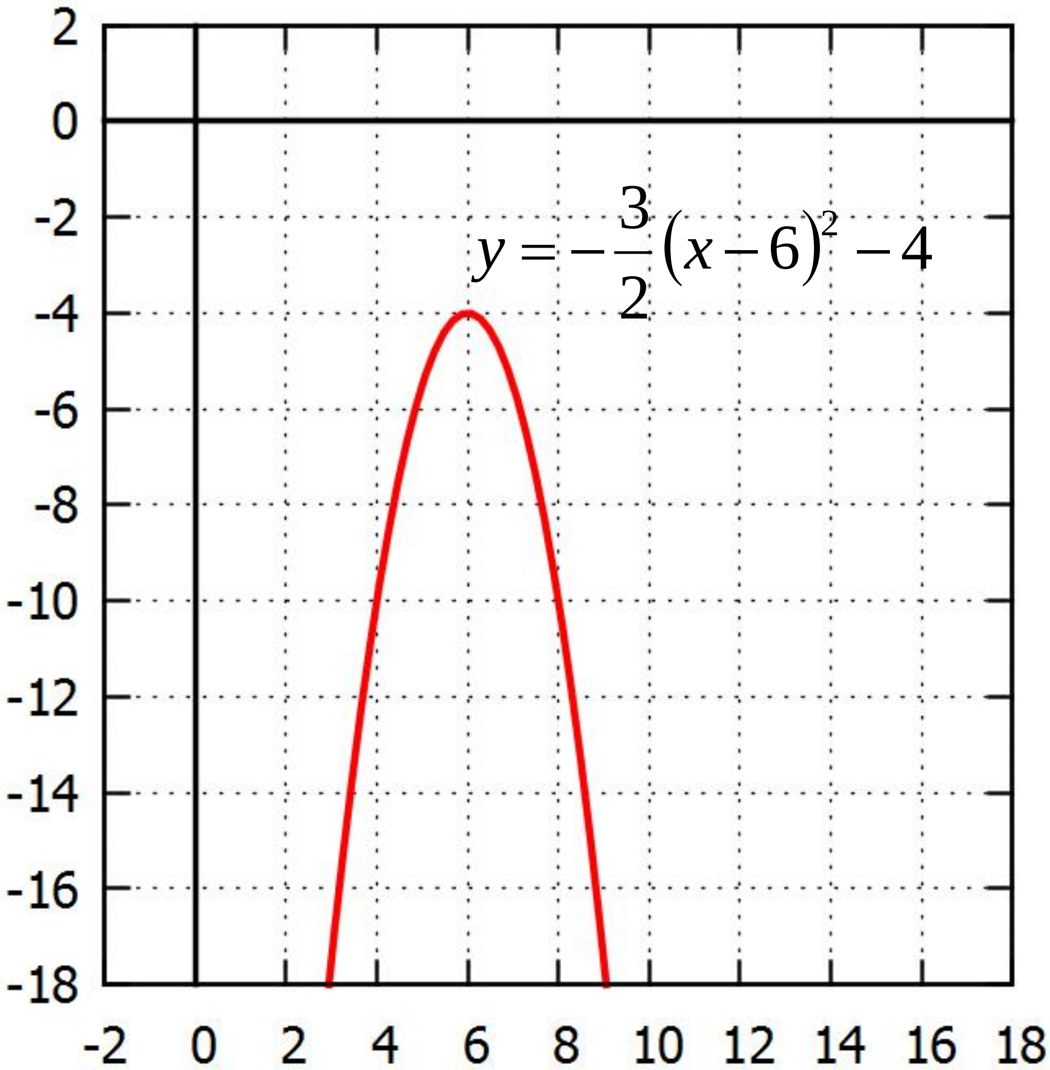
A. No solutions

B. 1 solution

C. 2 solutions

D. Infinite solutions

E. Cannot be determined



# Solution

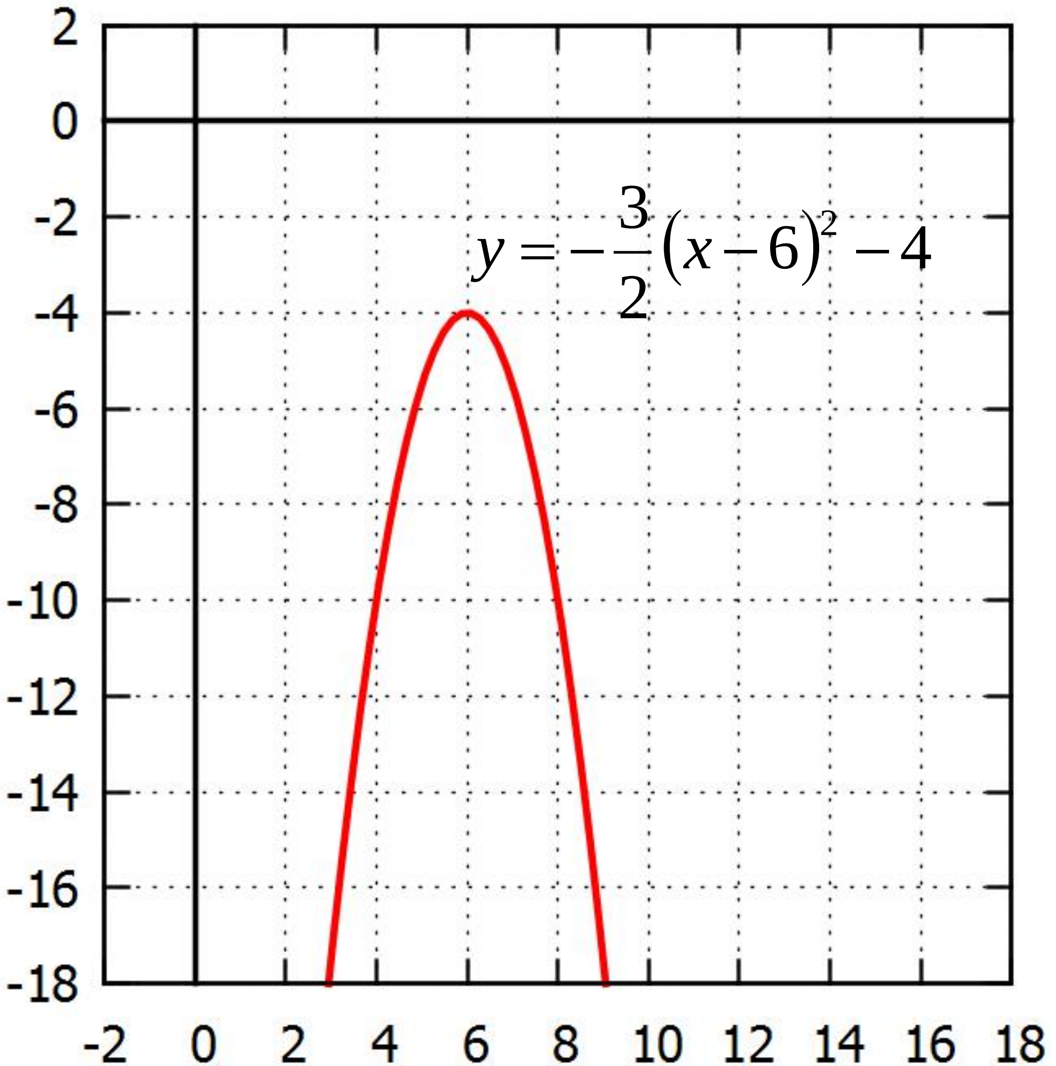
