a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

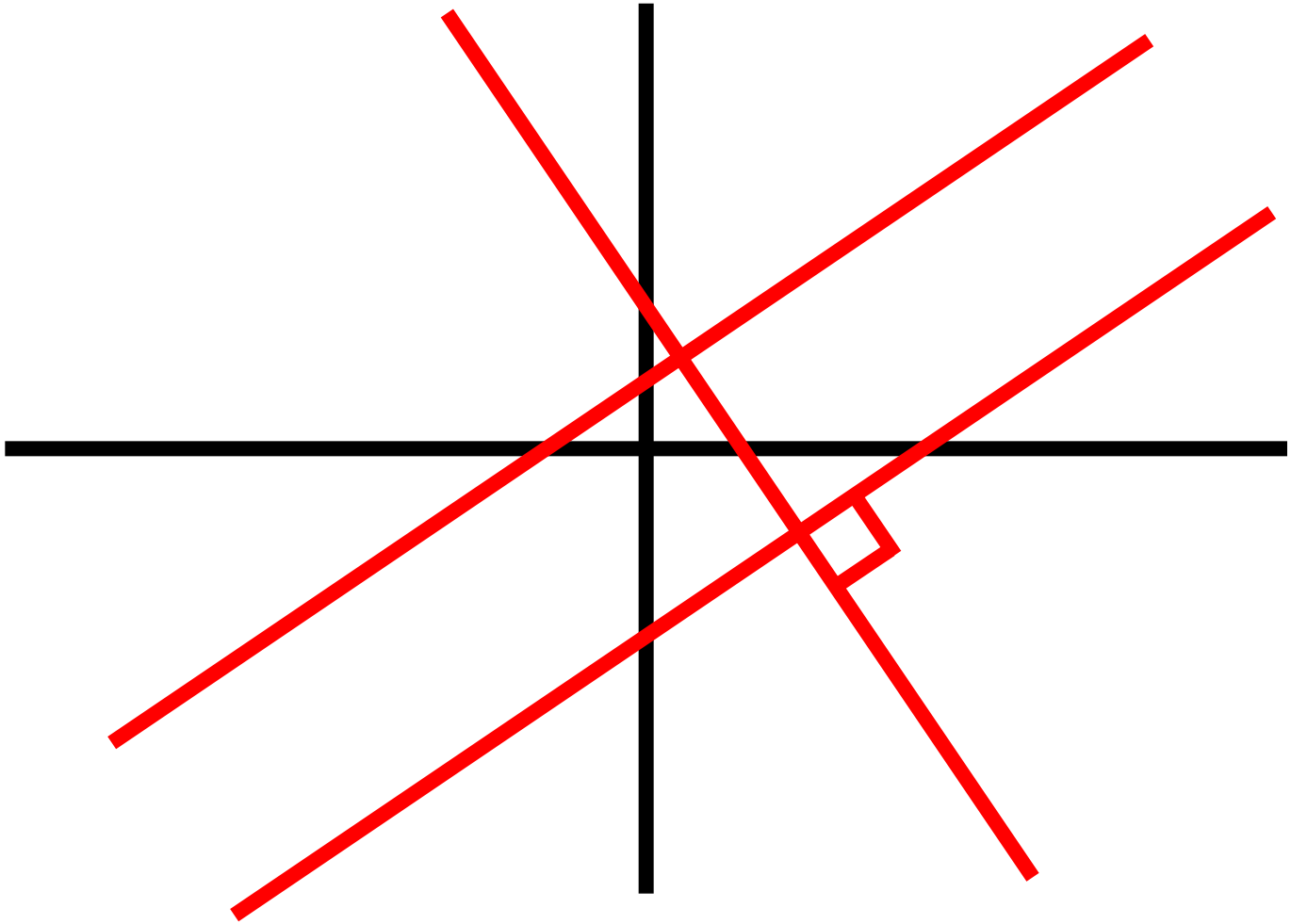
Mathematics

Parallel and Perpendicular Lines

Science and Mathematics
Education Research Group

Supported by UBC Teaching and Learning Enhancement Fund 2012-2013

Parallel and Perpendicular Lines



Parallel Lines

Let the slope of Line 1 be m_1 and the slope of Line 2 be m_2. Line 1 and Line 2 are parallel. Which one of the following statements is true?

