a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

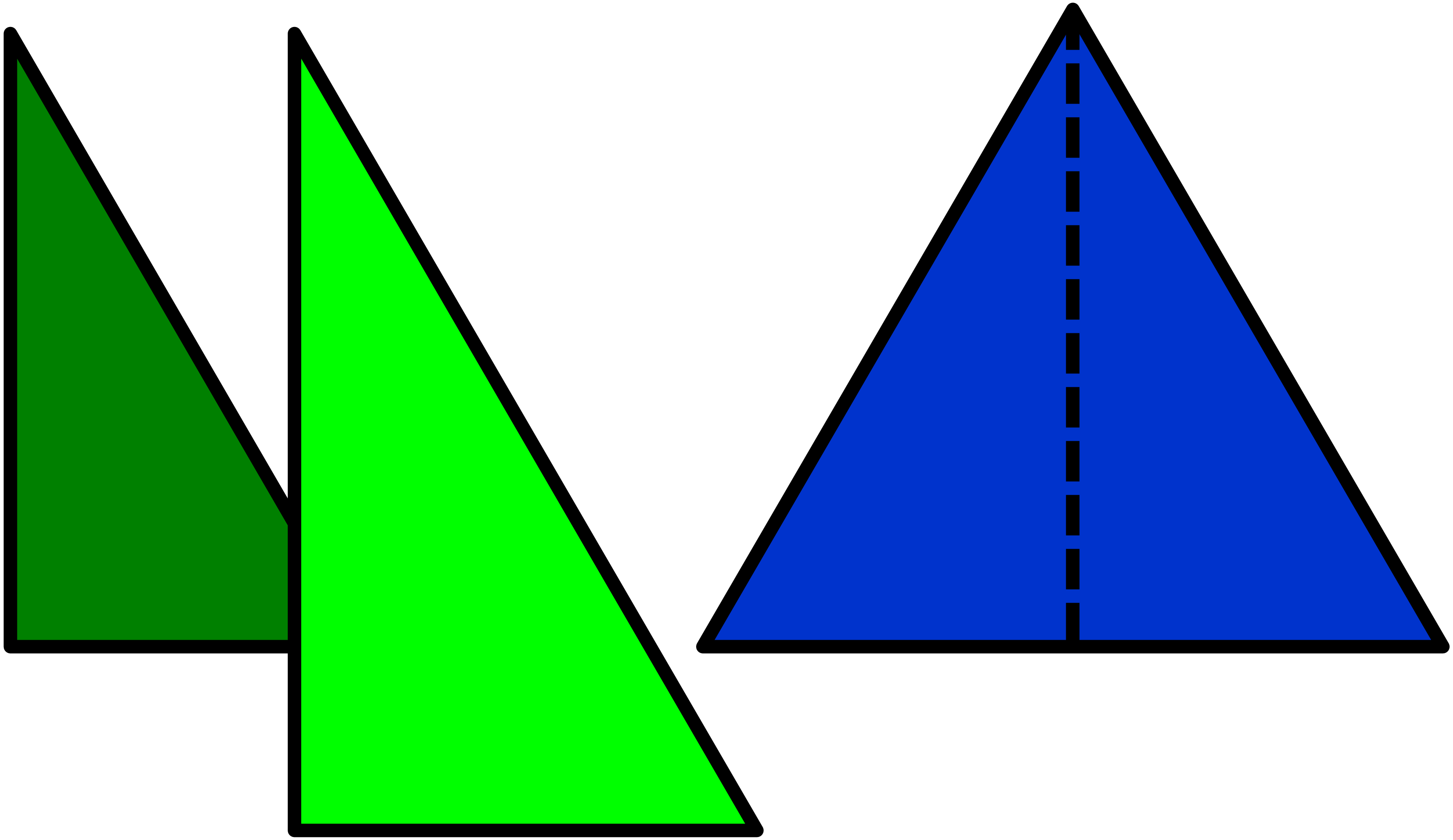
Mathematics

Trigonometry: Special Triangles (30-60-90)

