UBC

a place of mind

FACULTY OF EDUCATION

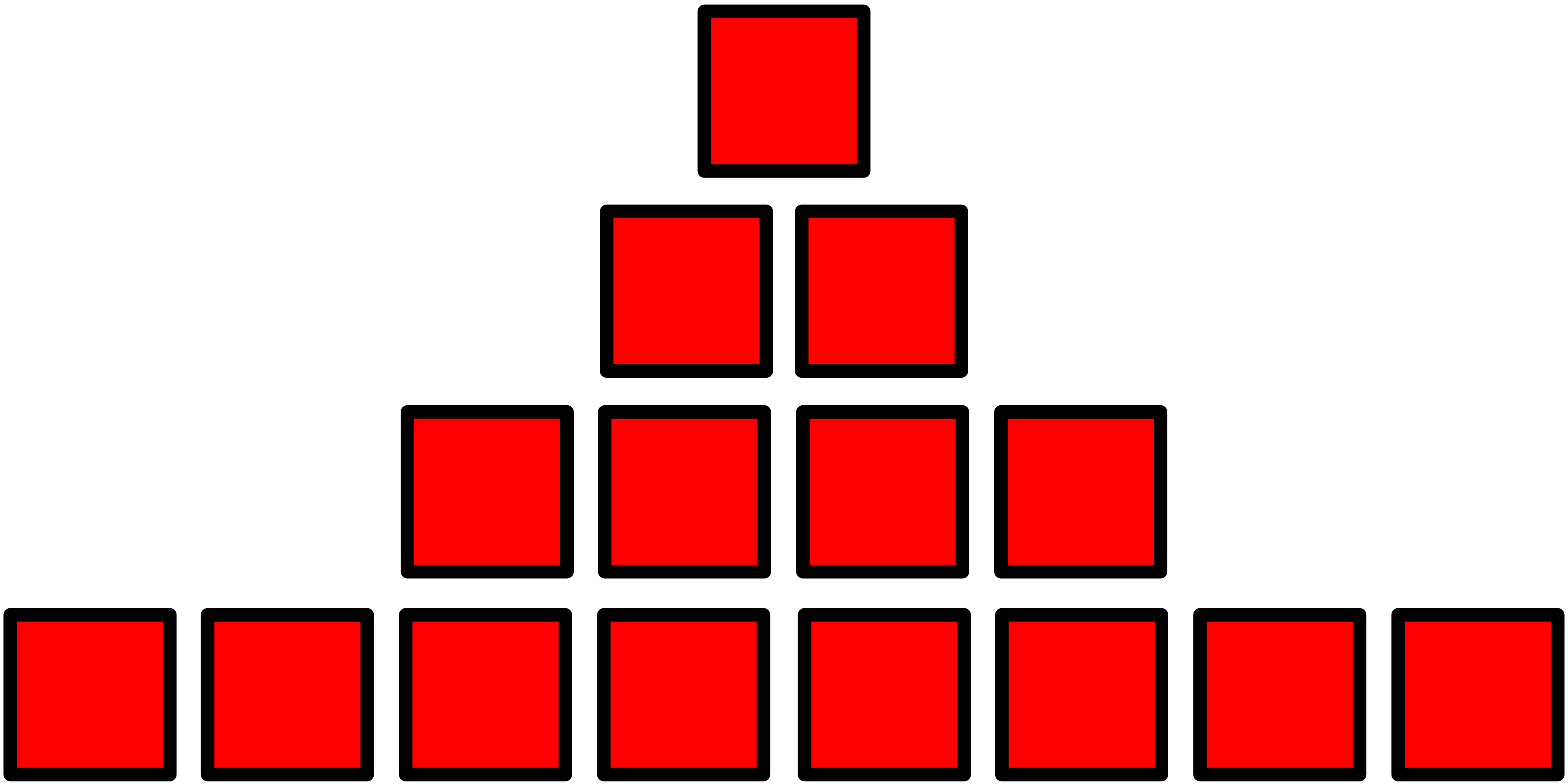
Department of
Curriculum and Pedagogy

Mathematics
Geometric Sequences

Science and Mathematics
Education Research Group

Supported by UBC Teaching and Learning Enhancement Fund 2012-2013

Geometric Sequences



Geometric Sequences I

Consider the following sequence of numbers:

1, 2, 4, 8, 16 ....

The first 5 terms are shown. What is the 8^{th} term in the arithmetic sequence?