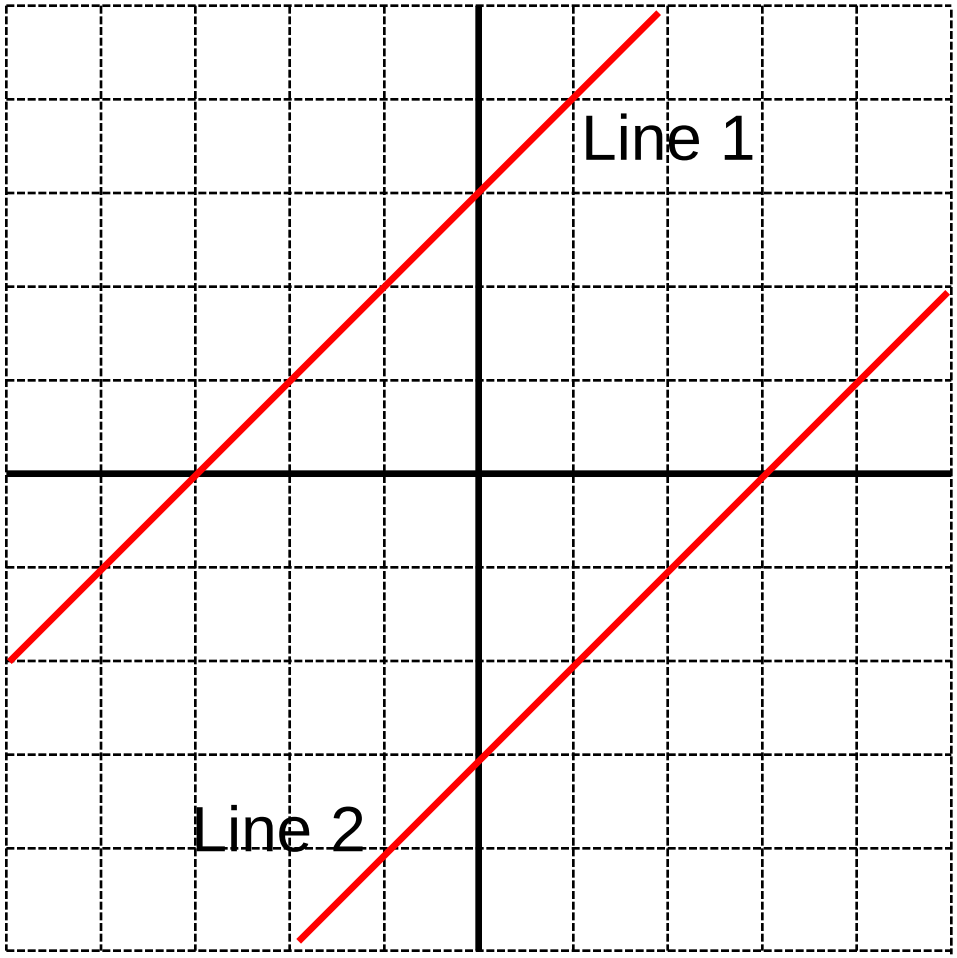
A. $m_1 = m_2$

B. $m_1 > m_2$

C. $m_1 < m_2$

D. $m_1 = -m_2$

E. $m_1 = \frac{1}{m_2}$

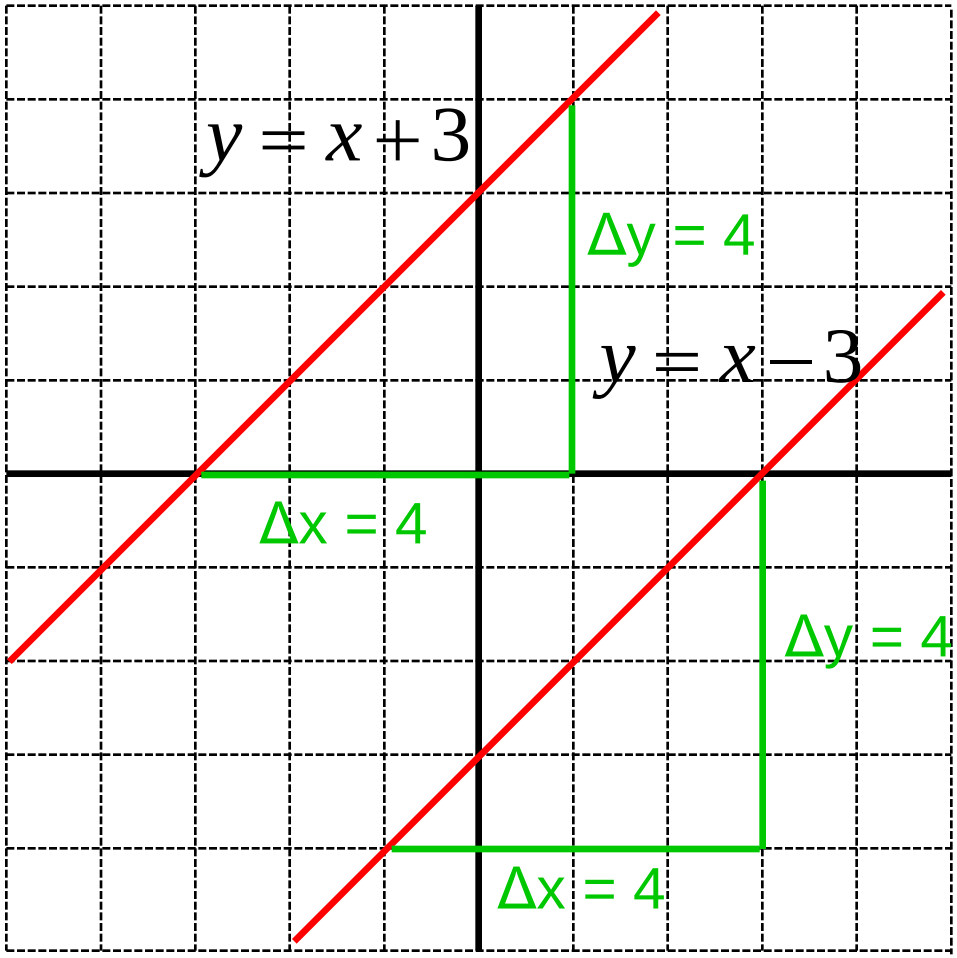


Solution

Answer: A

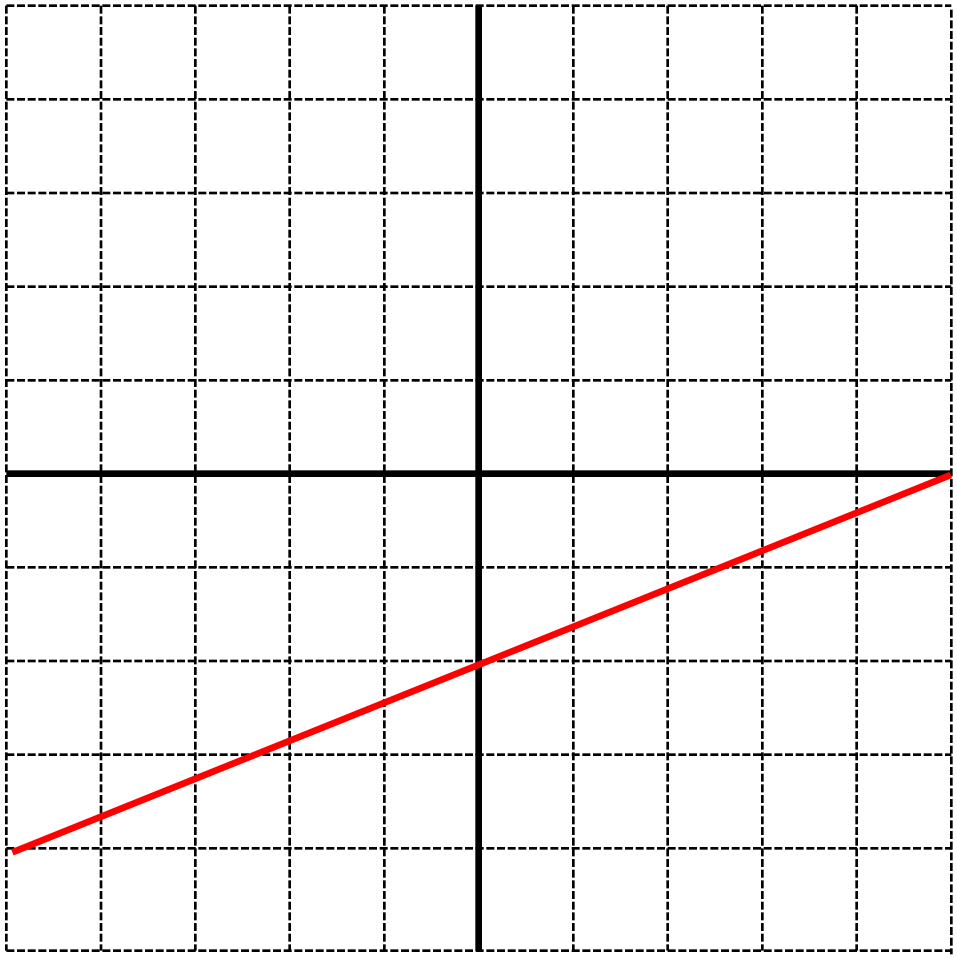
Justification: Both the lines have the same rise (Δy) and run (Δx), so they have an equal slope.

Parallel lines have the same slope.



Parallel Lines II

Which one of the following lines is parallel to line shown in the graph?



A. $y = \frac{2}{5}x$

B. $y = \frac{5}{2}x$

C. $y = -\frac{2}{5}x$

D. $y = -\frac{5}{2}x$

E. None of the above

Solution

Answer: A

Justification: Recall that the general equation of a line is

$$y = mx + b$$

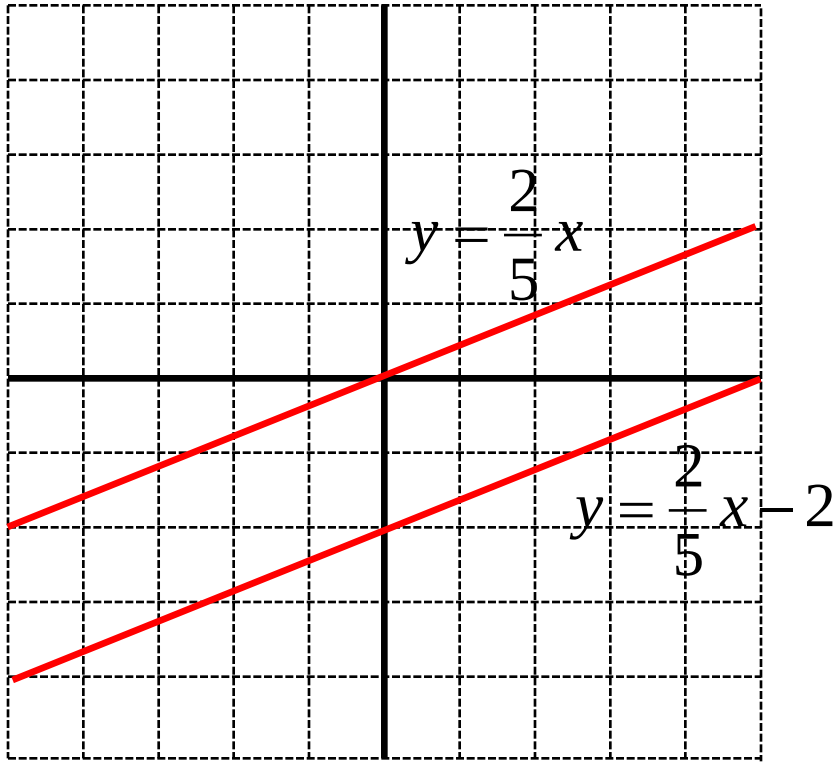
where m is the slope and b is the y-intercept.