**Answer:** A

**Justification:** The parabola never crosses the x-axis, which means it has no zeroes.

Therefore

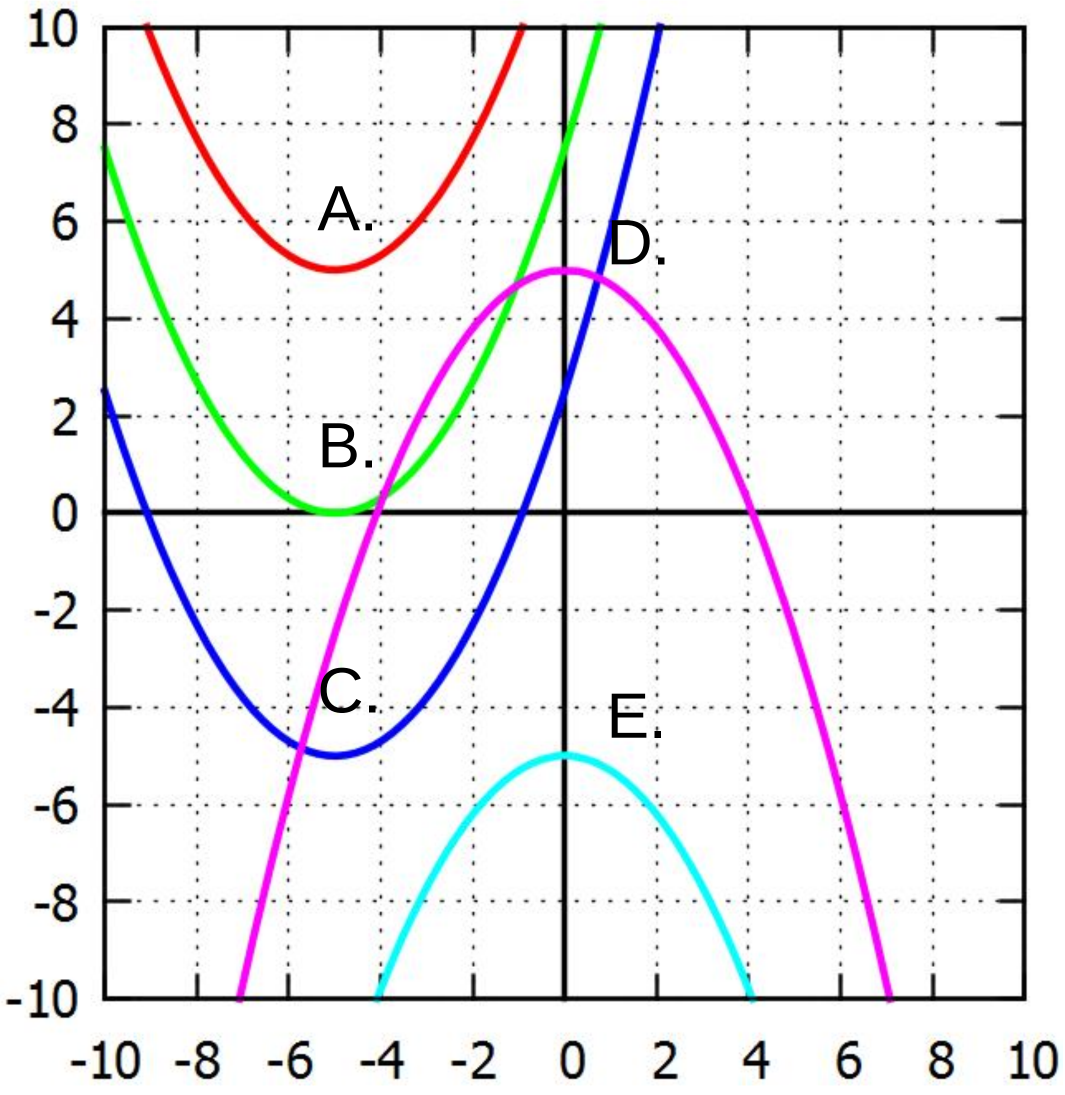
$$0 = -\frac{3}{2}(x-6)^2 - 4$$

has no solutions.