A. 24
B. 32
C. 64
D. 128
E. 256

Solution

Answer: D

Justification: The sequence is called a geometric sequence because every two consecutive terms follow the same ratio. In this case, the next number is always twice the previous number. Therefore, the 8^{th} term can be found by multiplying the 5^{th} term by 2 three times.

x2 x2 x2

1, 2, 4, 8, 16, 32, 64, 128

↑ 5^{th} *term*

↑ 8^{th} *term*

Geometric Sequences II

Consider the following sequence of numbers:

a_1, a_2, a_3, a_4, a_5, ... a_n

where a_n is the n^{th} term of the sequence. The next term is 3 times larger than the previous term (the *common ratio* is 3). Which of the following is a correct value for a_7?

A. $a_7 = 3a_5$

B. $a_7 = 6a_5$

C. $a_7 = 9a_4$

D. $a_7 = 27a_4$

E. $a_7 = 3a_5 + 3a_6$

Solution

Answer: D

Justification: The next term is always 3 times larger than the previous term. Therefore,

$$a_7 = 3(a_6)$$

Although this is a correct value for a_7, none of the provided answers are in terms of a_6. Breaking down a_6 in terms of a_5 gives another expression for a_7:

$$a_7 = 3(a_6) = 3(3a_5) = 9a_5 \qquad a_6 = 3(a_5)$$

This answer also does not match any of the given solutions. The value of a_7 can also be written in terms of a_4:

$$a_5 = 3(a_4) \qquad a_7 = 9a_5 = 9(3a_4) = 27a_4$$

Geometric Sequences III

In a geometric sequence, the first term is a_1 and each term is r times the previous (the common ratio is r). What is the n^{th} term in the sequence?

$$a_1, \quad a_1(r), \quad a_1(r)(r), \quad a_1(r)(r)(r), \quad \ldots$$

Term: 1 2 3 4

A. $a_n = a_1(n)(r)$

B. $a_n = a_1(r^n)$

C. $a_n = a_1(r^{n-1})$

D. $a_n = a_1(r^{n+1})$

E. $a_n = a_1(n^r)$

Press for hint

Solution

Answer: C

Justification: The second term is the first term multiplied by r. The third term is the first term multiplied by r twice. Continuing this pattern to the n^{th} term:

Term:	*1*	*2*	*3*	*4*		*n*
	a_1,	$a_1(r)$,	$a_1(r)(r)$,	$a_1(r)(r)(r)$,	...	
	a_1,	$a_1(r)$,	$a_1(r^2)$,	$a_1(r^3)$,	...	$a_1(r^{n-1})$

Notice that to find the nth term, we multiply the first term by the common ratio n-1 times, not n times.

$$a_n = a_1 r^{n-1}$$

Geometric Sequences IV

Consider a geometric sequence with first term a_1, common ratio r, and $a_4 = 24$.

The first term of this sequence is multiplied by 2, while the common ratio is kept the same. What is a_4 in this new sequence?

$$a_n = a_1 r^{n-1}$$

A. $a_4 = 24$

B. $a_4 = 24(2) = 48$

C. $a_4 = 24(4) = 96$

D. $a_4 = 24(8) = 192$

E. The answer depends on the value of the first term.

Solution

Answer: B

Justification: The fourth term of any sequence expressed in terms of the first term a_1 and common ratio r is:

$$a_4 = a_1\left(r^{4-1}\right) = a_1\left(r^3\right)$$

If a_1 is doubled, then a_4 is also multiplied by 2:

$$\left(2a_1\right)\left(r^3\right) = 2a_4 = 2(24) = 48$$

Geometric Sequences V

Consider a geometric sequence with first term a_1, common ratio 2, and $a_4 = 24$.

The common ratio of this sequence is now increased from 2 to 3, while the first term is kept the same. What is a_4 in this new sequence?

A. 24(3) = 72
B. 24(4) = 96
C. 24(8) = 192
D. 24(27) = 648
E. None of the above

$$a_n = a_1 r^{n-1}$$

Solution

Answer: E

Justification: The fourth term of any sequence expressed in terms of the first term a_1 and common ratio r is:

$$a_4 = a_1\left(r^{4-1}\right) = a_1\left(r^3\right)$$

The sequences will look like:

r = 2: 3, 6, 12, 24, ...

r = 3: 3, 9, 27, 81, ...