Since parallel lines have the same slope, we must find a line where:

$$m = \frac{2}{5}$$

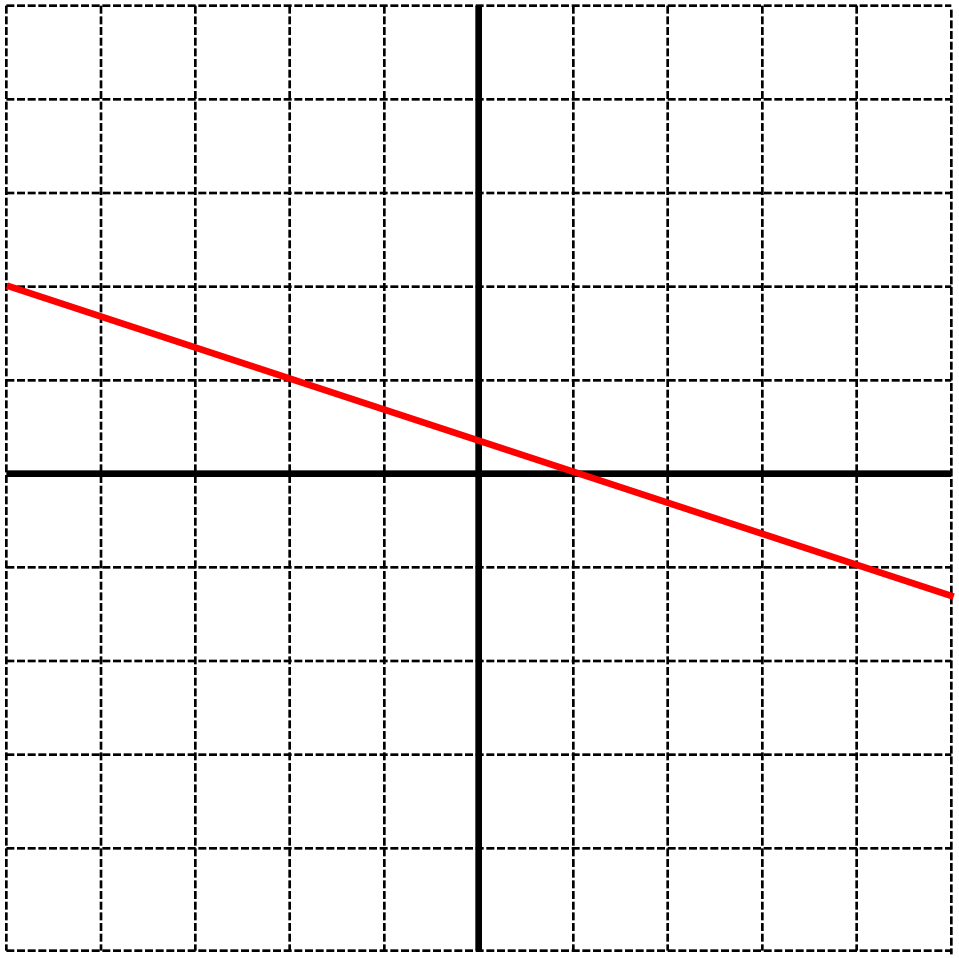
This corresponds to the equation:

$$y = \frac{2}{5}x$$



Parallel Lines III

Which line does not intersect the line shown in the graph?



A. $y = \frac{1}{3}x + \frac{1}{3}$

B. $y = -\frac{1}{3}x + \frac{1}{3}$

C. $y = \frac{1}{3}x + 3$

D. $y = -\frac{1}{3}x + 3$

E. Both B and D

Solution

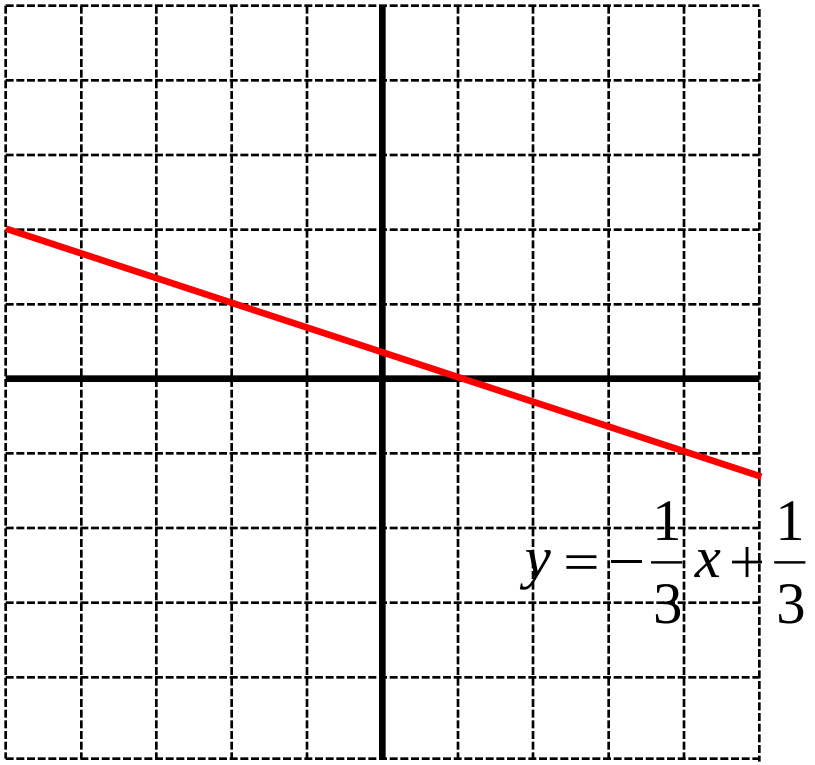
Answer: D

Justification: The two lines that are parallel to the red line are:

B. $y = -\frac{1}{3}x + \frac{1}{3}$

D. $y = -\frac{1}{3}x + 3$

In general, parallel lines do not intersect. However, line B is the same line as the one in the graph (they have the same slope and y-intercept). In this case, the 2 lines intersect at all points. Therefore, the only line that doesn't intersect the red line is D.



Parallel Lines IV

Which of the following is parallel to the line shown?

