a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Mathematics
Trigonometry: Reference Angles

Science and Mathematics
Education Research Group

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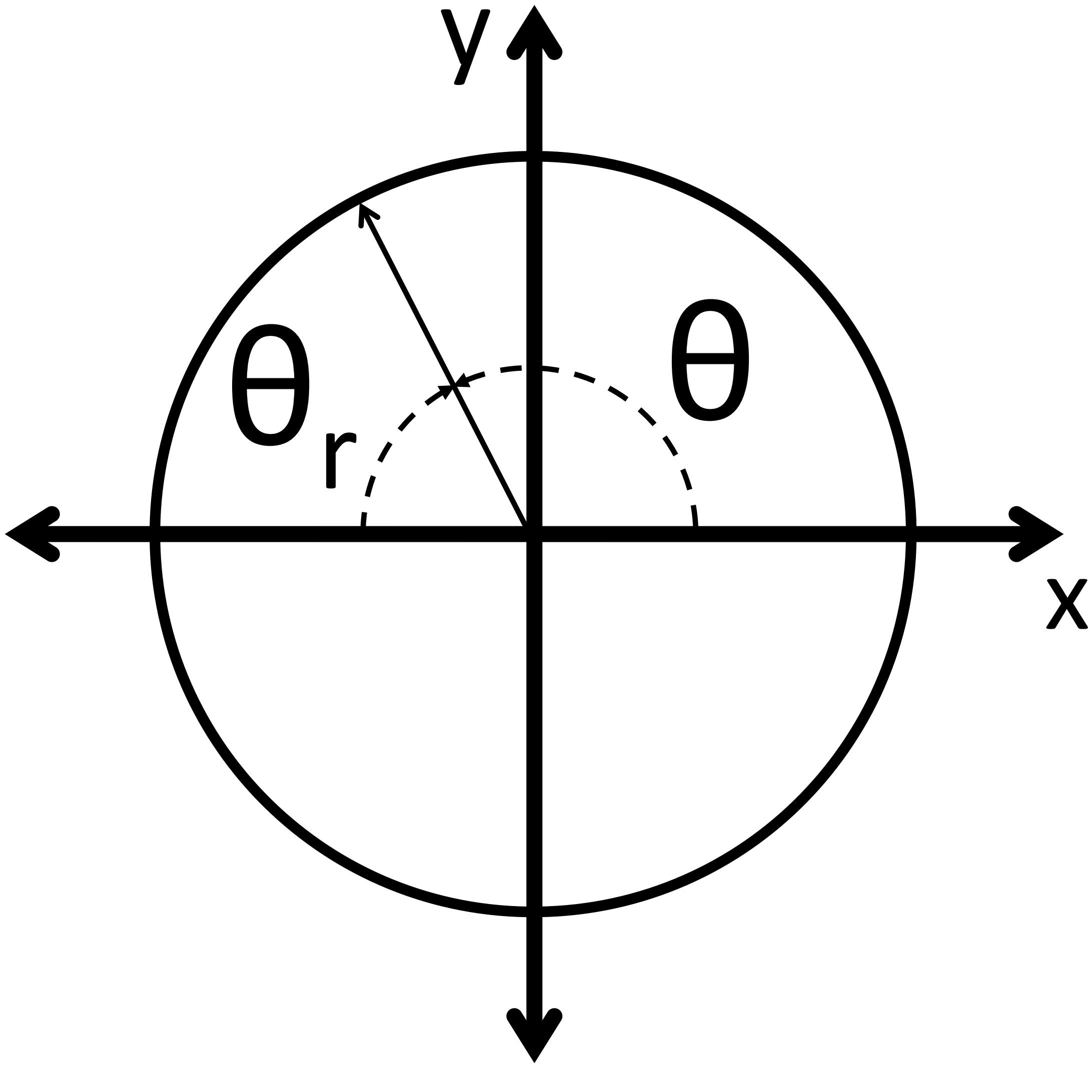
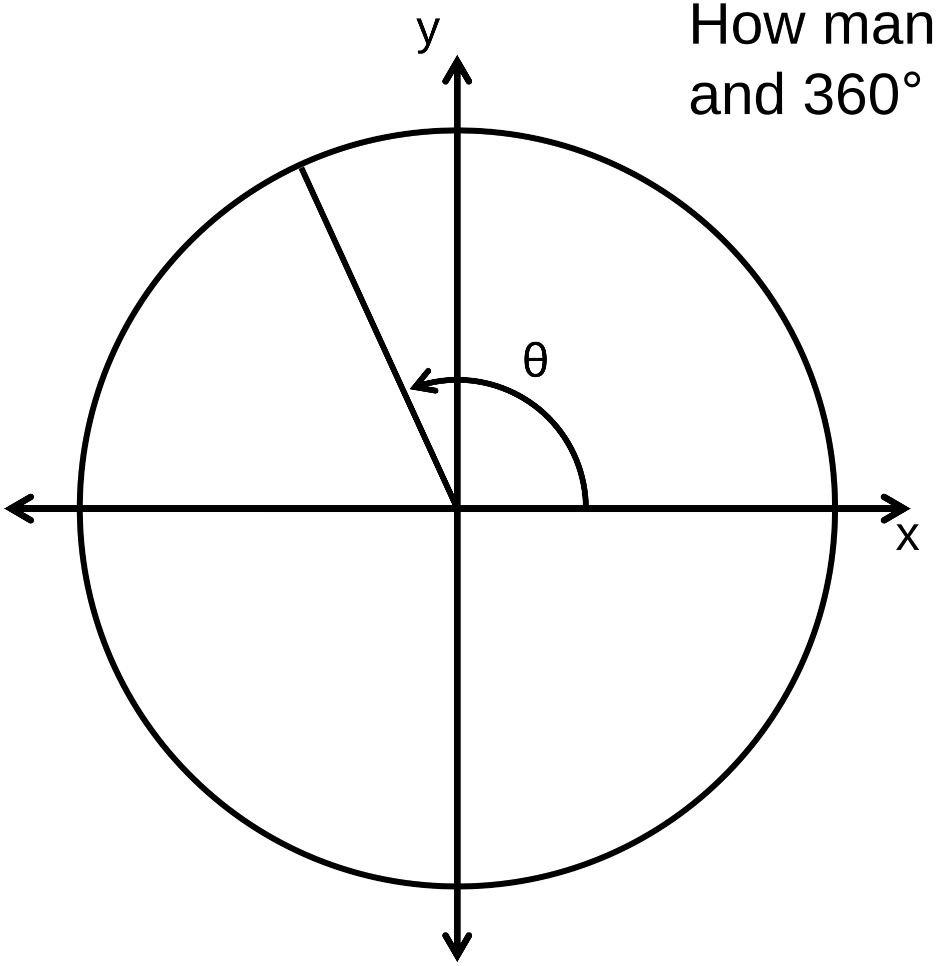
Trigonometric Ratios Using Reference Angles