# Parabolas VI

Which one of the graphs shown has 1 unique zero?




A. Red graph
B. Green graph
C. Blue graph
D. Purple graph
E. Cyan graph

# Solution

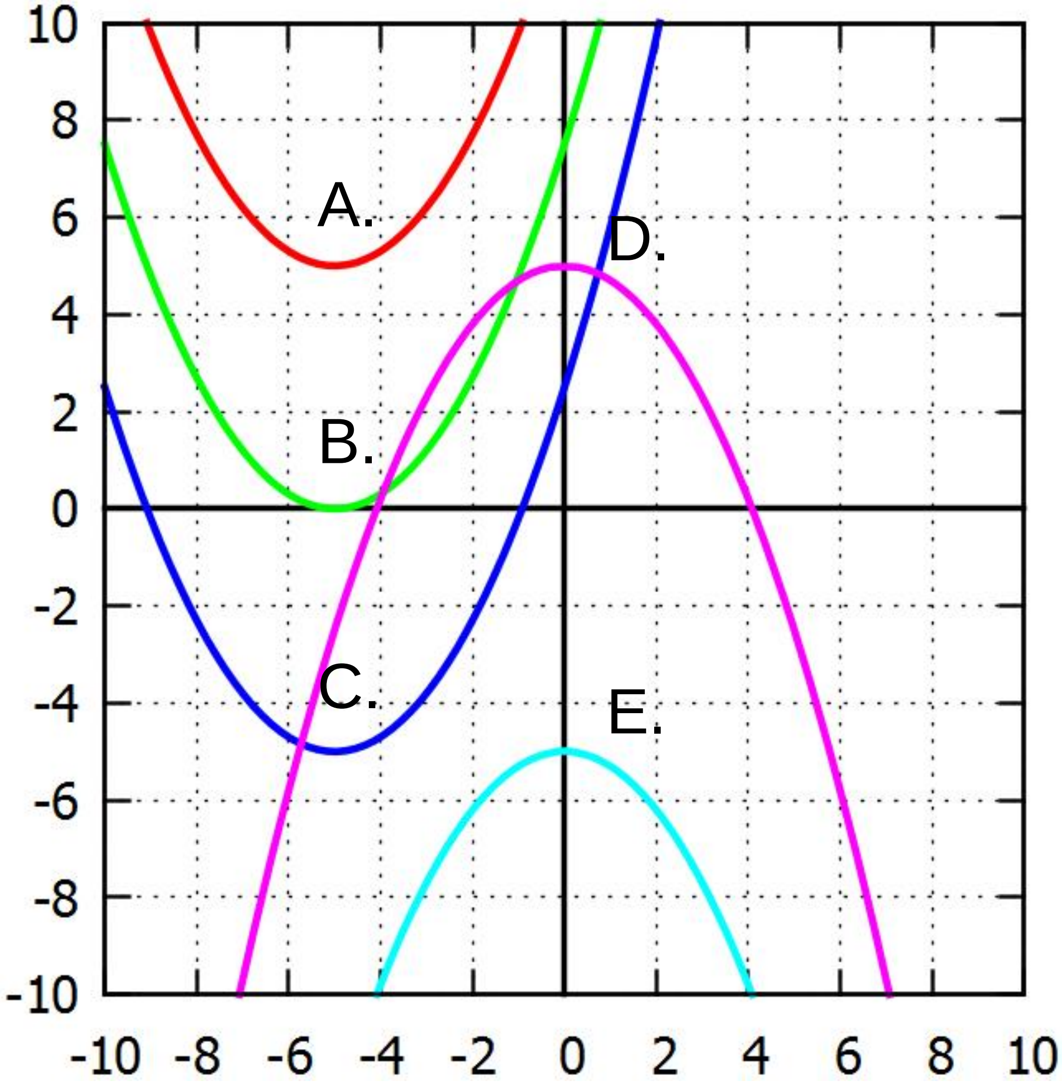
**Answer:** B (Green)