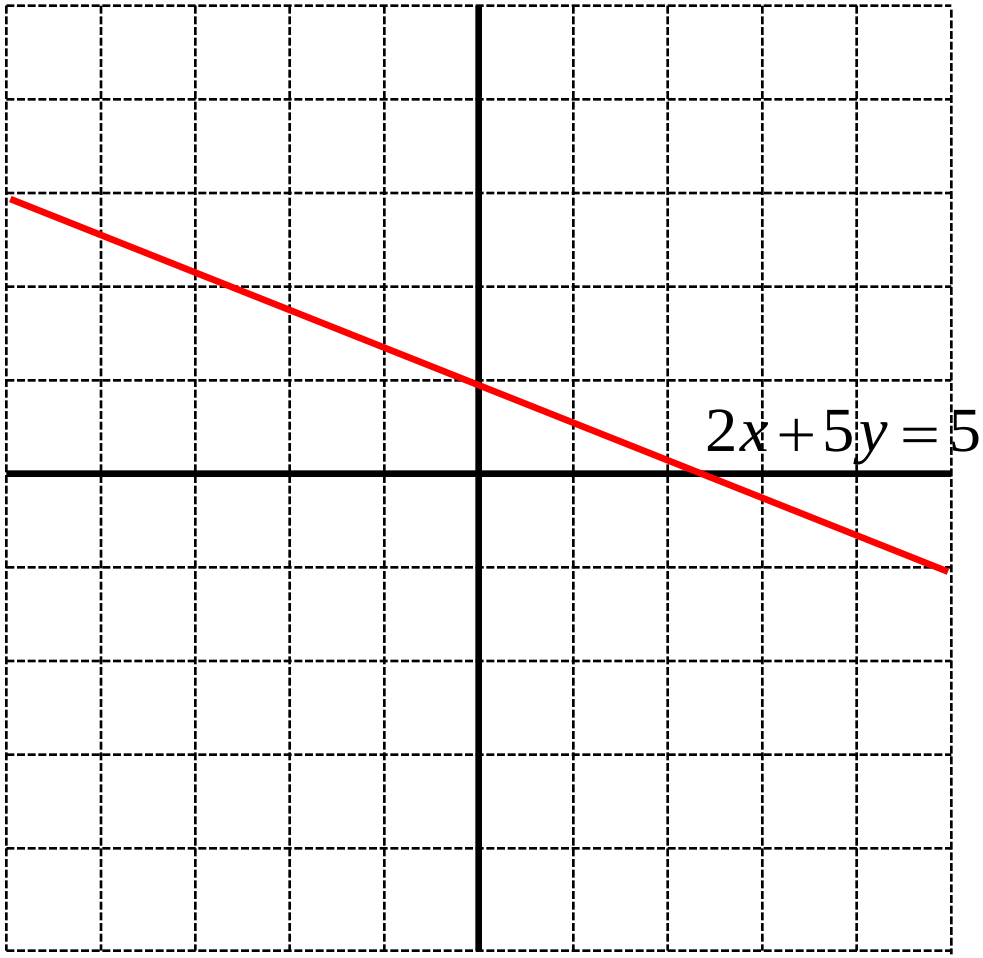
A. $2x-5y=7$

B. $-2x+5y=7$

C. $4x-10y=7$

D. $-4x+10y=7$

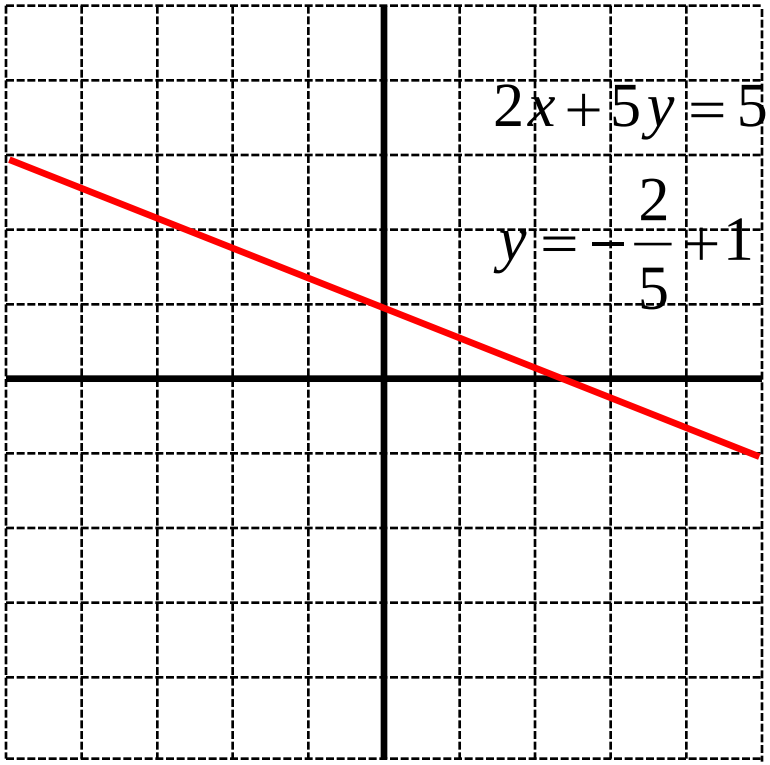
E. $-4x-10y=7$



Solution

Answer: E

Justification: The line that is parallel to the line shown in the diagram must have a slope of : $-\frac{2}{5}$



A. $2x-5y=7 \Rightarrow m=\frac{2}{5}$

B. $-2x+5y=7 \Rightarrow m=\frac{2}{5}$

C. $4x-10y=7 \Rightarrow m=\frac{4}{10}=\frac{2}{5}$

D. $-4x+10y=7 \Rightarrow m=\frac{4}{10}=\frac{2}{5}$

E. $-4x-10y=7 \Rightarrow m=-\frac{4}{10}=-\frac{2}{5}$

The constant term 7 does not affect the slope of the line. Find the slope of each line by converting to the form y=mx+b

Parallel Lines V

Which line is parallel to the line shown and passes through the point (2,3)?