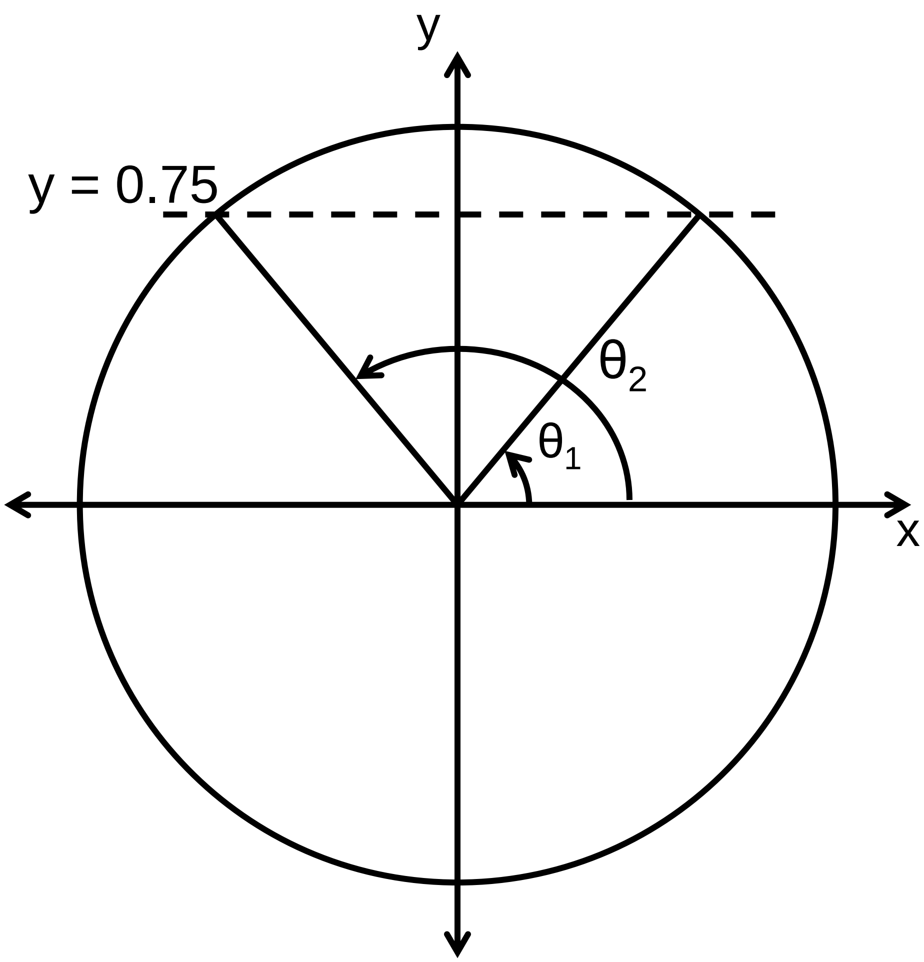
Reference Angles I



How many different values of θ between 0° and 360° are there such that sin(θ) = 0.75?

A. 0 values of θ
B. 1 value of θ
C. 2 values of θ
D. 3 values of θ
E. 4 values of θ

Solution



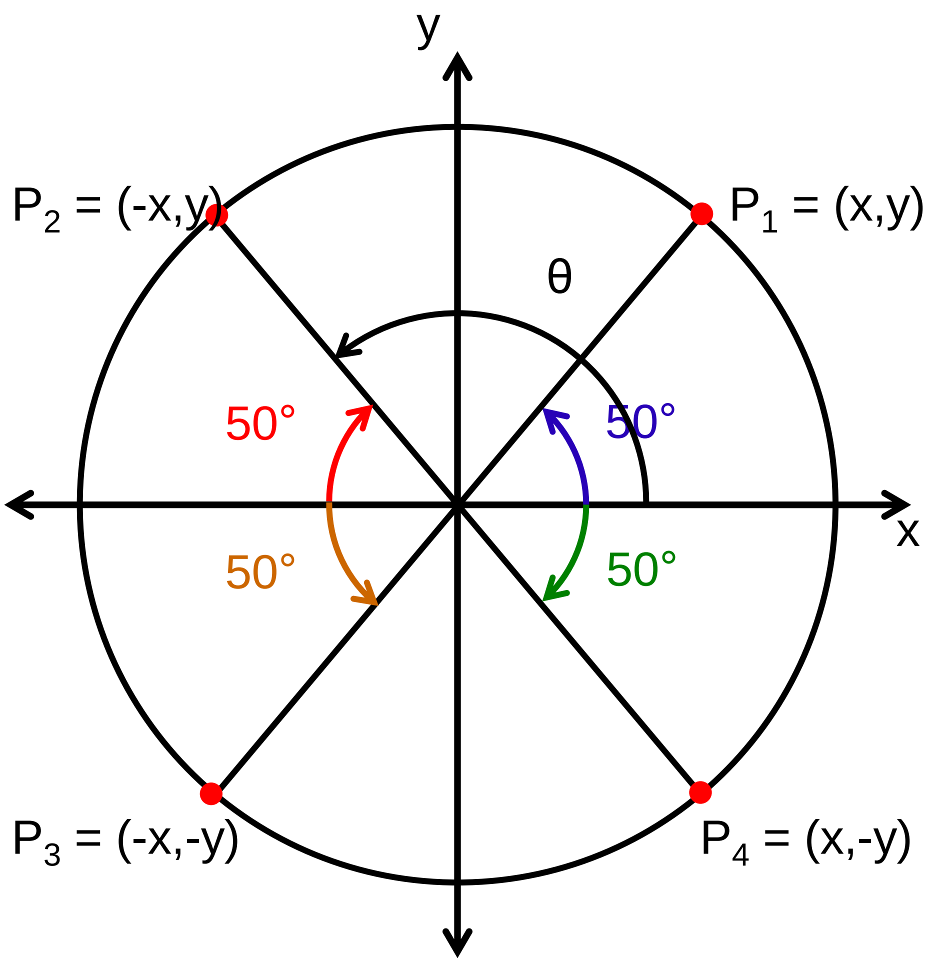
Answer: C

Justification: If sin(θ) = 0.75, then the y-coordinate on the unit circle must be 0.75. This occurs where the line y = 0.75 crosses the unit circle.

The diagram shows that the line crosses the unit circle at 2 points, so there are 2 values for θ where sin(θ) = 0.75.

This problem set will go over how to find the other angle of θ.

Reference Angles II

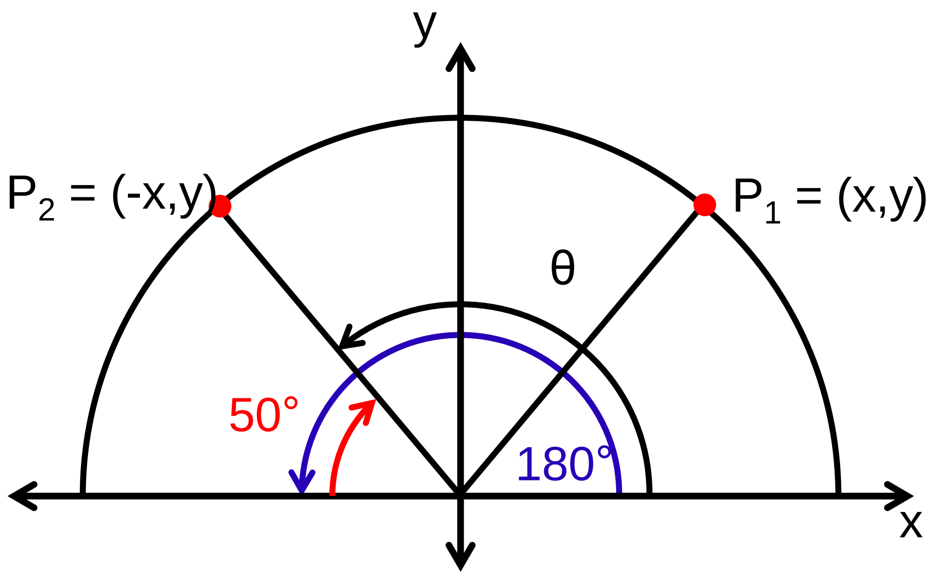


Consider the 4 points that are 50° from the x-axis. What is the angle θ (the angle to P_2)?

A. 100°
B. 110°
C. 120°
D. 130°
E. 150 °

Solution

Answer: D

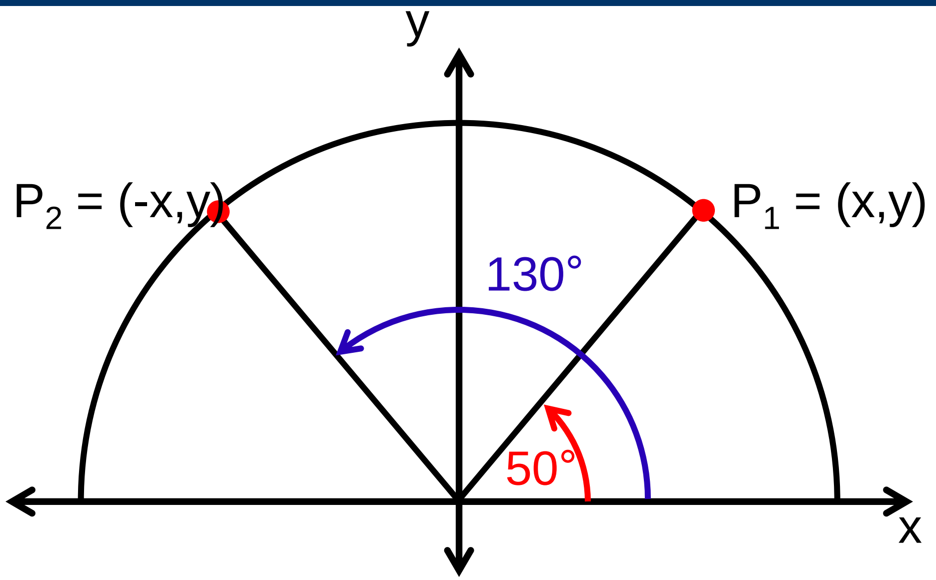


Justification: The diagram above shows that the angle θ can be calculated by: θ = 180° – 50° = 130°.