**Justification:** The vertex of the green graph is located at the point (-5,0). Only when x = -5 does y = 0, so it has 1 zero.

Quadratics in the form

$$y = a(x - p)^2$$

have 1 zero. The vertices of these quadratics lie on the x-axis at ($p$,0).



# Parabolas VII

Consider a parabola with a vertex at (2, 5) and one x-intercept at $\left(-\frac{2}{7}, 0\right)$.

What are the coordinates of the other x-intercept?

A. $\left(\frac{2}{7},0\right)$

B. $\left(\frac{16}{7},0\right)$

C. $(4,0)$

D. $\left(\frac{30}{7},0\right)$

E. $\left(\frac{32}{7}, 0\right)$

# Solution

**Answer:** D

**Justification:** The parabola is symmetric around the line x = 2.

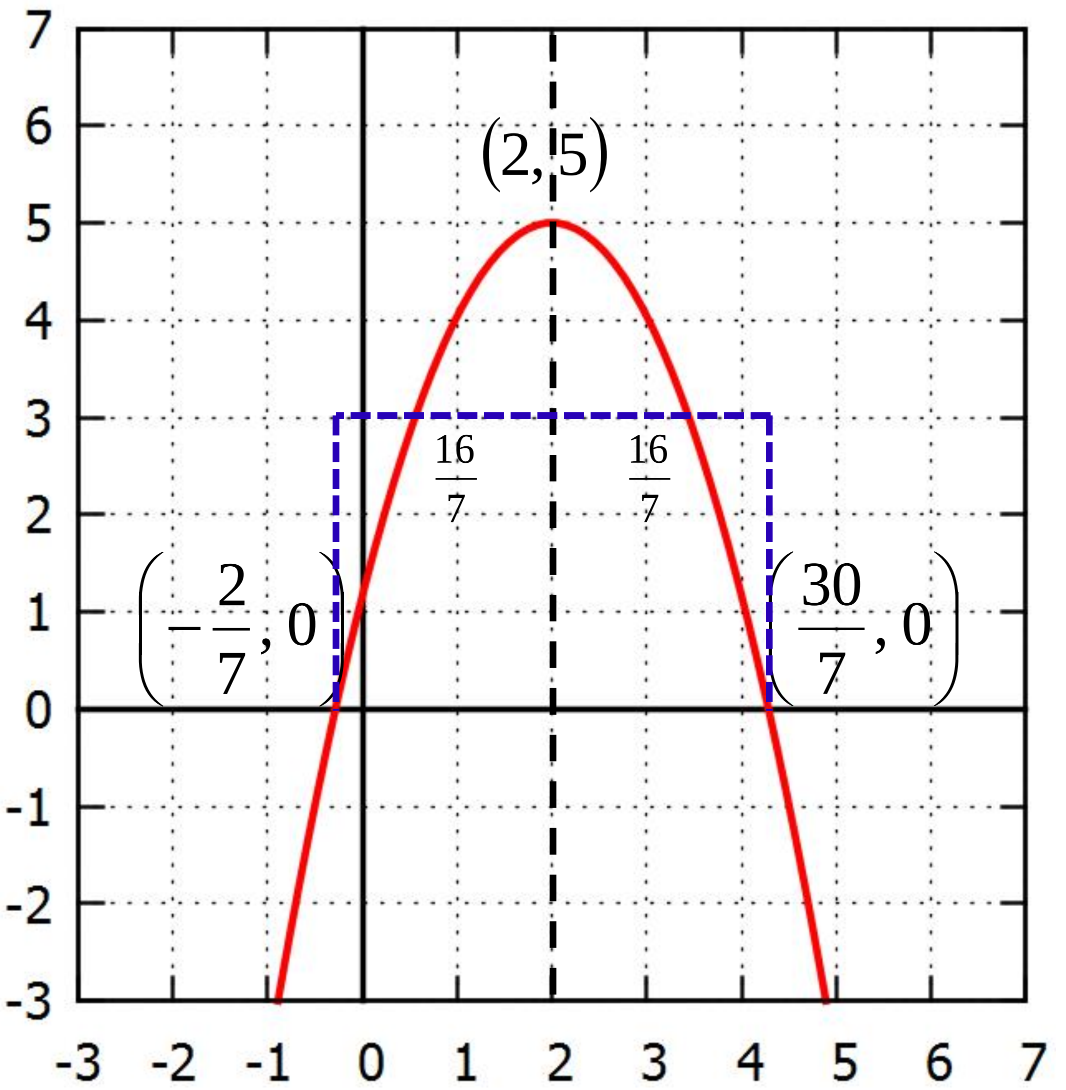
The two x-intercepts should therefore be the same distance from the line x = 2.