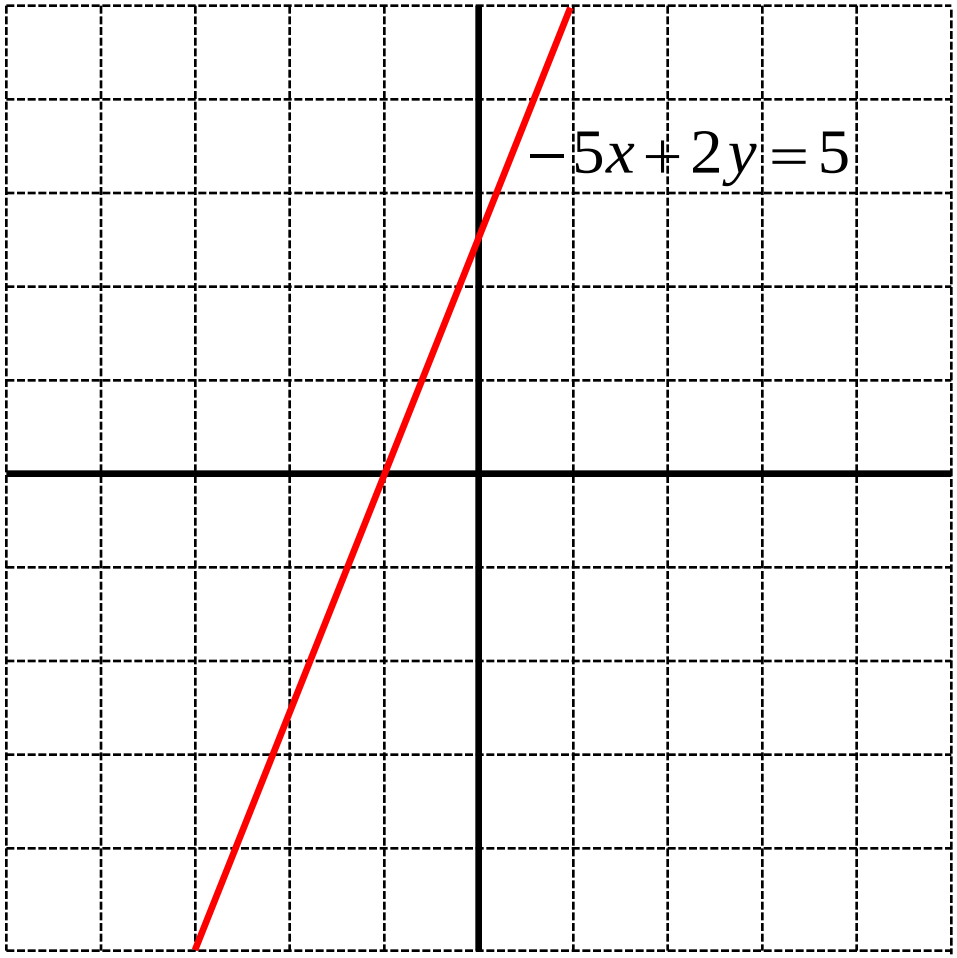
A. $-5x+2y=-5$

B. $10x-4y=-8$

C. $-10x+4y=-8$

D. $\frac{5}{2}x-y=2$

E. $-\frac{5}{2}x+y=2$



Solution

Answer: C

Justification: All of the given lines are parallel. (Check by converting to y=mx+b or analyzing the ratio of the coefficients in front of the x and y).

A. $-5x+2y=-5$

B. $10x-4y=-8$

C. $-10x+4y=-8$

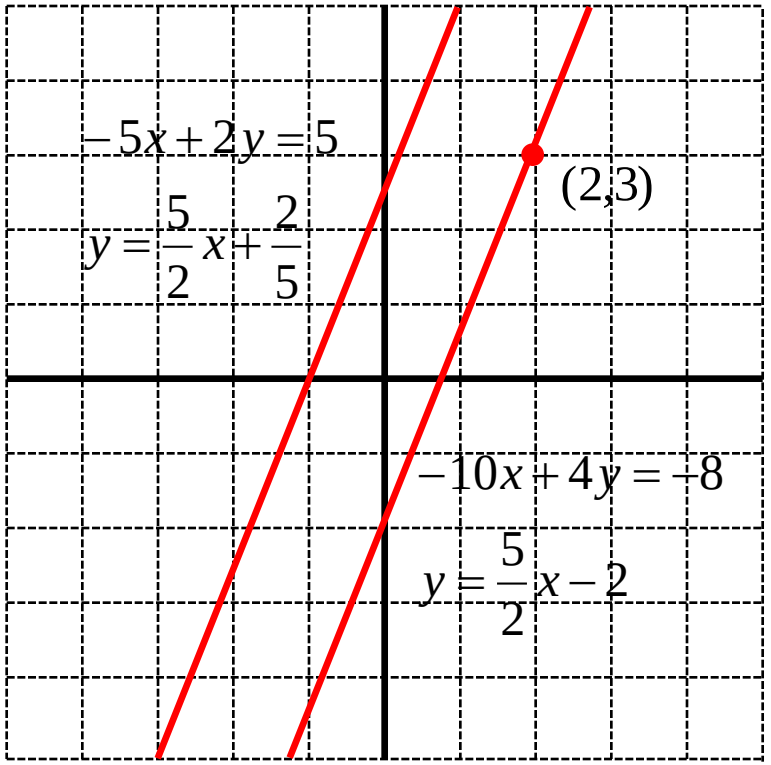
D. $\frac{5}{2}x-y=2$

E. $-\frac{5}{2}x+y=2$

C. $-10x+4y=-8$

$-10(2)+4(3)=-8$

$-8=-8$



Plug x=2 and y=3 into each equation to determine which gives a true statement. This line will pass through the point (2,3).

Perpendicular Lines I

Consider a line with a positive slope. Which of the following must be true about a line that is perpendicular to this line?