The acute angle to the x-axis from 130° is 50°, which is known as the reference angle of 130°.

Reference Angles III

Which of the statements below is true, if any?



A. sin(50°) = sin(130°) and cos(50°) = cos(130°)

B. sin(50°) = -sin(130°) and cos(50°) = cos(130°)

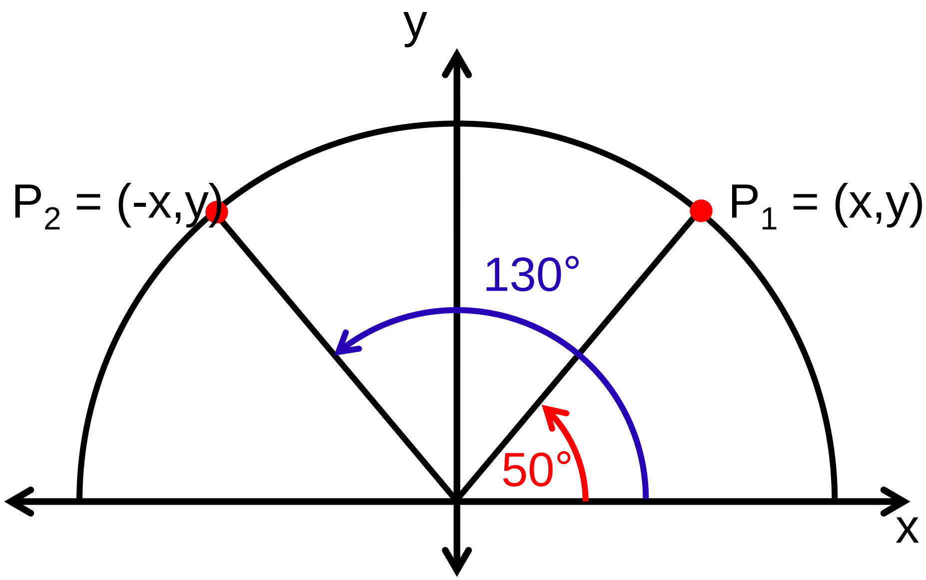
C. sin(50°) = sin(130°) and cos(50°) = -cos(130°)

D. sin(50°) = -sin(130°) and cos(50°) = -cos(130°)

E. None of the above are true

Solution

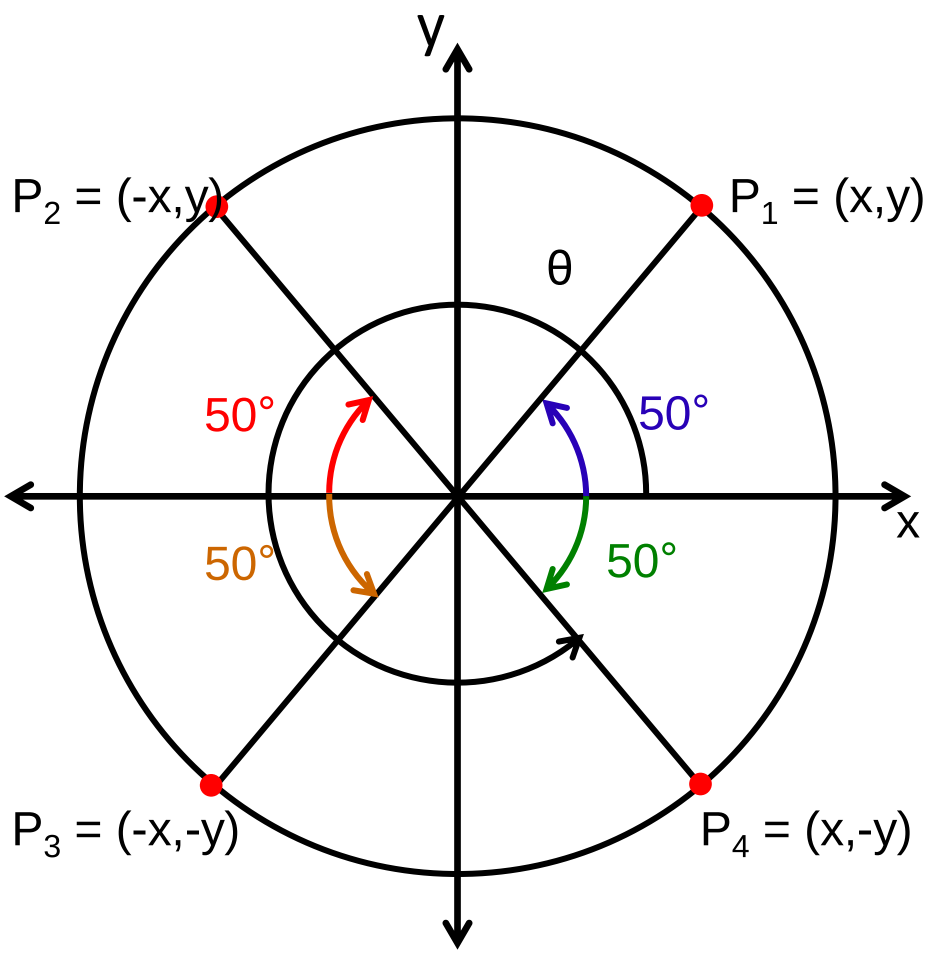
Answer: C



Justification: The y-coordinates of P_1 and P_2 are the same. Therefore sin(50°) = sin(130°).

The x-coordinate of P_2 is the same as P_1 except negative. Therefore cos(50°) = -cos(130°).

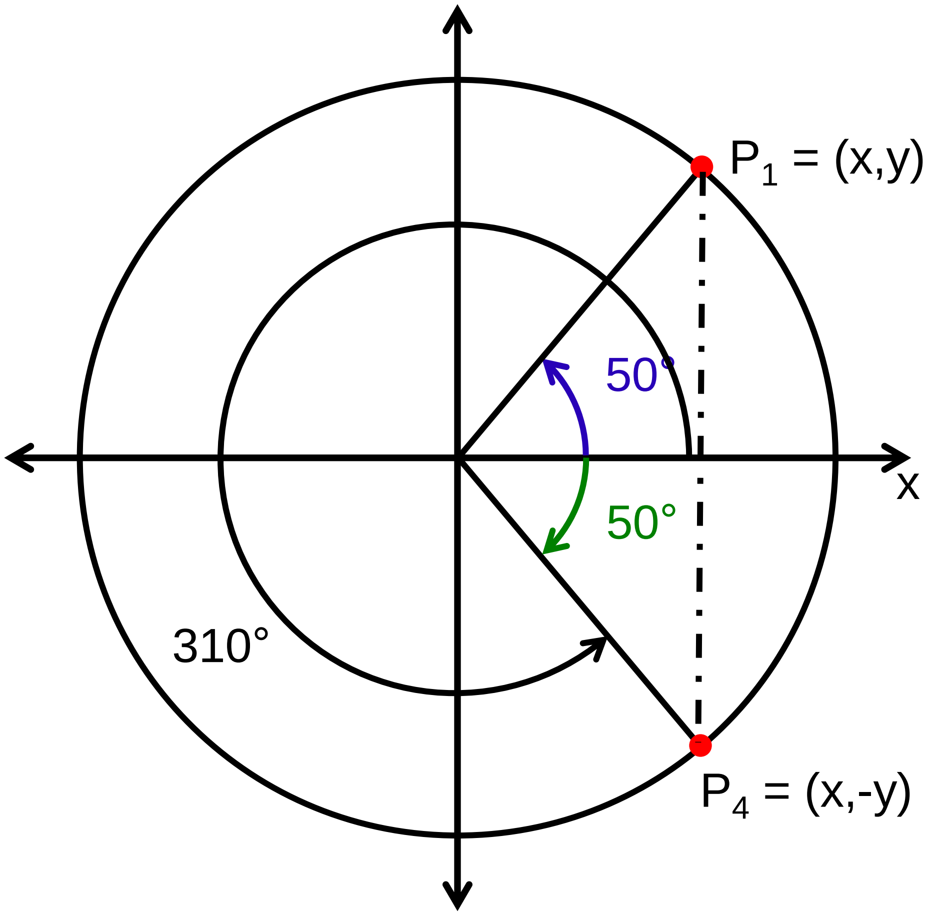
Reference Angles IV



For what value of θ in the 4th quadrant does cos(θ) = cos(50°)?