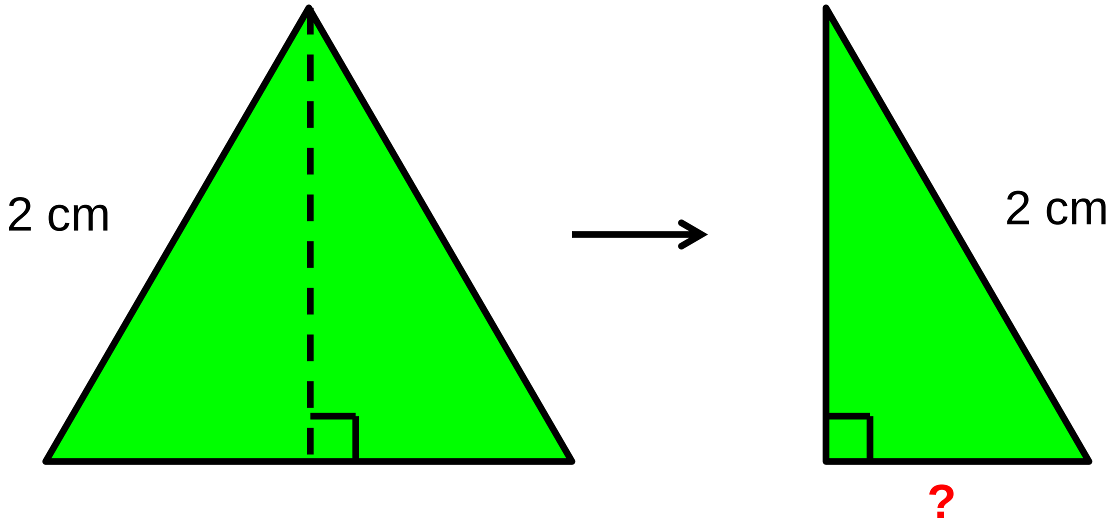
Science and Mathematics
Education Research Group

Supported by UBC Teaching and Learning Enhancement Fund 2012-2013

Special Triangles 30-60-90



The 30-60-90 Triangle I



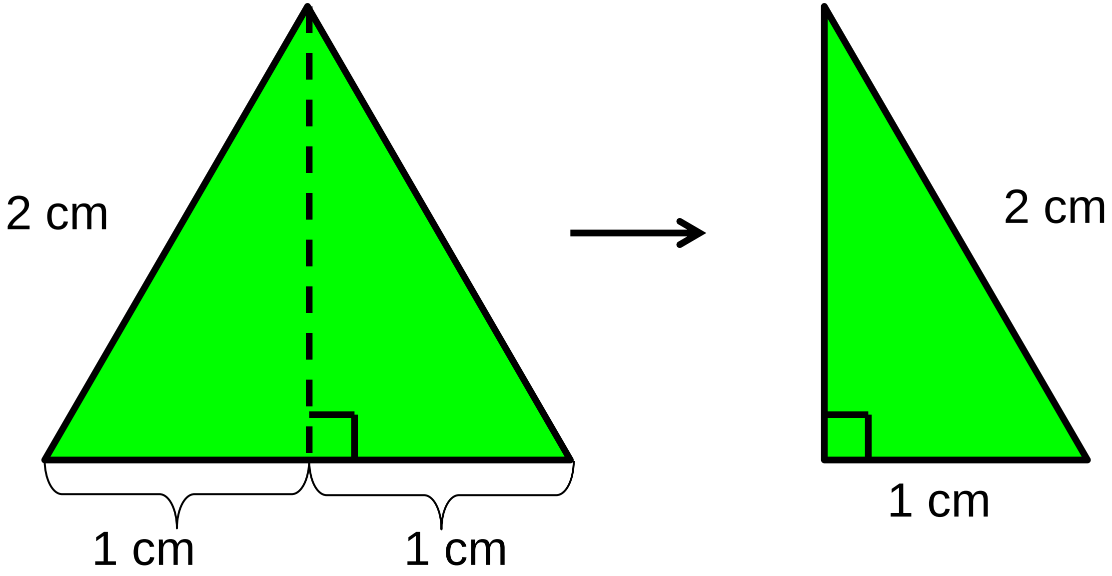
A. 1 cm
B. $\sqrt{2}$ cm
C. 1.5 cm
D. 2 cm
E. Not enough information

Consider an equilateral triangle with side length of 2 cm. The triangle is cut in half as shown in the figure above. What is the length of the smaller triangle's base?