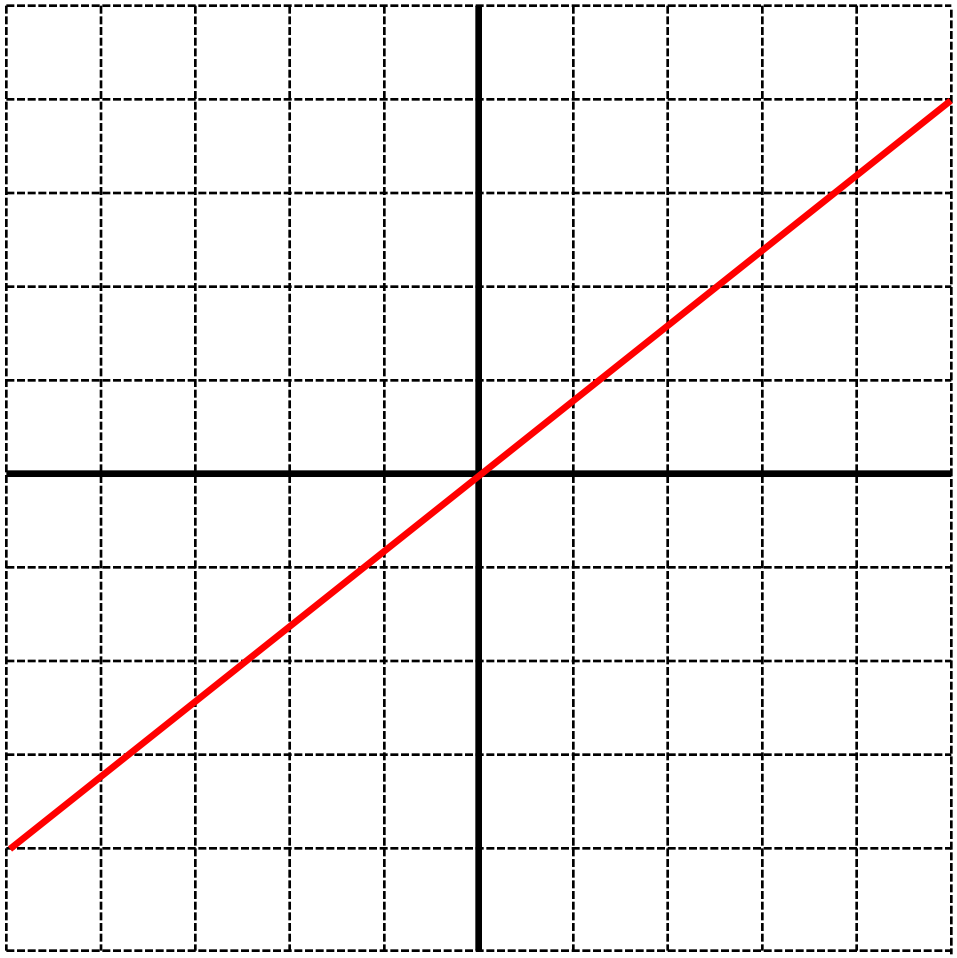
A. $m < 0$

B. $m > 0$

C. $m = 0$

D. $-1 \le m \le 1$

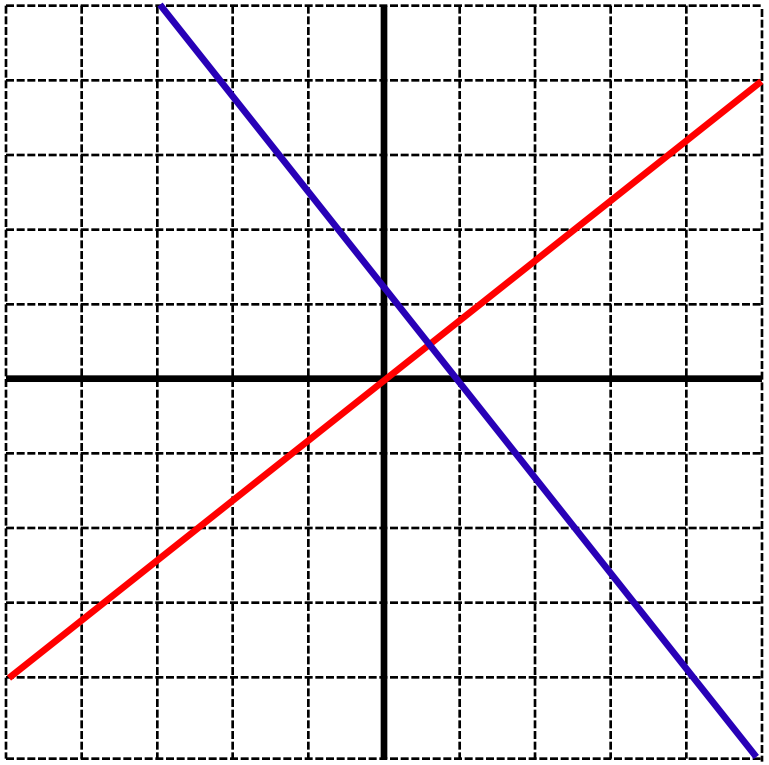
E. None of the above



Solution

Answer: A

Justification: If a line has a positive slope, the line perpendicular to this line must have a negative slope. Since perpendicular lines form a 90° angle between each other, if one line is increasing, the other should be decreasing.



Perpendicular Lines II

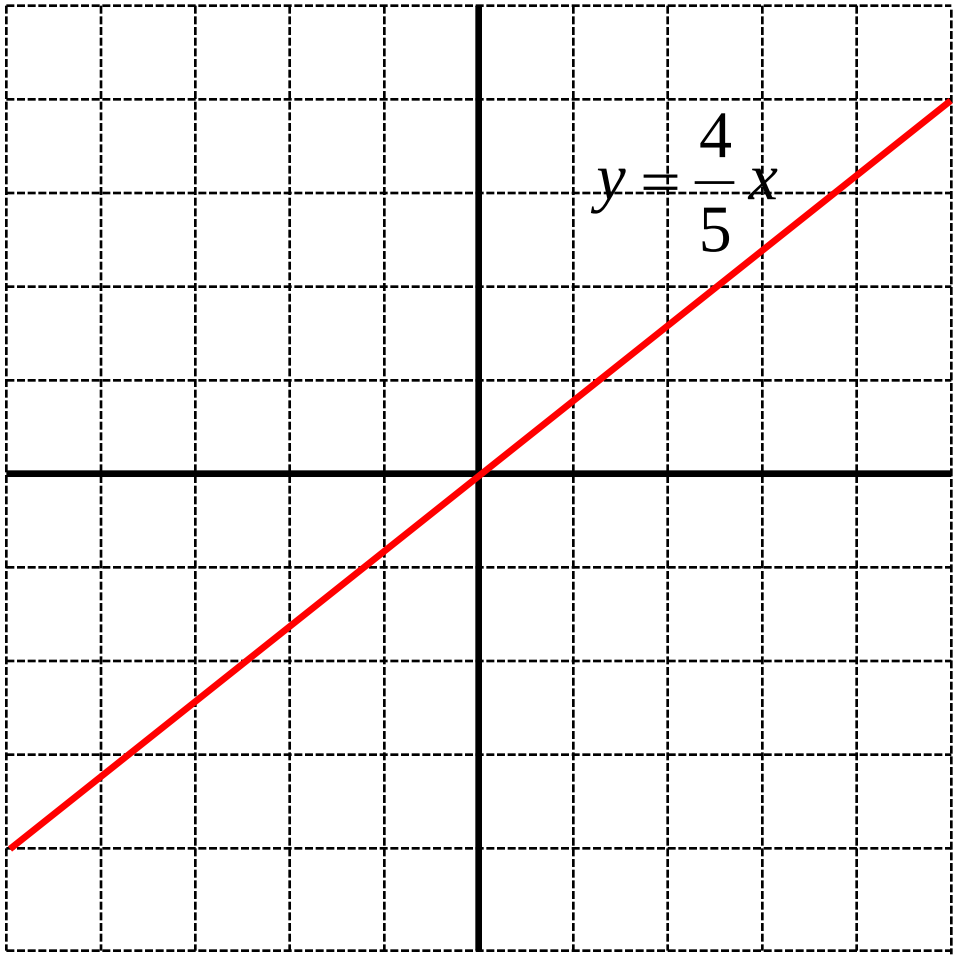
Is the line

$$y = -\frac{4}{5}x$$

perpendicular to the line shown in the graph?

A. Yes

B. No



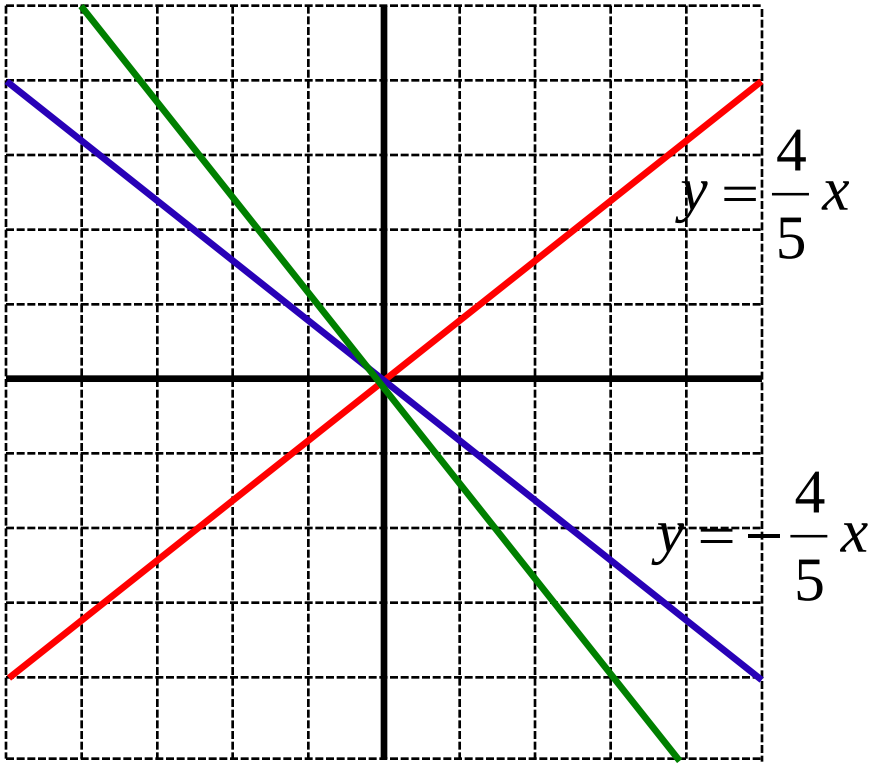
Solution

Answer: B

Justification: The blue and red lines shown in the graph have opposite slopes.

The angle between these lines is not 90°. Simply changing the sign of the slope of a line does not give the slope of a perpendicular line.

The green line is perpendicular to the red line. The next few questions will explore the slope of perpendicular lines.



Perpendicular Lines III

The two lines shown are perpendicular. Which statement is true about the slopes of 2 perpendicular lines?