A. θ = 230°
B. θ = 290°
C. θ = 300°
D. θ = 310°
E. None of the above

Solution

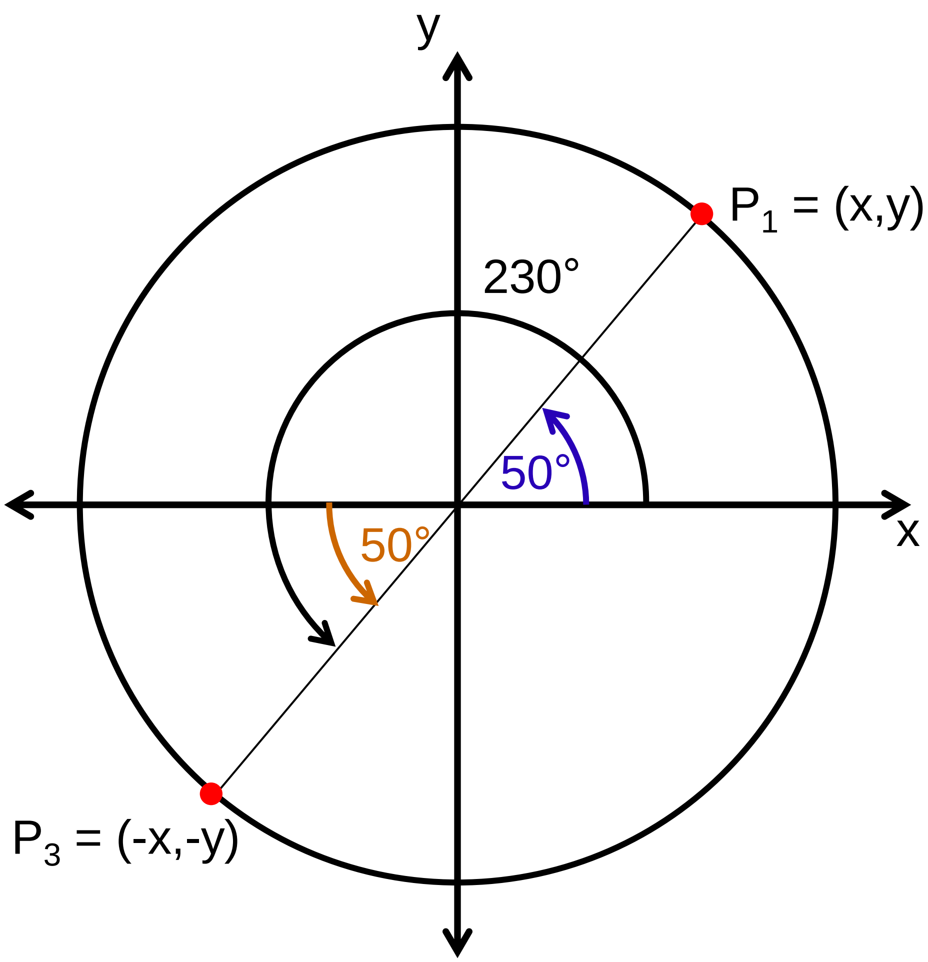


Answer: D

The angle to P_4 is 360° - 50° = 310° (the reference angle to 310° is 50°). At this point we can see that the x-coordinate of P_1 and P_4 are equal, so:

$$\cos(310°) = \cos(50°)$$

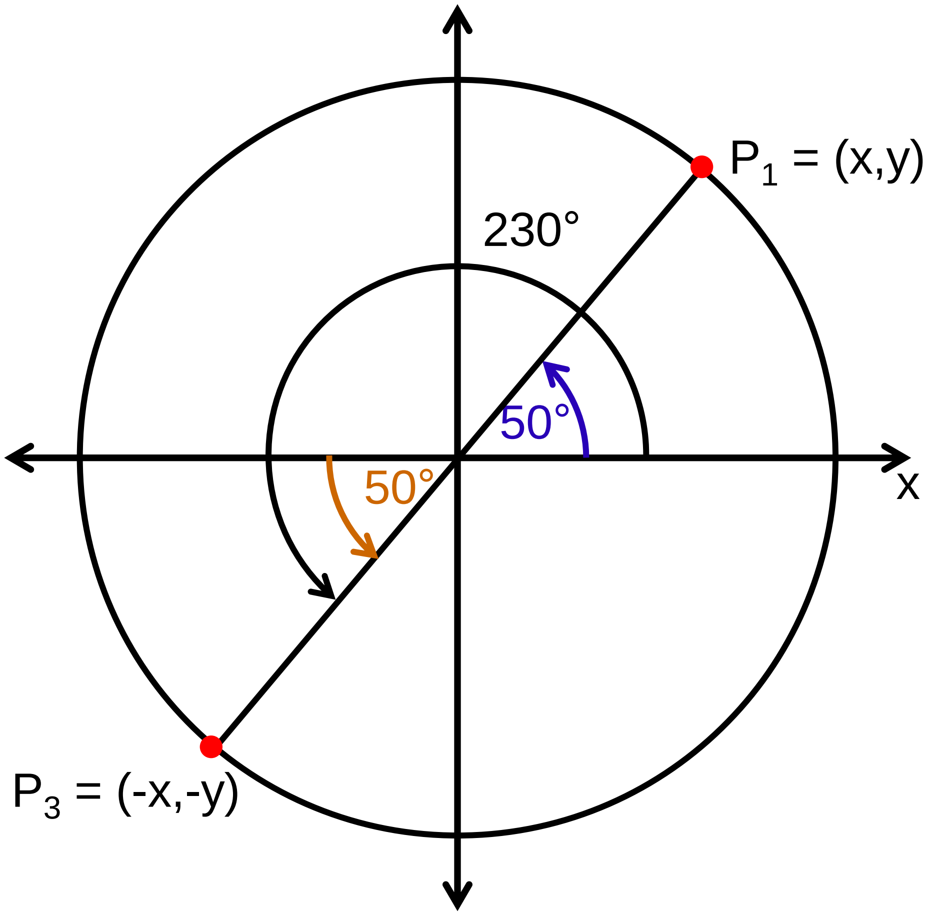
Reference Angles V



The angle to P_3 is 230°. The reference angle of 230° is 50°. Which of the following statements is true?

A. sin(50°) = sin(230°)
B. cos(50°) = cos(230°)
C. tan(50°) = tan(230°)
D. A and B are true
E. A, B and C are true

Solution



Answer: C

All 3 trigonometric ratios are positive in the first quadrant. The only trigonometric ratio that is positive in the 3rd quadrant is tangent. Only tan(50°) = tan(230°) is true.