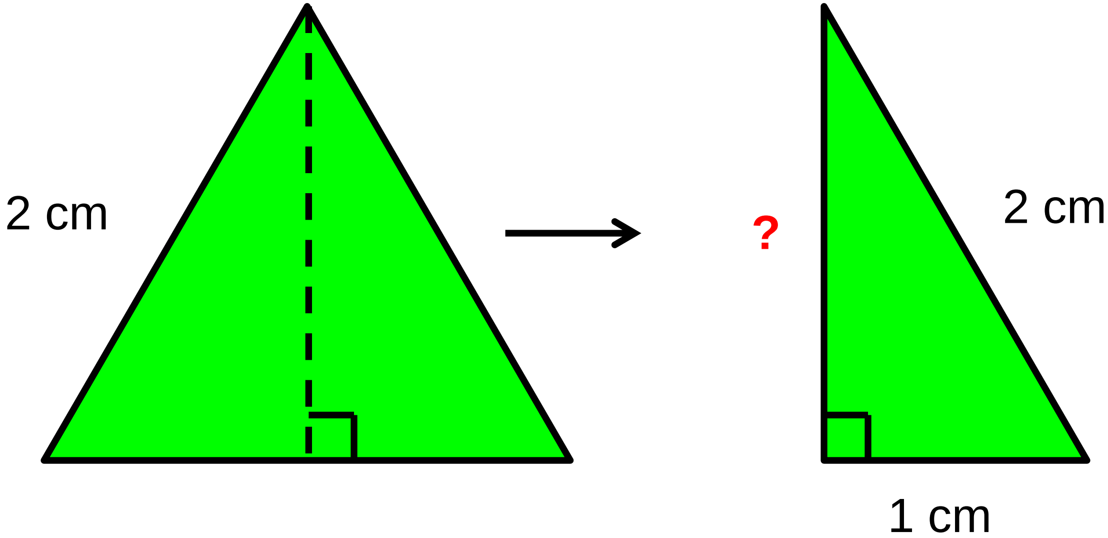
Solution

Answer: A

Justification: The equilateral triangle is cut in half and splits the base at the midpoint. The base of the new triangle should be half the side length of the original triangle, or 1 cm.



The 30-60-90 Triangle II



What is the height of the smaller triangle?

A. $\sqrt{2}$

B. $\sqrt{3}$

C. 2

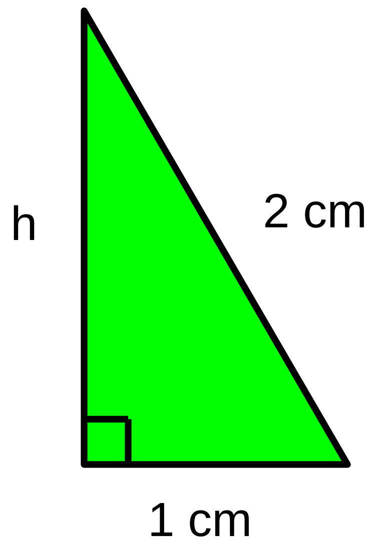
D. $\sqrt{5}$

E. Not enough information

Solution

Answer: B

Justification: Using the Pythagorean Theorem:



$$h^2 + 1^2 = 2^2$$

$$h^2 = 4 - 1$$

$$h^2 = 3$$

$$h = \pm\sqrt{3}$$ (reject negative solution)

$$h = \sqrt{3}\text{ cm}$$ (include units)

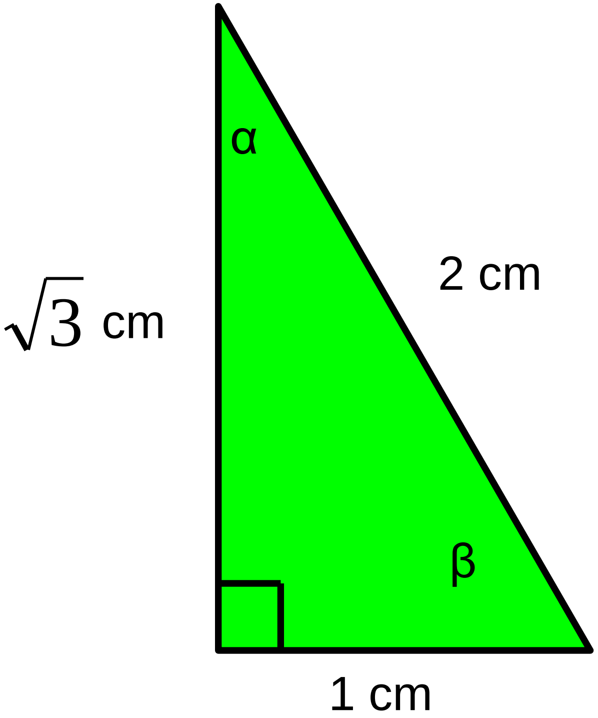
Note: We reject the negative solution because lengths of geometric shapes must always be positive.

The 30-60-90 Triangle III

What are the angles alpha (α) and beta (β)?

