a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

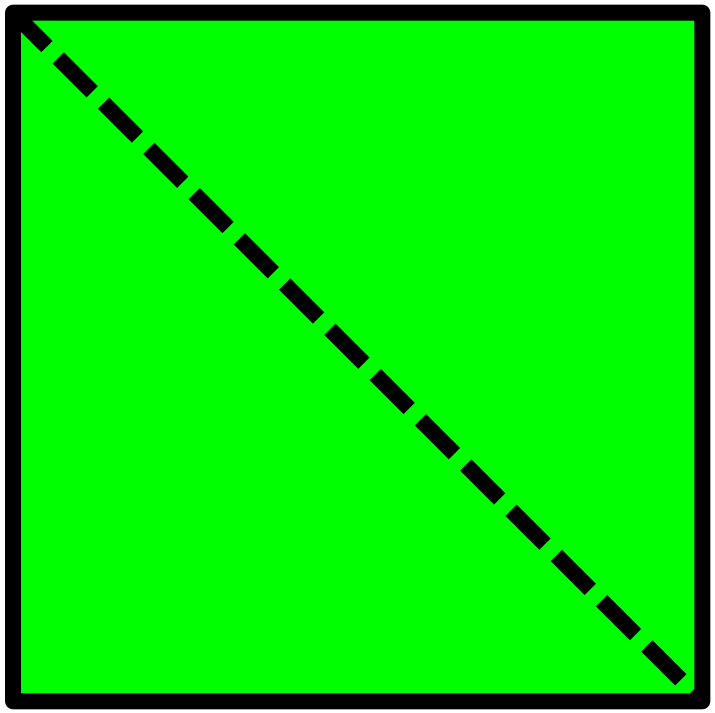
Mathematics

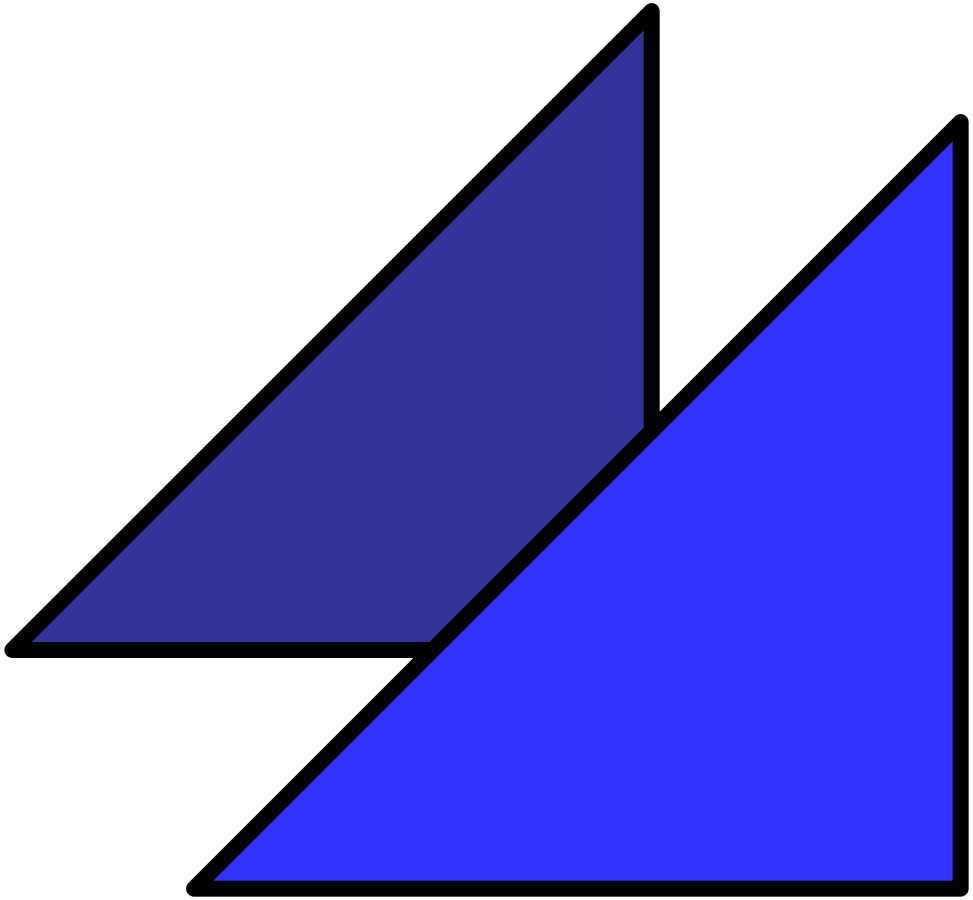
Trigonometry: Special Triangles (45-45-90)