However, since the x and y coordinates of P_3 are negative, we can also conclude that:

sin(50°) = -sin(230°)

cos(50°) = -cos(230°)

Summary

Quadrant II

$\sin(\theta) > 0$

$\cos(\theta) < 0$

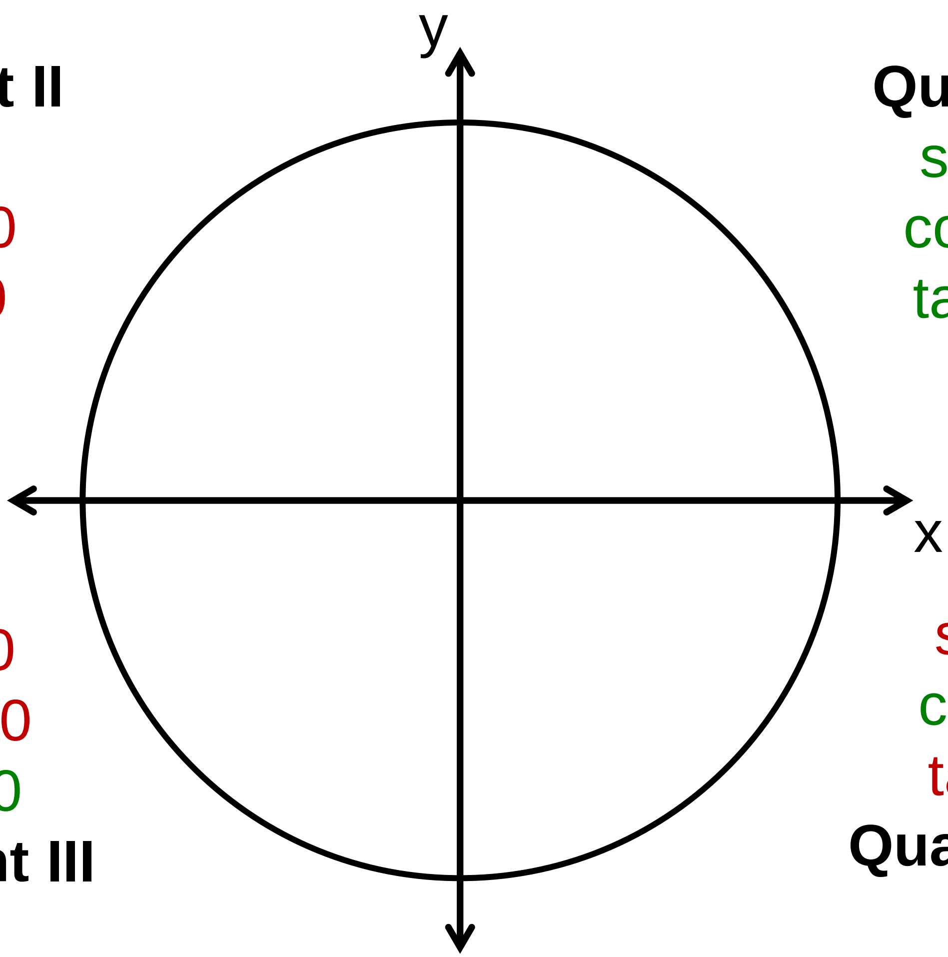
$\tan(\theta) < 0$

Quadrant I

$\sin(\theta) > 0$

$\cos(\theta) > 0$

$\tan(\theta) > 0$



$\sin(\theta) < 0$

$\cos(\theta) < 0$

$\tan(\theta) > 0$

Quadrant III

$\sin(\theta) < 0$

$\cos(\theta) > 0$

$\tan(\theta) < 0$

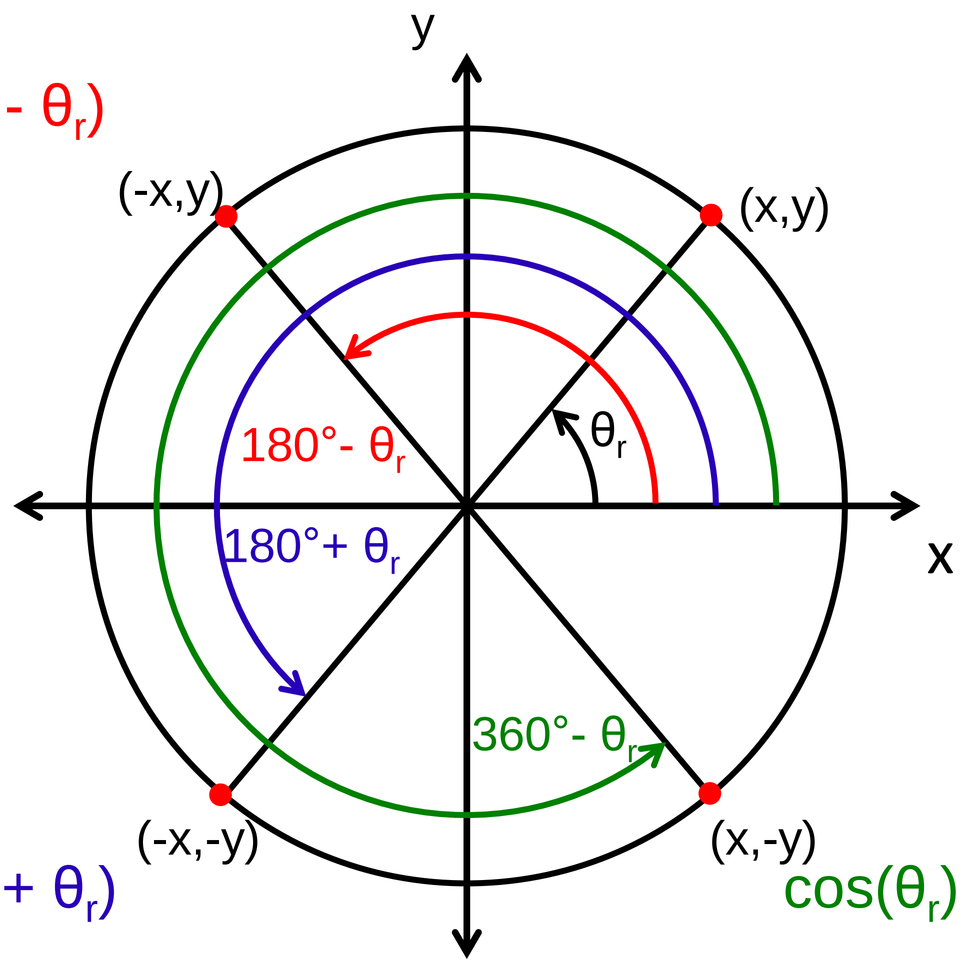
Quadrant IV

Summary

Quadrant II

$\sin(\theta_r) = \sin(180° - \theta_r)$

Quadrant I



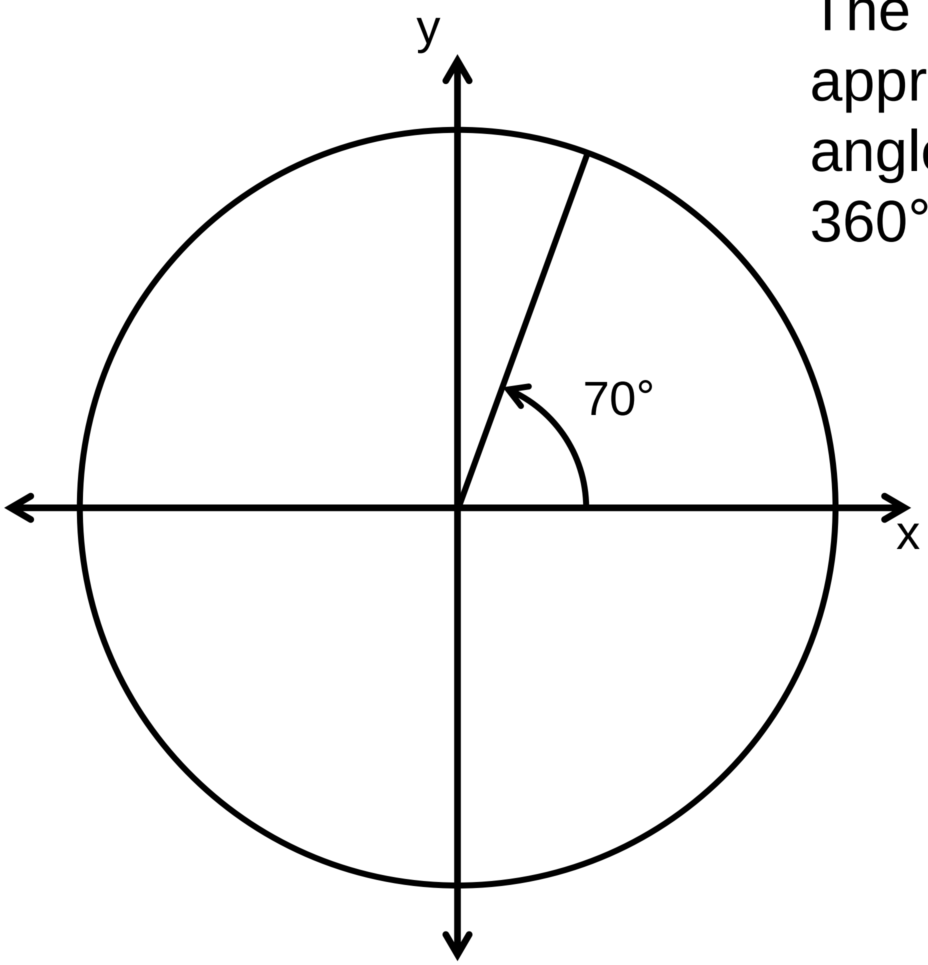
$\tan(\theta_r) = \tan(180° + \theta_r)$

Quadrant III

$\cos(\theta_r) = \cos(360° - \theta_r)$

Quadrant IV

Reference Angles VI



The value of cos(70°) is approximately 0.34. At what other angle does cos(θ) = 0.34, for 0° ≤ θ ≤ 360°?