When r = 2,

$$a_4 = a_1\left(2^3\right) = 8a_1$$

When r = 3, the new 4th term is:

$$a_4 = a_1\left(3^3\right) = 27a_1$$

The new value for a_4 is therefore $\frac{27}{8}$ times larger, giving

$\frac{27}{8}(24) = 81$ The answer is therefore “None of the above.”

Geometric Sequences VI

The first term of a sequence is 40 and the common difference is $-\frac{1}{2}$. Which of the following correctly displays this sequence?

A. 40, 80, 160, 320, …
B. 40, -80, 160, -320, …
C. 40, 20, 10, 5, …
D. -40, -20, -10, -5, …
E. 40, -20, 10, -5, …

Solution

Answer: E

Justification: The common ratio is a fraction less than 1. Instead of terms getting larger than the previous, each term is smaller than the previous. In this case, since the common ratio is a half, each term is half as large as the previous.

The common ratio is also negative. Repeatedly multiplying by a negative number results in a number alternating from positive to negative. The expected geometric sequence is therefore:

40, -20, 10, -5, 2.5, -1.25, 0.75, ...

Geometric Sequences VII

In a geometric sequence, $a_{41} = 29$ and $a_{43} = 32$. What is the common ratio of this sequence?

A. $r = \pm 3$

B. $r = \pm \frac{32}{29}$

C. $r = \pm \frac{29}{32}$

D. $r = \pm \sqrt{\frac{32}{29}}$

E. $r = \pm \sqrt{\frac{29}{32}}$

Press for hint

$$r = \frac{a_n}{a_{n-1}} \qquad a_{43} = a_{41} r^2$$

Solution

Answer: D

Justification: An equation must be found that states a_{43} in terms of a_{41}. Multiplying a_{41} by r gives a_{42}, and multiplying a_{41} by r twice gives a_{43}:

$$a_{41}r = a_{42}$$

$$a_{41}r^2 = a_{43}$$

Since both a_{41} and a_{43} are known, the equation can be solved for r:

$$29r^2 = 32$$

$$r = \pm\sqrt{\frac{32}{29}}$$

Note that if the common difference is negative, the 41st and 43rd term will be positive and the 42nd term will be negative.

Geometric Sequences VIII

How would the 999th term of a geometric sequence be expressed in terms of the 99th term?

(Express a_{999} in terms of a_{99} and r)

A. $a_{999} = r^{99} a_{99}$

B. $a_{999} = r^{100} a_{99}$

C. $a_{999} = r^{900} a_{99}$

D. $a_{999} = r^{999} a_{99}$

E. $a_{999} = r^{\frac{999}{99}} a_{99}$

Solution

Answer: C

Justification: Write both a_{999} and a_{99} in terms of a_1:

$$a_{999} = r^{998} a_1$$

$$a_{99} = r^{98} a_1$$

Dividing these equations cancel a_1, leaving:

$$\frac{a_{999}}{a_{99}} = \frac{r^{998} a_1}{r^{98} a_1}$$

$$a_{999} = r^{998-98} a_{99}$$

$$a_{999} = r^{900} a_{99}$$

In general, the n^{th} term in a sequence written in terms of the b^{th} term (where $n > b$) is:

$$a_n = r^{n-b} a_b$$

Geometric Sequences IX

Consider the four geometric sequences shown below:

1. $a_1 = 10, r = 2$
2. $a_1 = -10, r = 2$
3. $a_1 = 10, r = -2$
4. $a_1 = -10, r = -2$

In how many of the sequences is the 100^{th} term positive?

A. The 100^{th} term is positive in all of the sequences

B. The 100^{th} term is positive in 3 of the sequences

C. The 100^{th} term is positive in 2 of the sequences

D. The 100^{th} term is positive in 1 of the sequences

E. The 100^{th} term is positive in none of the sequences

Solution

Answer: C

Justification:

Sequence 1: $a_1 > 0, r > 0$
Every term in this sequence is positive because positive numbers are multiplied by positive numbers.

Sequence 2: $a_1 < 0, r > 0$
Every term in this sequence is negative. From the formula $a_n = a_1 r^{n-1}$, a_1 is negative but r^{n-1} is always positive.

Sequence 3: $a_1 > 0, r < 0$
Every odd term in this sequence is positive. Since $a_{100} = a_1(r^{99})$, a_{100} is negative because r^{99} is negative and a_1 is positive.

Sequence 4: $a_1 < 0, r < 0$
Every even term in this sequence is positive. Since $a_{100} = a_1(r^{99})$, a_{100} is positive because r^{99} is negative and a_1 is negative.