Science and Mathematics Education Research Group

Supported by UBC Teaching and Learning Enhancement Fund 2012-2013

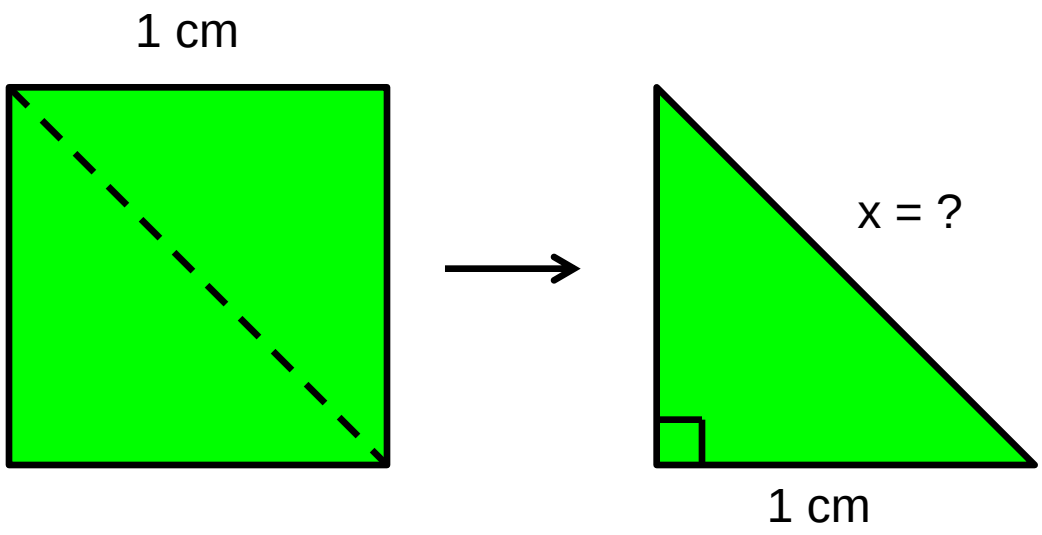
Special Triangles 45-45-90





The 45-45-90 Triangle I

Consider a square with side length 1 cm. The square is cut along its diagonal. What is the length of the hypotenuse of the resulting triangle?



A. 1

B. $\frac{3}{2}$

C. 2

D. $\sqrt{2}$

E. $\sqrt{3}$

Solution

Answer: D

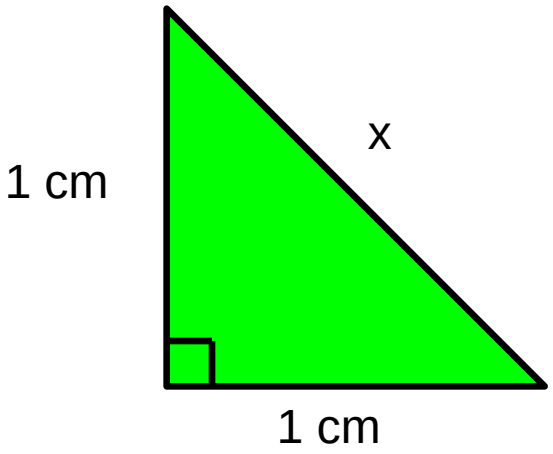
Justification: Using the Pythagorean Theorem:

$$1^2 + 1^2 = x^2$$

$$x^2 = 2$$

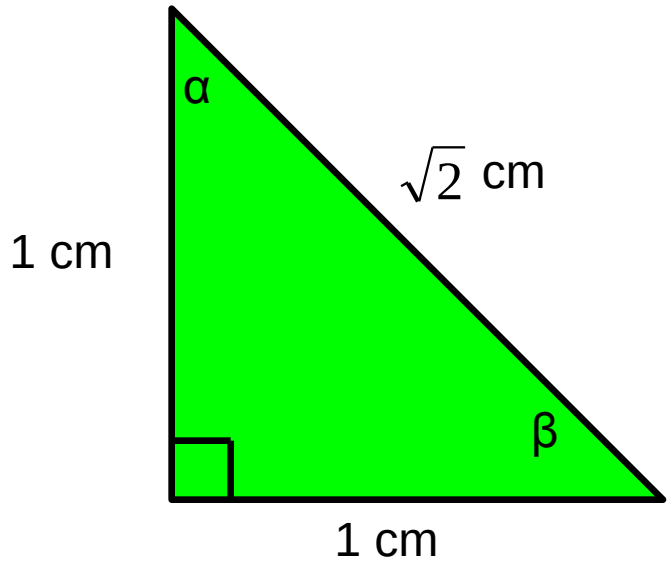
$$x = \pm\sqrt{2}$$

$$x = \sqrt{2} \text{ cm}$$



The 45-45-90 Triangle II

Consider a square with side length 1 cm. The square is cut along its diagonal. What are the angles alpha (α) and beta (β)?



A. α = 30°, β = 30°

B. α = 45°, β = 45°

C. α = 50°, β = 50°

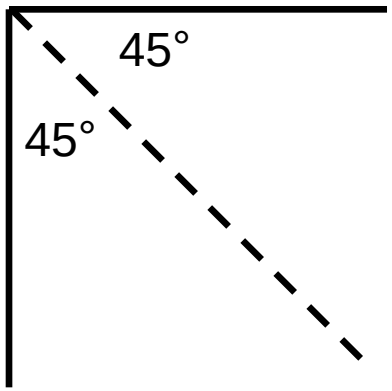
D. α = 60°, β = 60°

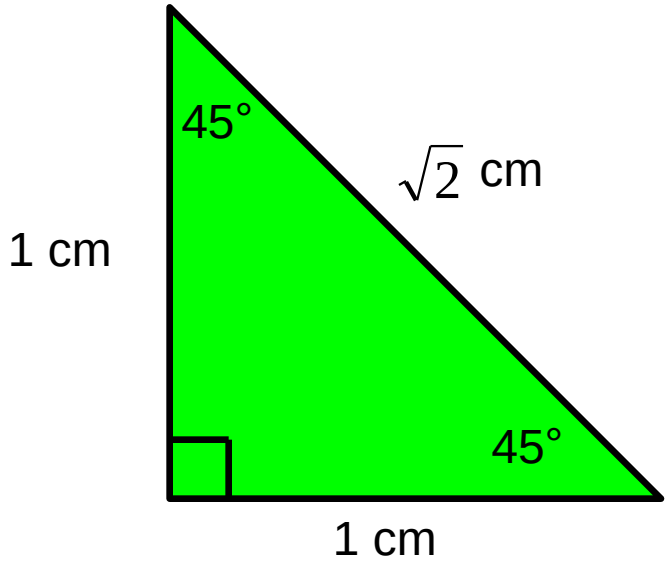
E. α = 90°, β = 90°

Solution

Answer: B

Justification: Cutting the square along its diagonal divides the 90° corner of the square into two smaller 45° angles. (Note that this is only true for a square and not rectangles)