A. θ = 110°

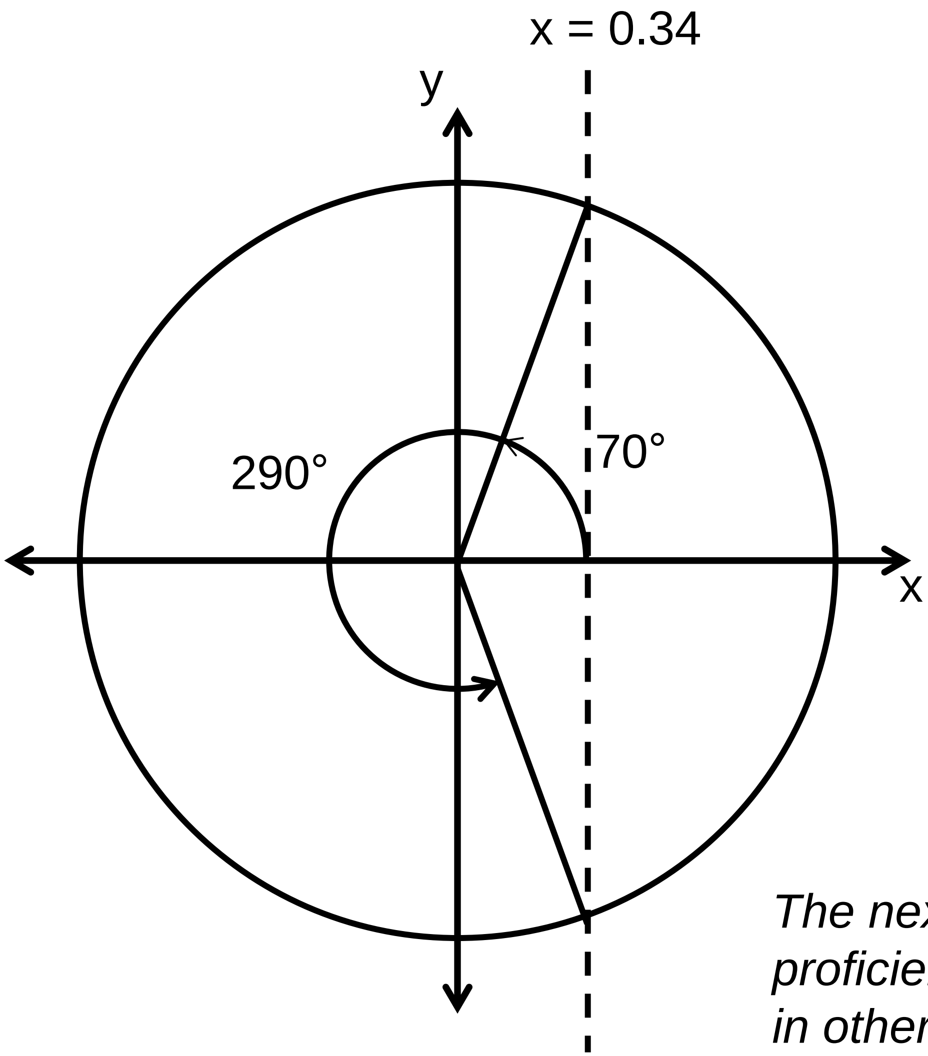
B. θ = 250°

C. θ = 290°

D. θ = 340°

E. cos(70°) = 0.34 for only 1 value of θ

Solution



Answer: C

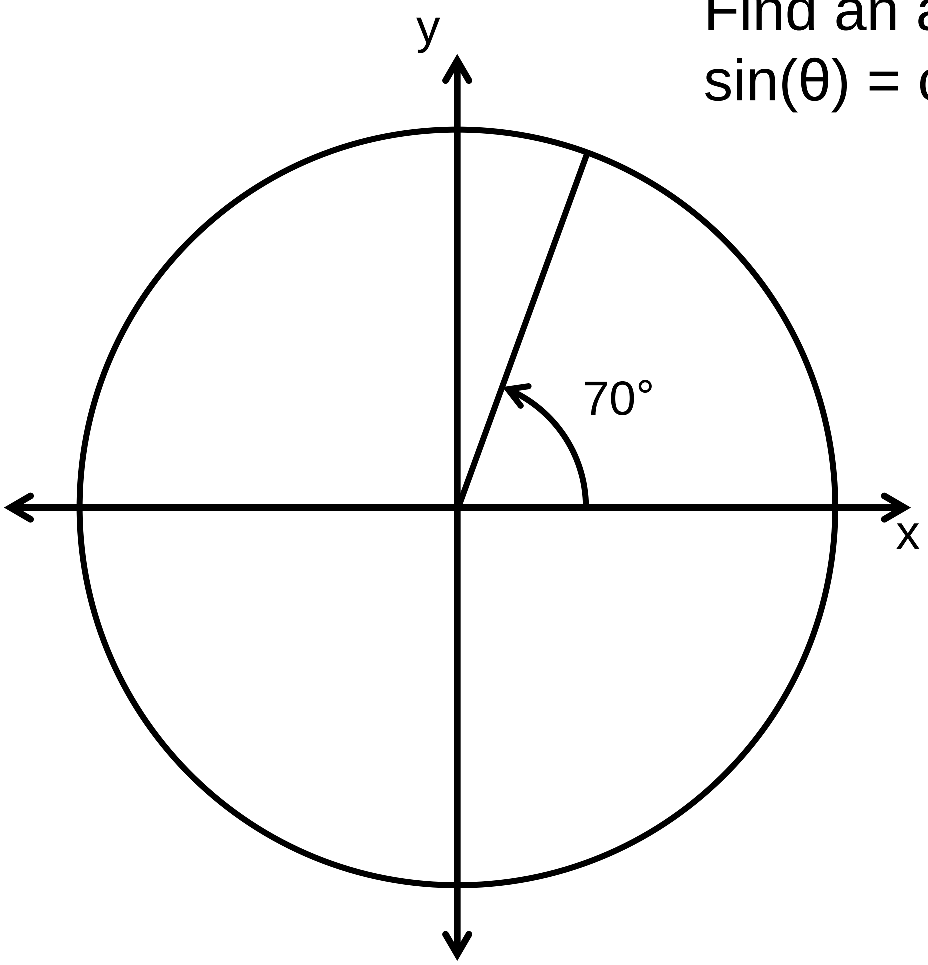
Justification: The value of cos(θ) is the same where the line x = 0.34 intersects the unit circle (these 2 points have the same x-coordinate).

Cosine is positive in the 1st and 4th quadrants. The angle whose reference angle is 70° in the 4th quadrant is 360° - 70° = 290°.

cos(270°) = cos(70°) = 0.34

The next questions expect students to be proficient at finding equivalent trigonometric ratios in other quadrants.

Reference Angles VII



Find an angle between 90° and 180° where $\sin(\theta) = \cos(70°)$.