Distance from x = 2:

$$2-\left(-\frac{2}{7}\right)=\frac{16}{7}$$

Other x-intercept:

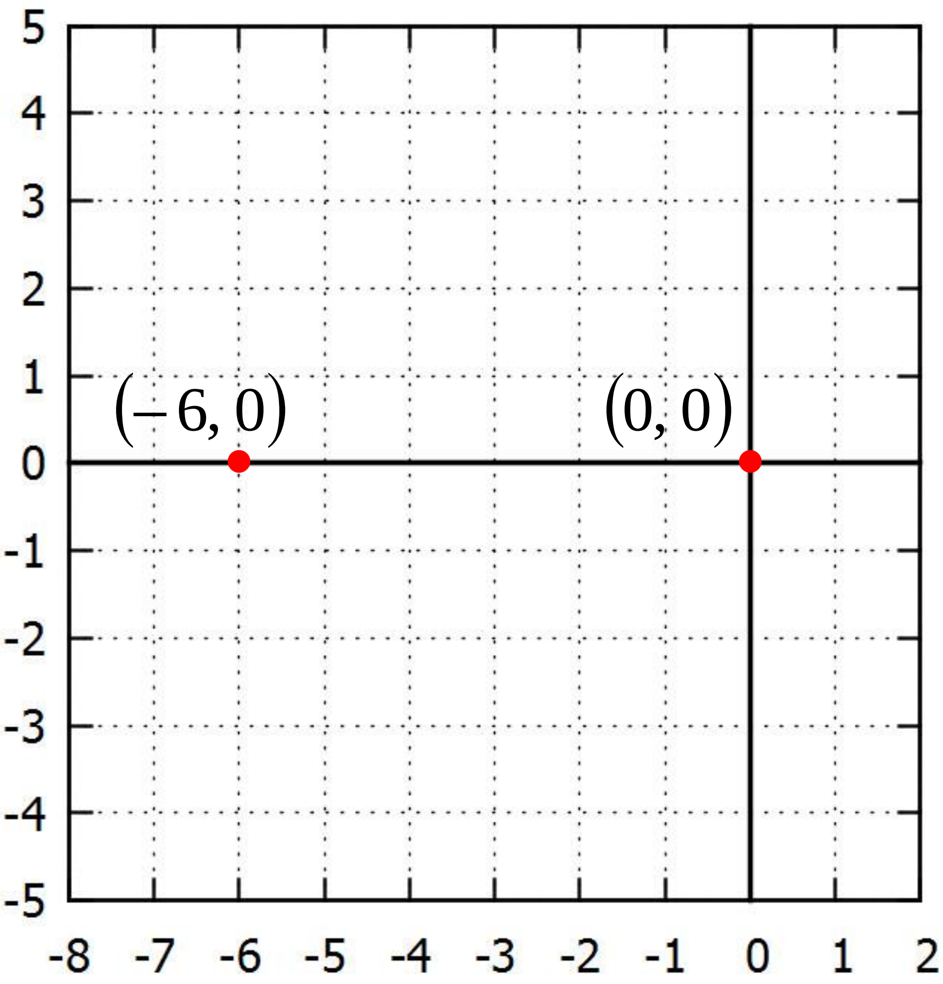
$$2+\frac{16}{7}=\frac{30}{7}$$



# Parabolas VIII

Consider a parabola with x-intercepts at (0,0) and (-6,0).

What are the coordinates of its vertex?