Press for hint

The triangle is half a equilateral triangle, which contains three 60° angles



A. α = 30°, β = 30°

B. α = 30°, β = 60°

C. α = 30°, β = 90°

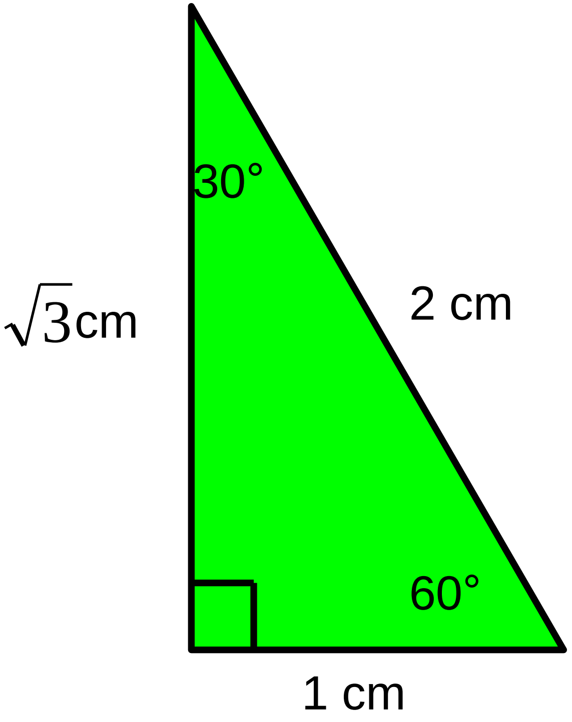
D. α = 60°, β = 30°

E. α = 60°, β = 60°

Solution

Answer: B

Justification: The triangle was originally an equilateral triangle with three 60° angles.



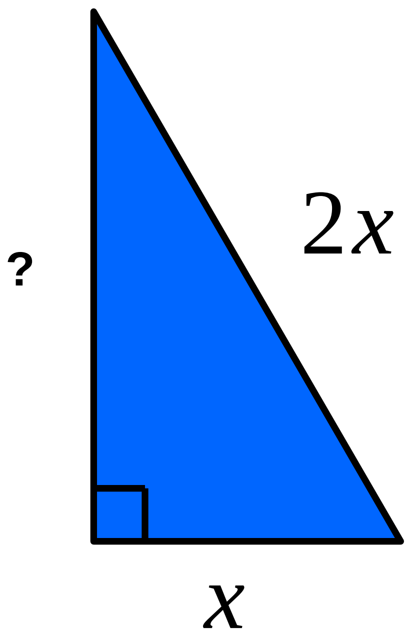
The equilateral triangle was split down the middle, so α = 30°. The other two angles on the side were not changed, so β = 60°.

Remember that the angles in a triangle must sum up to 180 °. Notice that:

$$30° + 60° + 90° = 180°.$$

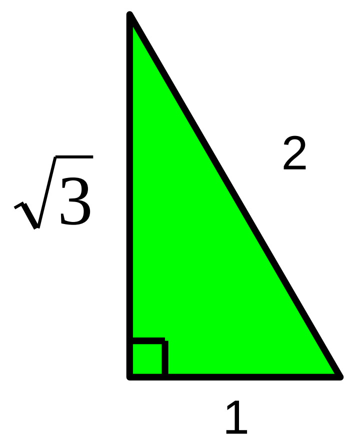
The 30-60-90 Triangle IV

The length of the hypotenuse of the 30-60-90 triangle is now 2x. What is the height of the triangle?



Press for hint

Remember:



A. $3\sqrt{x}$

B. $\sqrt{3x}$

C. $x\sqrt{3}$

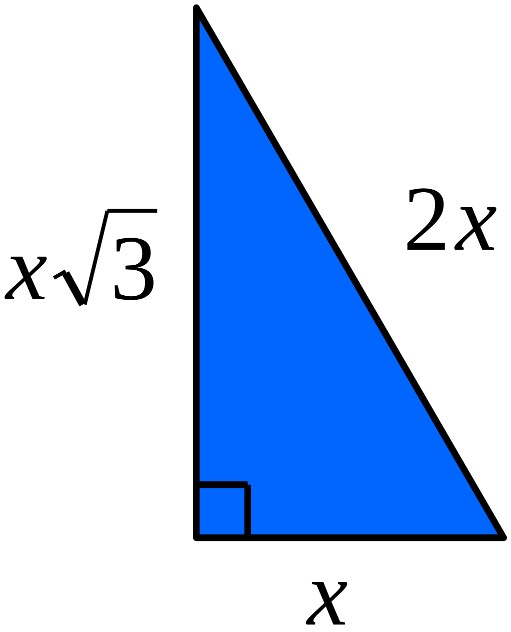
D. $\dfrac{x}{\sqrt{3}}$

E. $\dfrac{\sqrt{3}}{x}$

Solution

Answer: C

Justification: The ratio of the length of sides are $1:\sqrt{3}:2$. Multiplying this ratio by x gives $x : x\sqrt{3} : 2x$. Multiplying by x only rescales the triangle, so the ratio remains the same.