A. $\theta = 110°$

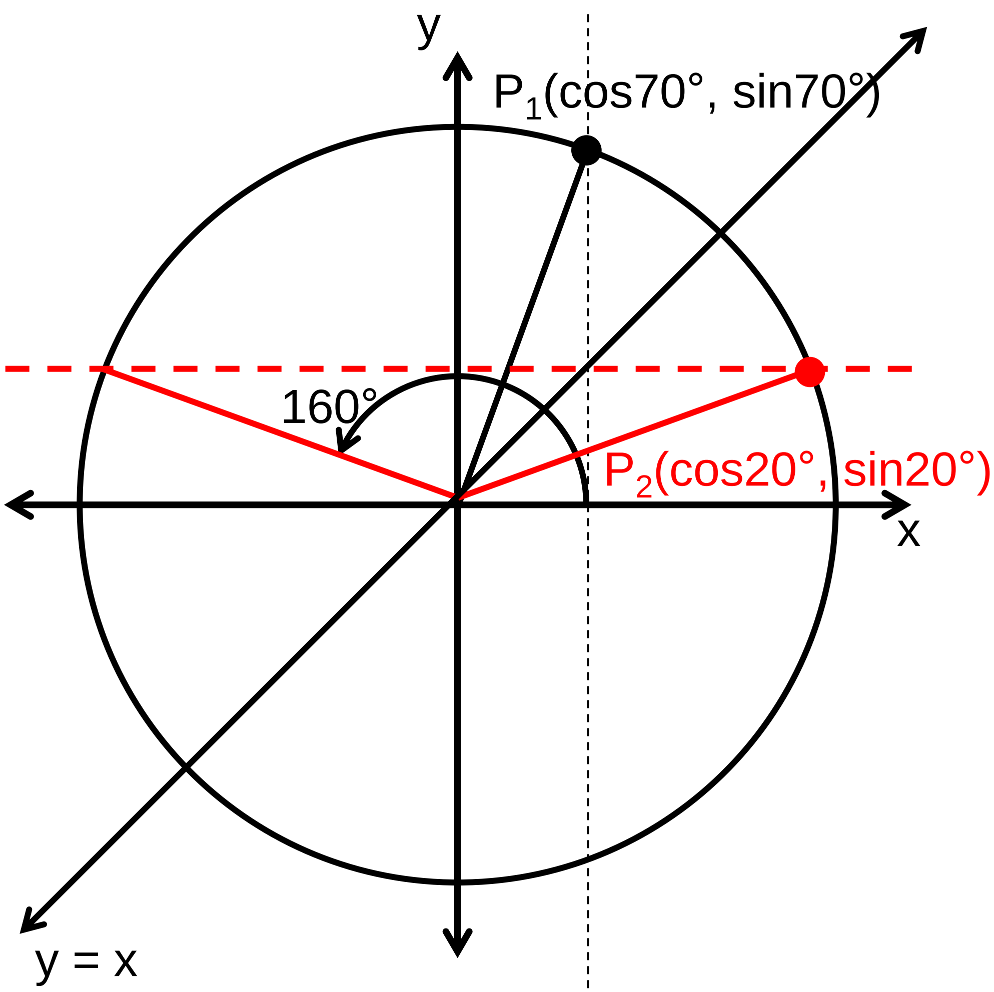
B. $\theta = 120°$

C. $\theta = 150°$

D. $\theta = 160°$

E. No such value of θ exists

Solution



Answer: D

Justification: Reflecting the point P_1 through the line y = x gives P_2 = (sin70°, cos70°) by interchanging the x and y coordinates. However, the diagram shows P_2 can be written as (cos20°, sin20°). Equating these two expressions for P_2 gives:

$$(\sin 70°, \cos 70°) = (\cos 20°, \sin 20°).$$

$$\cos(70°) = \sin(20°)$$

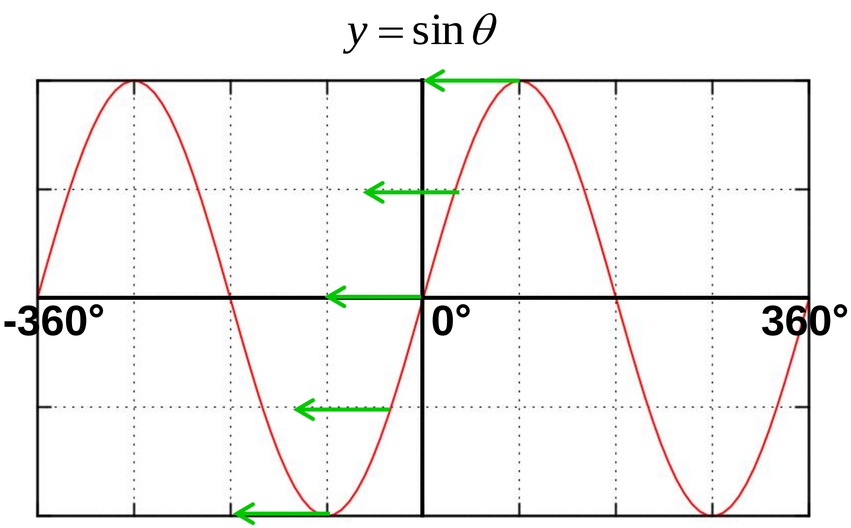
Finally, the equivalent of sin(20°) in the 2nd quadrant is sin(160°).

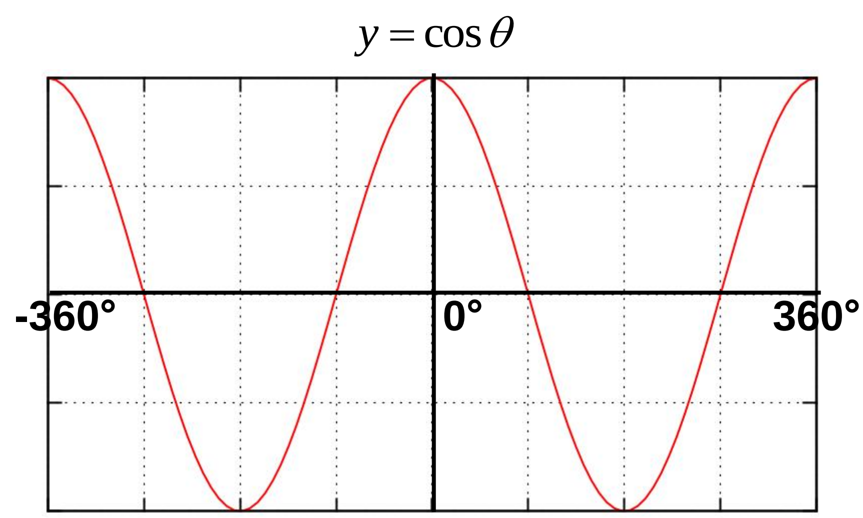
In general: $\cos(\theta) = \sin(90° - \theta)$

Alternative Solution

Answer: D

Justification: The graphs of sine and cosine are shown below:



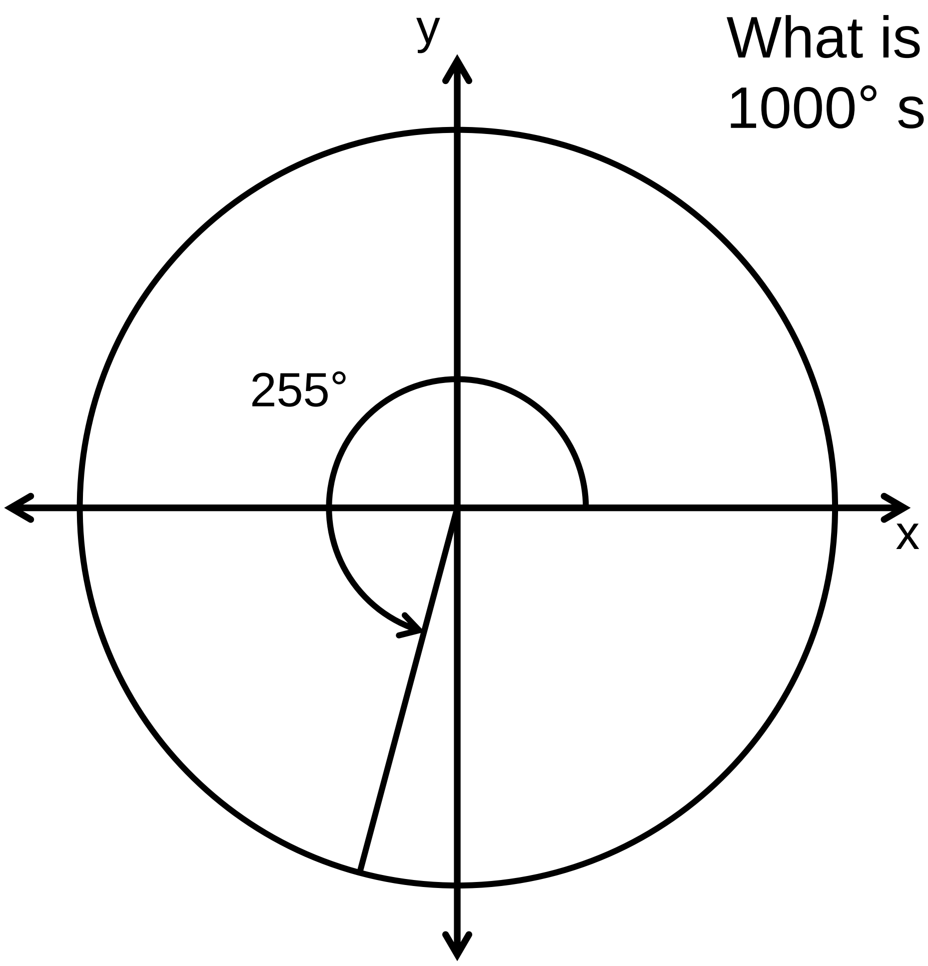


Phase shifting the sine graph to the left by 90° (by replacing θ with θ+90°) gives the cosine graph. This gives us the identity

$$\cos(\theta) = \sin(\theta + 90°).$$

When θ=70°, cos(70°) = sin(160°), which agrees with our previous solution.