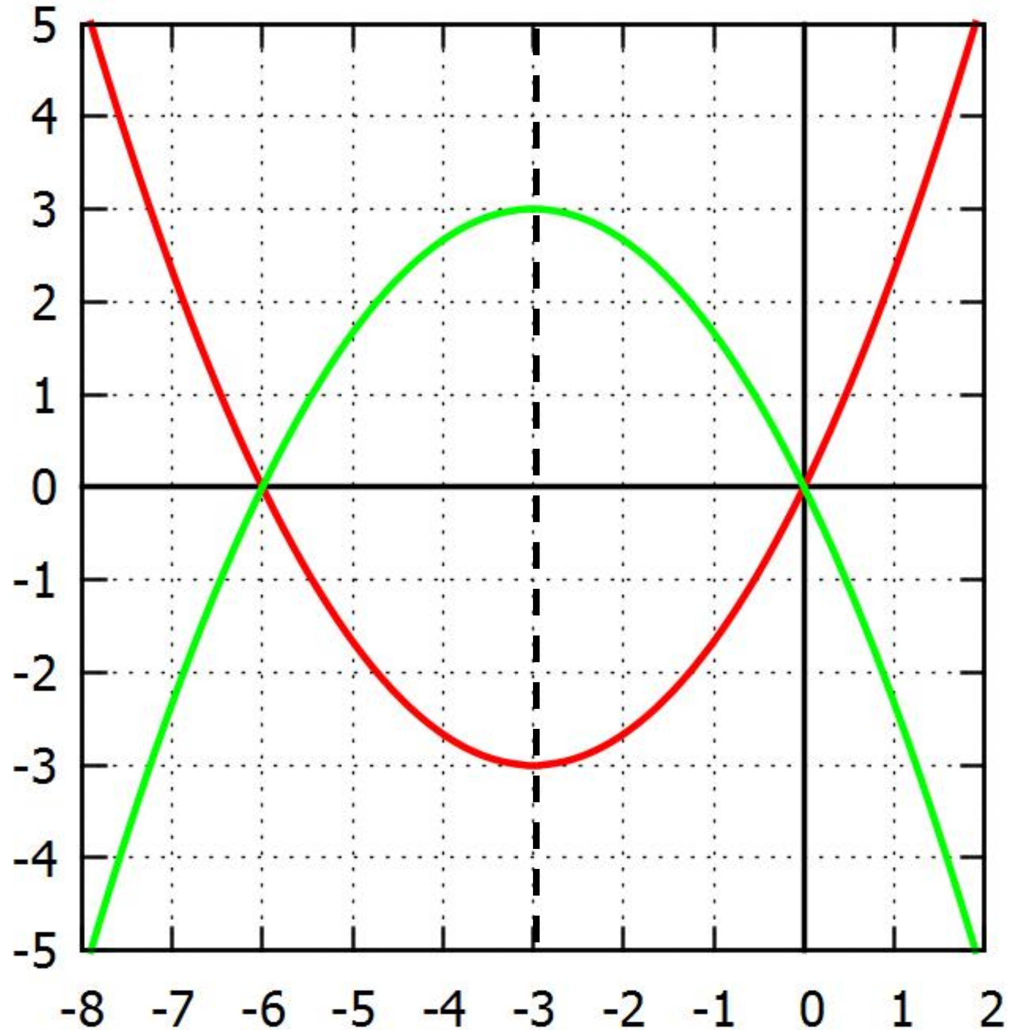

A. $(-3, 3)$
B. $(-3, -3)$
C. $(6, 0)$
D. $(3, 0)$
E. Cannot be determined

# Solution

**Answer:** E

**Justification:** It is not possible to determine the y-coordinate of the vertex only knowing two x-intercepts. The answer was therefore “Cannot be determined.”

It is possible to determine the x-value, however. The vertex must located at the midpoint between (-6,0) and (0,0), so its x-coordinate is -3. Both A and B were *possible* vertices.



# Parabolas IX

Consider a parabola with its vertex at (-3, -3) and one of its x-intercepts at (0, 0).

Which one of the following equations represents this parabola?