Alternative solution:

Using the Pythagorean Theorem:

$$h^2 + x^2 = (2x)^2$$

$$h^2 = 4x^2 - x^2$$

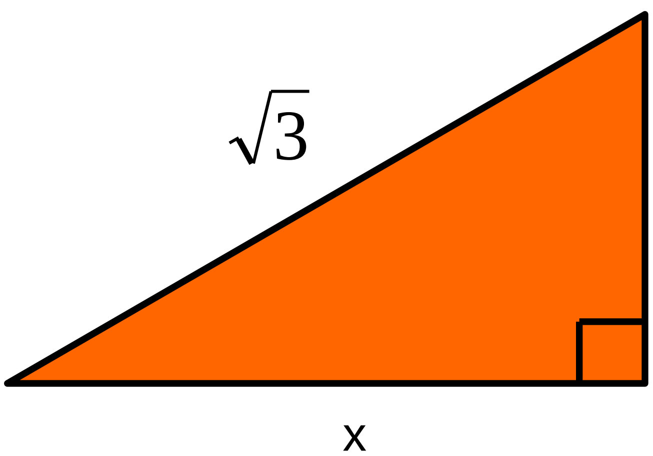
$$h^2 = 3x^2$$

$$h = \pm\sqrt{3x^2}$$

$$h = x\sqrt{3}$$

The 30-60-90 Triangle V

The orange triangle below is a 30-60-90 triangle. What is the length of the side labelled *x*?



A. $x = \frac{2}{3}$

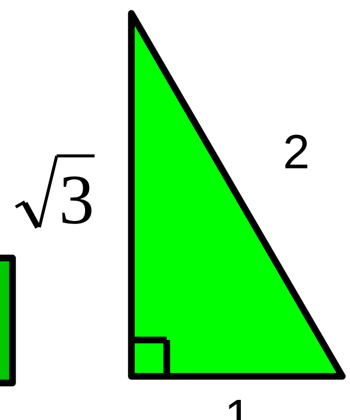
B. $x = \frac{3}{2}$

C. $x = 2$

D. $x = 3$

E. $x = 2\sqrt{3}$

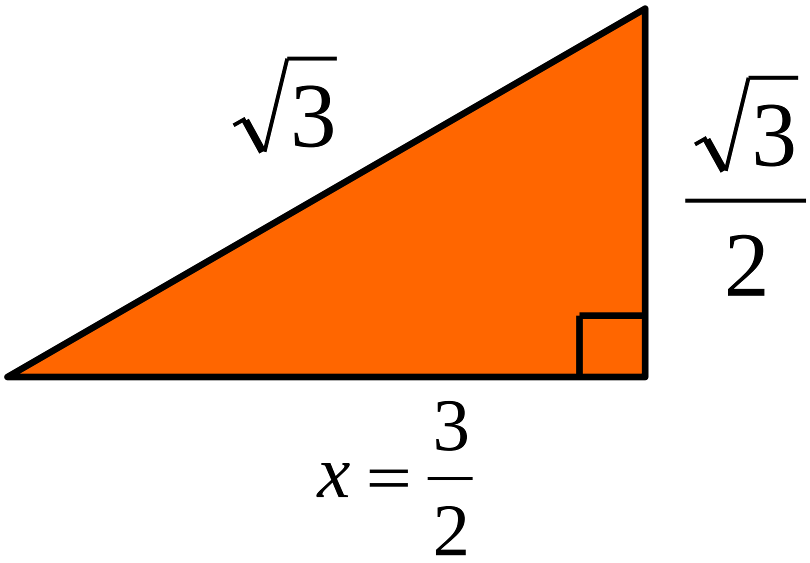
Press for hint



Solution

Answer: B

Justification: The ratio of the length of sides in a 30-60-90 triangle is $1:\sqrt{3}:2$. Multiplying this ratio by $\frac{\sqrt{3}}{2}$ so that the hypotenuse is $\sqrt{3}$ gives a ratio of $\frac{\sqrt{3}}{2}:\frac{3}{2}:\sqrt{3}$.



Alternative solution:

Using the Pythagorean Theorem:

$$x^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = (\sqrt{3})^2$$

$$x^2 = \frac{9}{4}$$

$$x = \pm\frac{3}{2}$$

$$x = \frac{3}{2}$$