Reference Angles VIII



What is the smallest angle θ greater than 1000° such that sin(θ) = sin(255°)?

A. $\theta = 1005°$

B. $\theta = 1155°$

C. $\theta = 1185°$

D. $\theta = 1335°$

E. No such value of θ exists

Solution

Answer: A

Justification: Adding multiples of 360° to θ does not change the value of sin(θ). So,

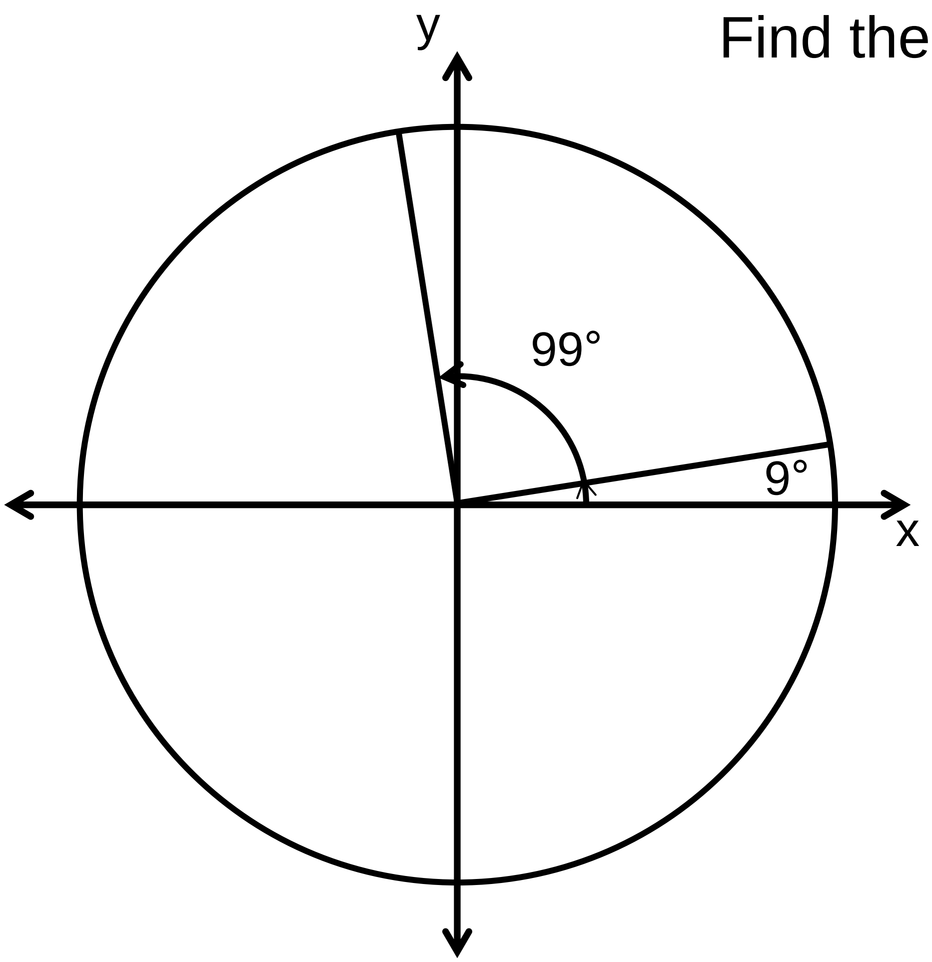
$$\sin(255°) = \sin(615°) = \sin(975°) = \sin(1335°)$$

However, this is not the smallest angle greater than 1000°. The equivalent of sin(255°) in the 4^{th} quadrant is sin(285°). Adding multiples of 360° to 285° gives:

$$\sin(285°) = \sin(645°) = \sin(1005°)$$

Therefore the smallest angle of θ greater than 1000° where sin(θ) = sin(255°) is θ = 1005°.

Reference Angles IX



Find the smallest positive angle θ where:

$$\tan\theta = \frac{\sin(99°)}{\cos(9°)}$$

A. θ = 0°

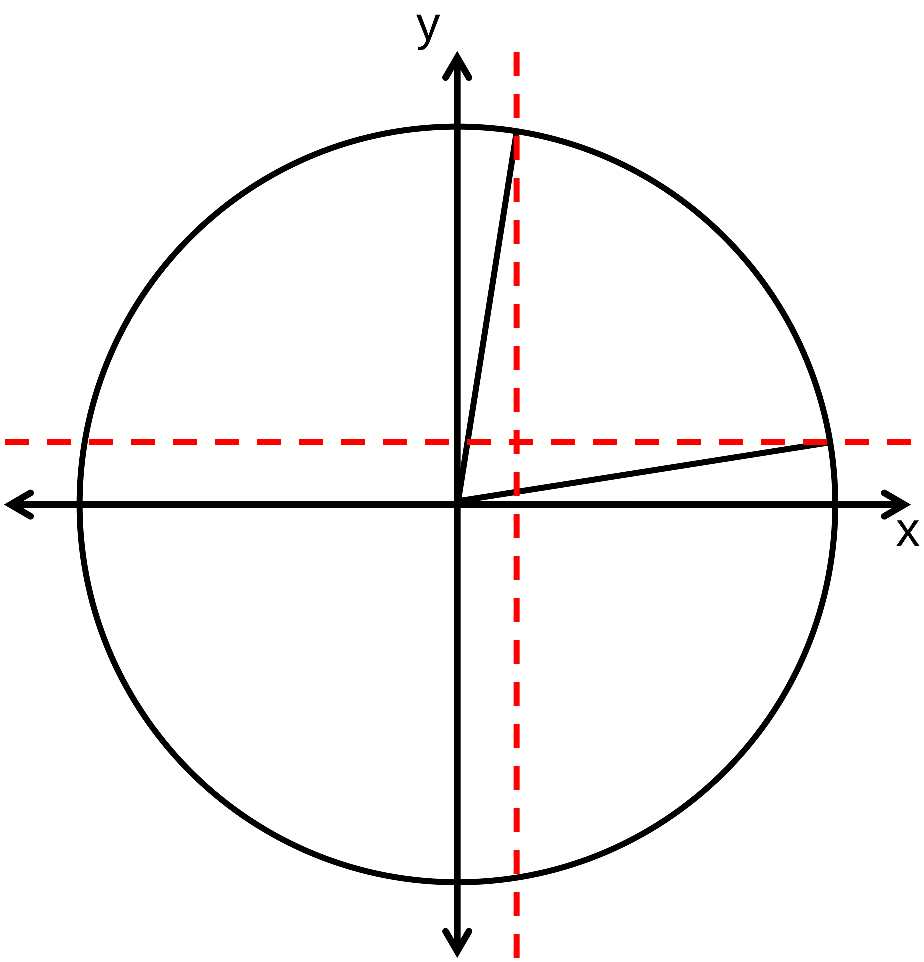
B. θ = 30°

C. θ = 45°

D. θ = 60°

E. θ = 90°

Solution



Answer: C

Justification: The equivalent of sin(99°) in the first quadrant is sin(81°).

Using the same argument from question 7, we can conclude that:

sin(81°) = cos(9°)

Therefore:

$$\frac{\sin(99°)}{\cos(9°)} = 1 = \tan(45°)$$