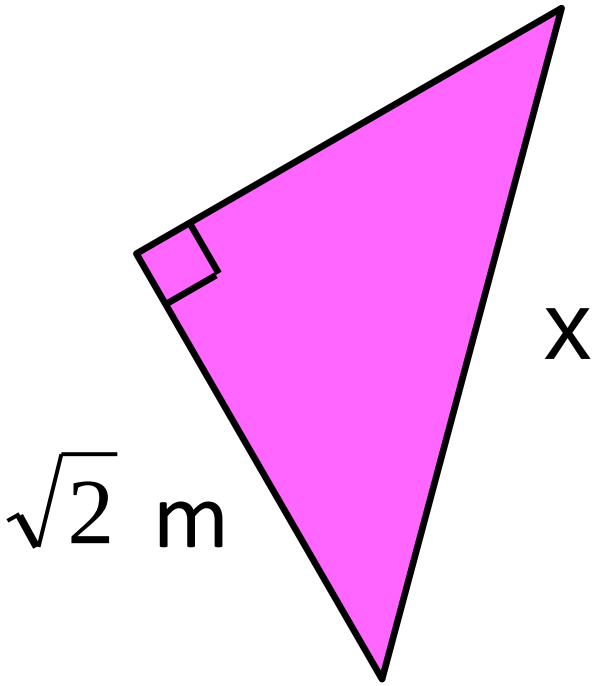




Check: 45° + 45° + 90° = 180°, as required for triangles.

The 45-45-90 Triangle III

The triangle below is a 45-45-90 triangle. What is the length of the side labelled x?



A. $x = 2$ m

B. $x = 4$ m

C. $x = \sqrt{2}$ m

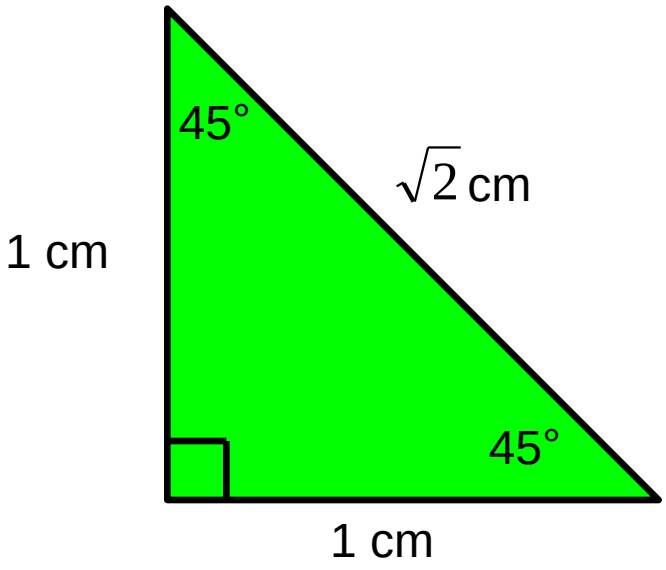
D. $x = 2\sqrt{2}$ m

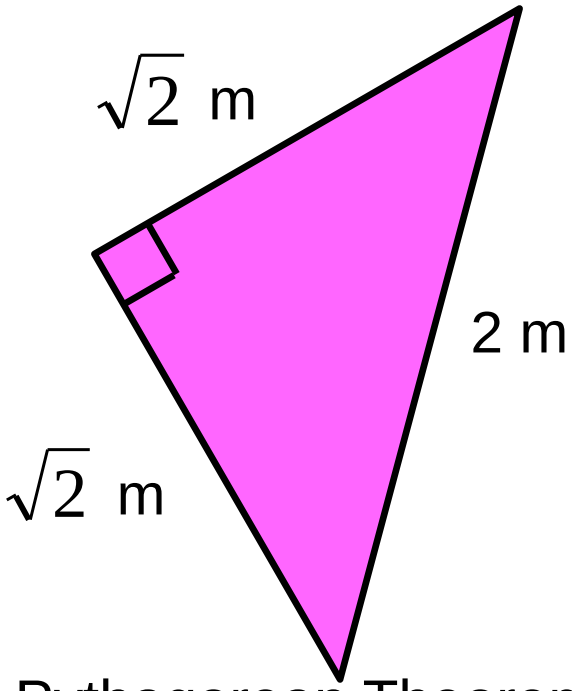
E. $x = \sqrt{\sqrt{2}}$ m

Solution

Answer: A

Justification: The ratio of the side of a 45-45-90 triangle is 1:1:$\sqrt{2}$. Multiplying this ratio by $\sqrt{2}$ so that the lengths of the shorter sides are $\sqrt{2}$. This gives a ratio of $\sqrt{2} : \sqrt{2} : 2$.