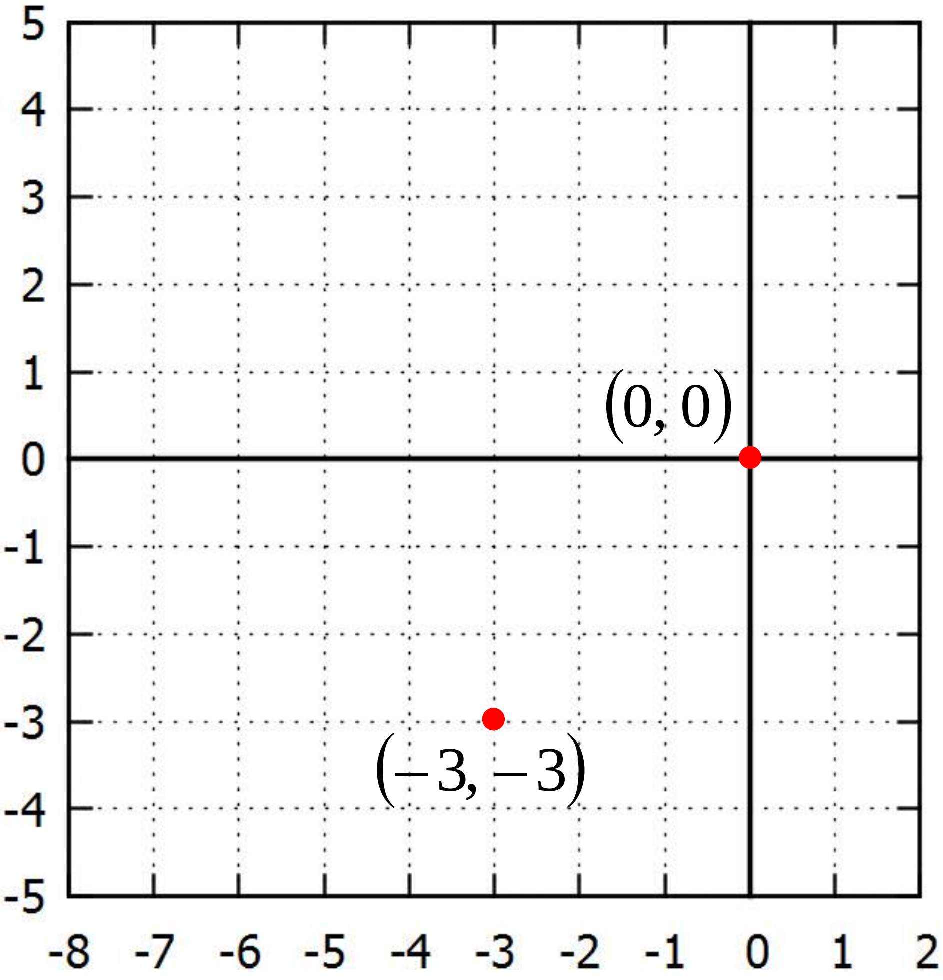
A. $y = (x+3)^2 - 3$

B. $y = 3(x+3)^2 - 3$

C. $y = \frac{1}{3}(x+3)^2 - 3$

D. $y = \frac{1}{9}(x+3)^2 - 3$

E. The parabola is not unique



# Solution

**Answer:** C

**Justification:** The vertex is given so the equation is in the form:

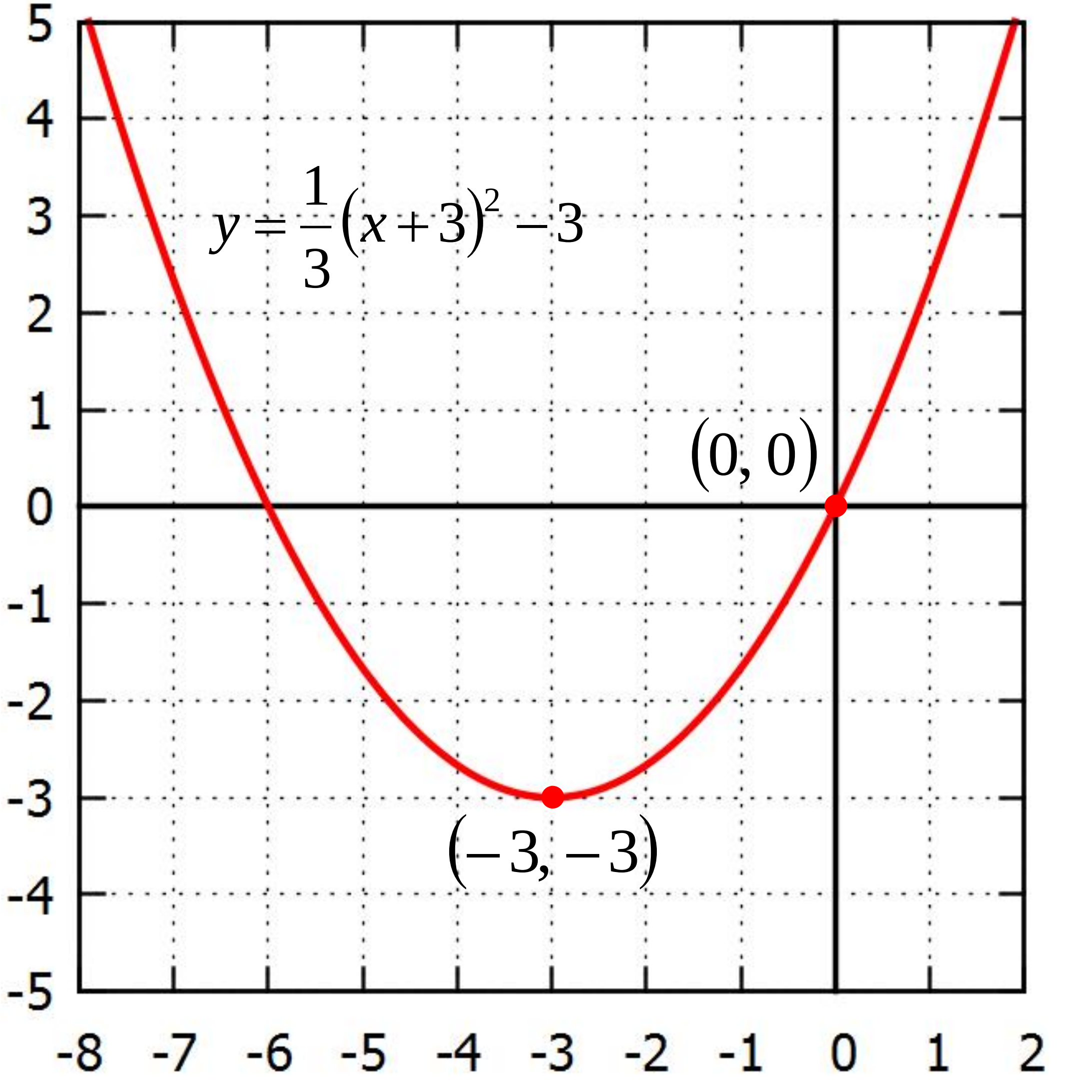
$$y = a(x+3)^2 - 3$$

Plugging in the point (0,0) will then find the value for *a*.