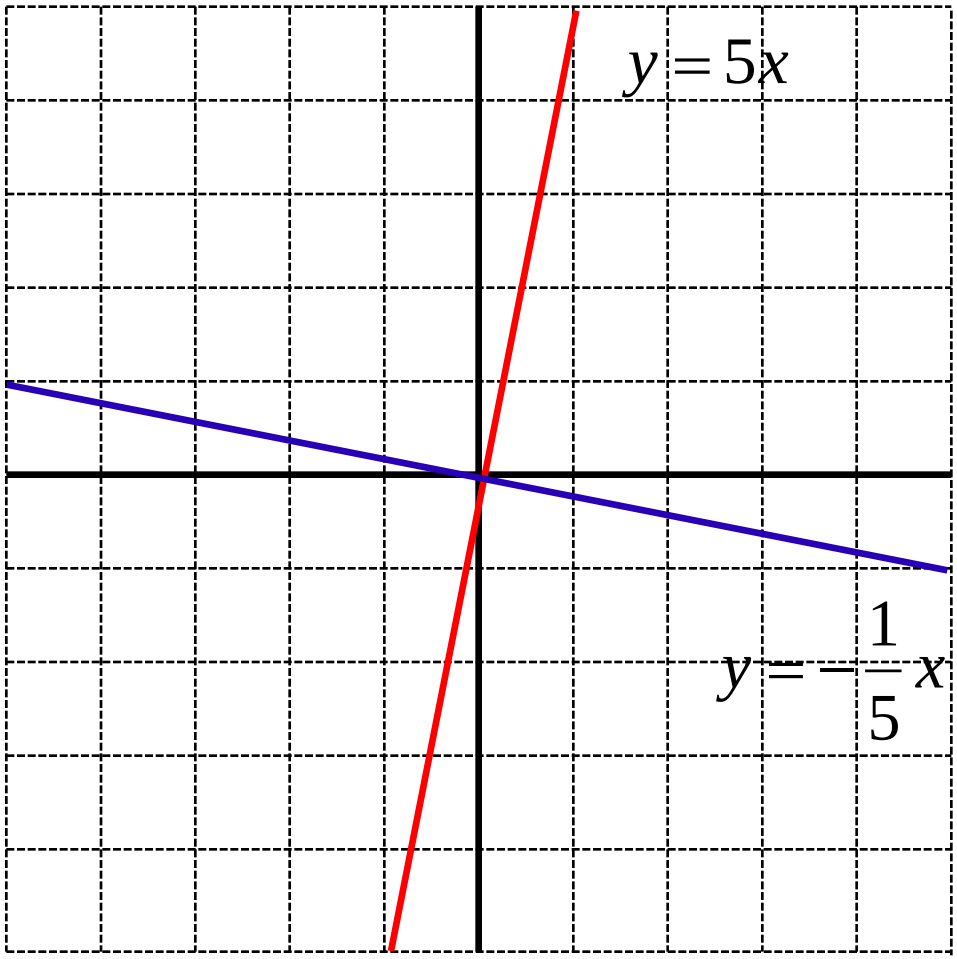
A. $m_1 = -m_2$

B. $m_1 - m_2 = 0$

C. $m_1 = -\frac{1}{m_2}$

D. $m_1 = \frac{1}{m_2}$

E. $m_1 m_2 = 0$



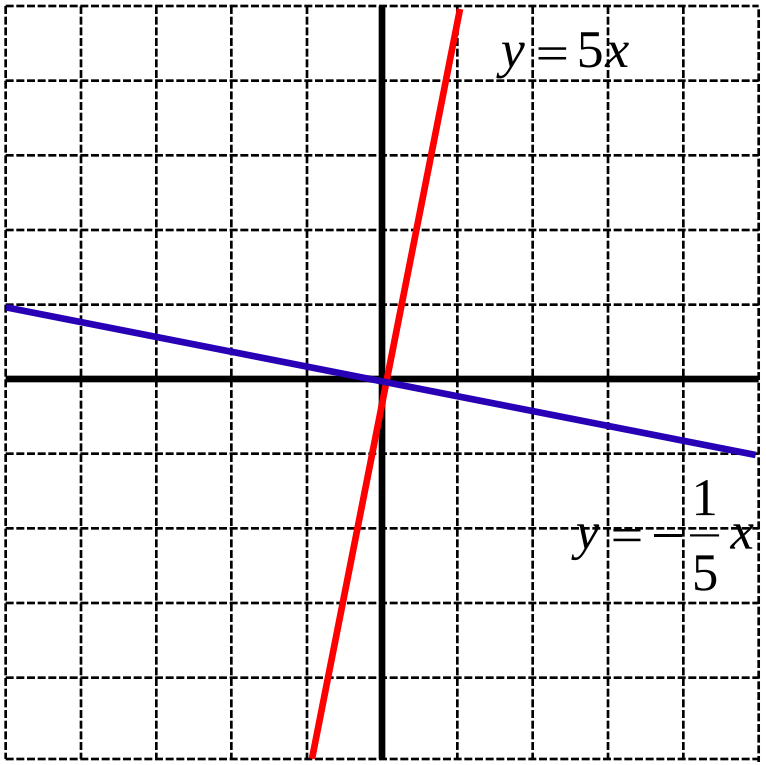
Solution

Answer: C

Justification: The line perpendicular to the red line must have a slope with the opposite sign.

In addition, the rise and run of the lines are also interchanged. An increase in x by 1 corresponds to an increase in y by 5 for the red line. For the blue line, an increase in y by 1 corresponds to a decrease in x by 5.

The slopes of perpendicular lines are therefore negative reciprocals.



$$m_1 = -\frac{1}{m_2}$$

Perpendicular Lines IV

Which line is perpendicular to the one in the graph?