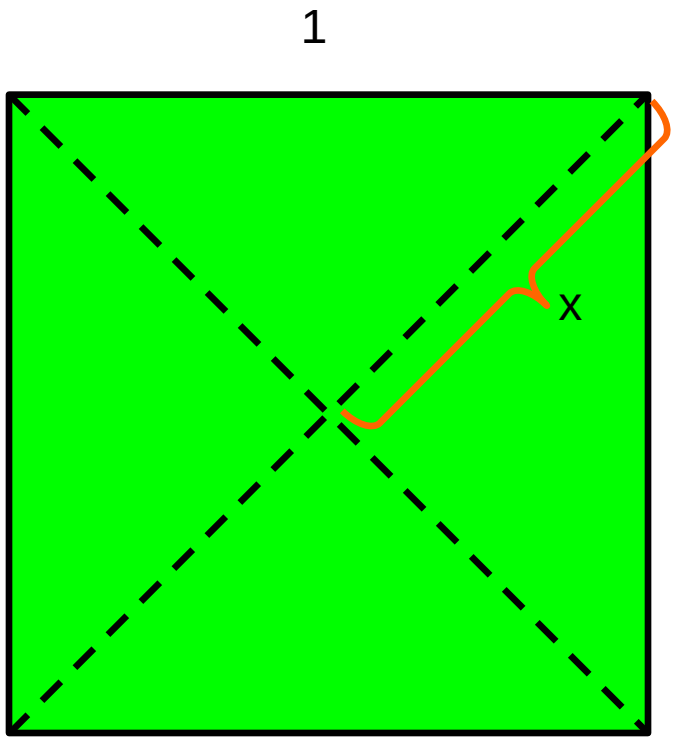




This may also be solved using the Pythagorean Theorem.

The 45-45-90 Triangle IV

A 1 by 1 square is cut along both its diagonals. What is the distance from the center of the square to one of its corners (the distance labelled x)?



A. $x = \frac{1}{2}$

B. $x = 2$

C. $x = \frac{\sqrt{2}}{2}$

D. $x = \sqrt{2}$

E. $x = 2\sqrt{2}$

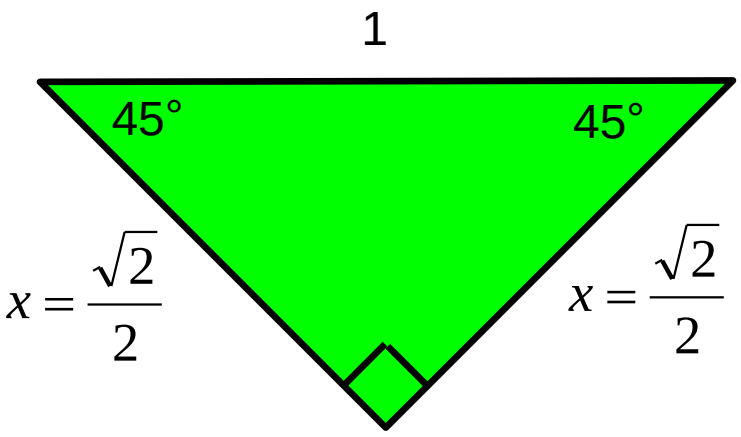
Solution

Answer: C

Justification: Cutting the 1 by 1 square along its diagonals gives a 45-45-90 triangle with hypotenuse 1, so the ratio of the lengths of the sides is x : x :1. The ratio 1:1:$\sqrt{2}$ must be scaled so that the hypotenuse is 1. Dividing by $\sqrt{2}$ gives

$$x : x : 1 = 1 : 1 : \sqrt{2} \quad \text{(Divide by } \sqrt{2}\text{)}$$

$$x : x : 1 = \frac{1}{\sqrt{2}} : \frac{1}{\sqrt{2}} : 1$$



Therefore, equating the x gives:

$$x = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$