$$0 = a(0+3)^2 - 3$$

$$a = \frac{1}{3}$$

*Note: When 2 points are given and 1 is the vertex, an unique parabola can be found.*



# Parabolas X

Consider the graph of the quadratic function:

$$y = (-4 - x)^2 + 3$$

What is the vertex of the parabola?

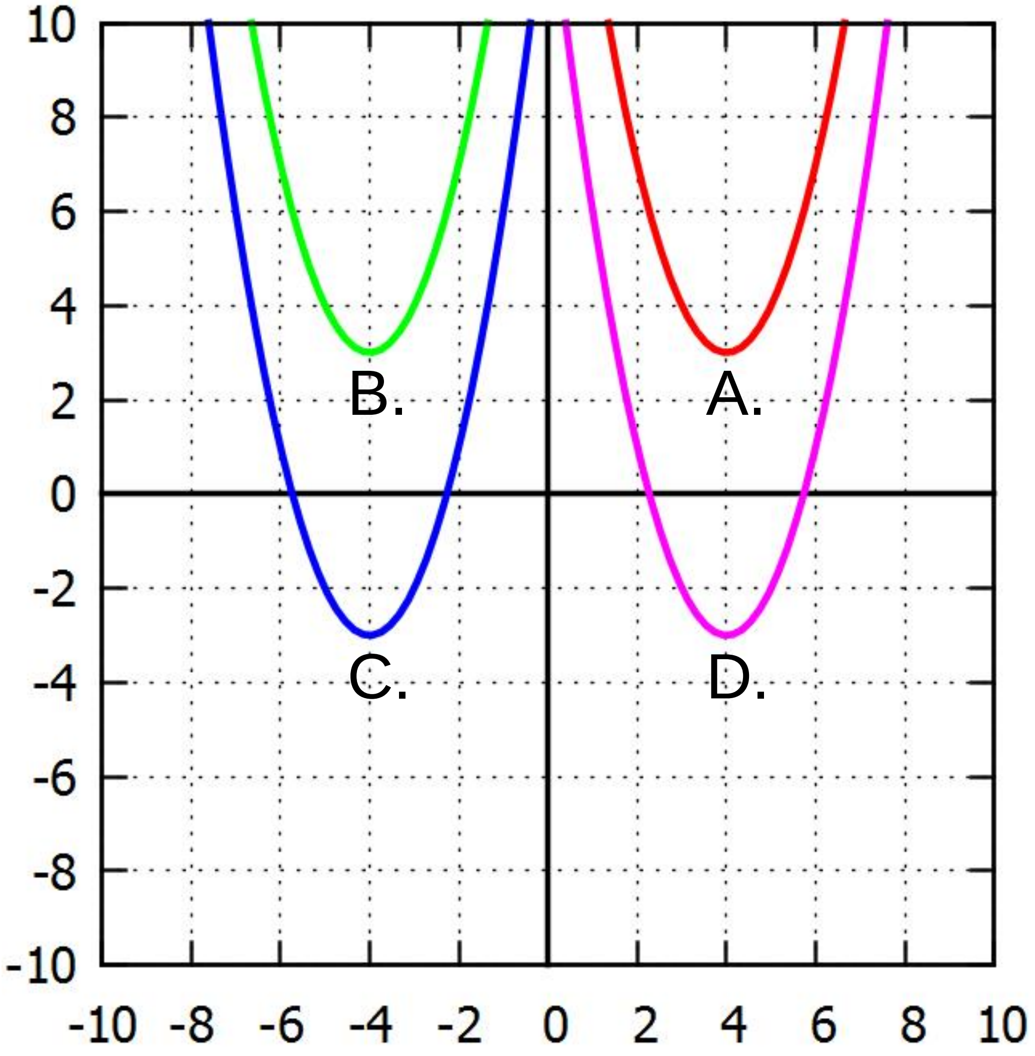
A. $(4, 3)$

B. $(-4, 3)$

C. $(-4, -3)$

D. $(4, -3)$

E. None of the above



# Solution

**Answer:** B

**Justification:** Factor out (-1) from (-4-x):

$$y = (-4 - x)^2 + 3$$

$$y = (-(4 + x))^2 + 3$$

This function is the same as

$$y = (4 + x)^2 + 3,$$

which has a vertex at (-4, 3). The extra negative does not affect the graph due to the square. (Try expanding the function and completing the square again)