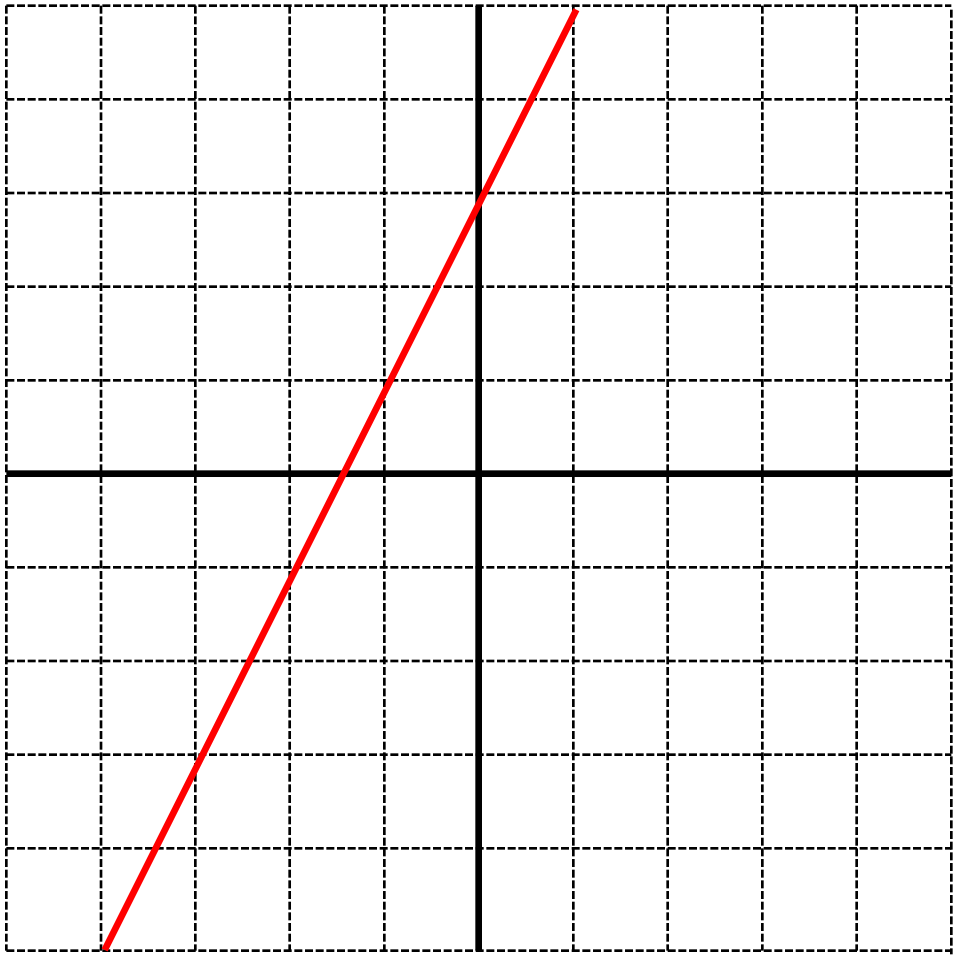
A. $y = 2x$

B. $y = \frac{1}{2}x$

C. $x = -2y$

D. $x = -\frac{1}{2}y$

E. None of the above



Solution

Answer: C

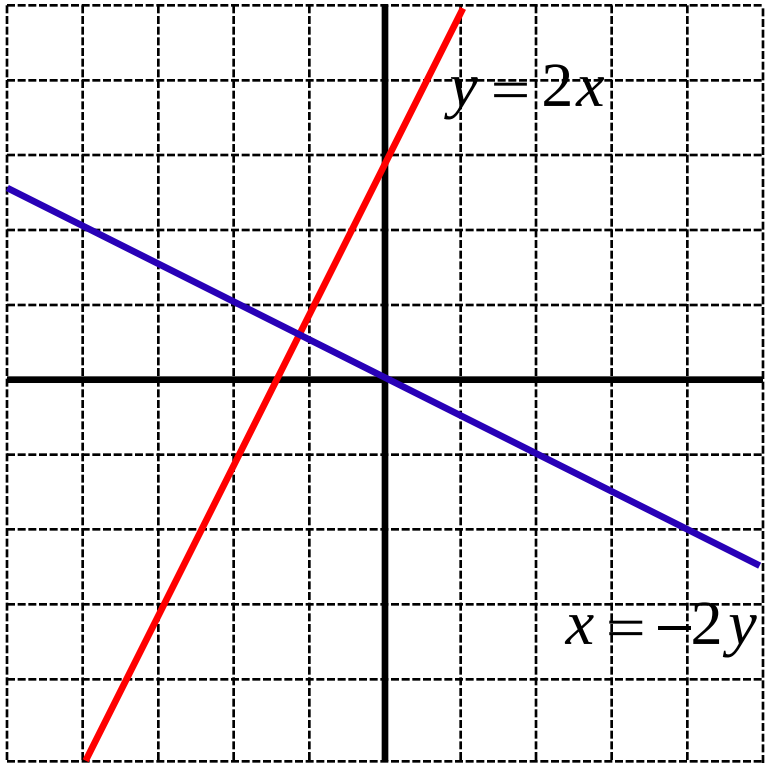
Justification: The slope of the red line is 2. The slope of the perpendicular line is therefore:

$$m_{\perp} = -\frac{1}{m} = -\frac{1}{2}$$

This corresponds to the line

$$y = -\frac{1}{2}x$$

$$x = -2y$$



Perpendicular Lines V

Consider a line with the equation:

$$462379x + 774334y - 43853 = 0$$

Which of the following lines is perpendicular to this line?

A. $-462379x - 774334y = 0$

B. $-774334x - 462379y = 0$

C. $774334x - 462379y = 0$

D. $-774334x + 462379y = 0$

E. Both C and D

Solution

Answer: E

Justification: The slope of the line $462379x + 774334y - 43853 = 0$ is:

$$m = -\frac{462379}{774334}$$

A line perpendicular to this must have a slope of: $m = \frac{774334}{462379}$

Taking the reciprocal of the slope interchanges the coefficients of x and y. Since we must also change the sign of the slope, either the x or y coefficient must change signs. Therefore, both C and D are perpendicular (they are the same line).

C. $774334x - 462379y = 0$

D. $-774334x + 462379y = 0$

Perpendicular Lines VI

Which line is perpendicular to the line shown, and passes through the point (-4,2)?

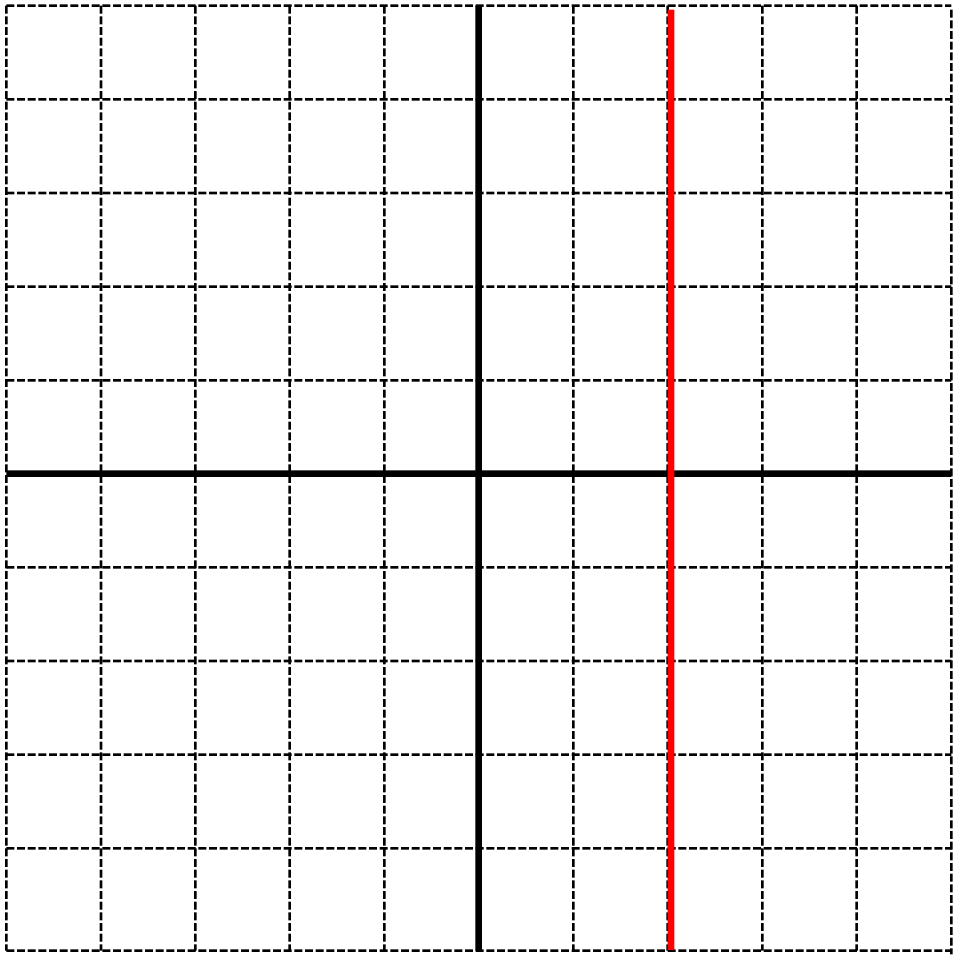
A. $y = -4$

B. $y = 2$

C. $x = -4$

D. $x = 2$

E. None of the above



Solution

Answer: B

Justification: A line perpendicular to a vertical line must be a horizontal line. The line must therefore be in the form y = b.

The horizontal line that passes through the point (-4,2) is y=2. The y- value of a horizontal line remains constant while the x-value changes.