a place of mind

FACULTY OF EDUCATION

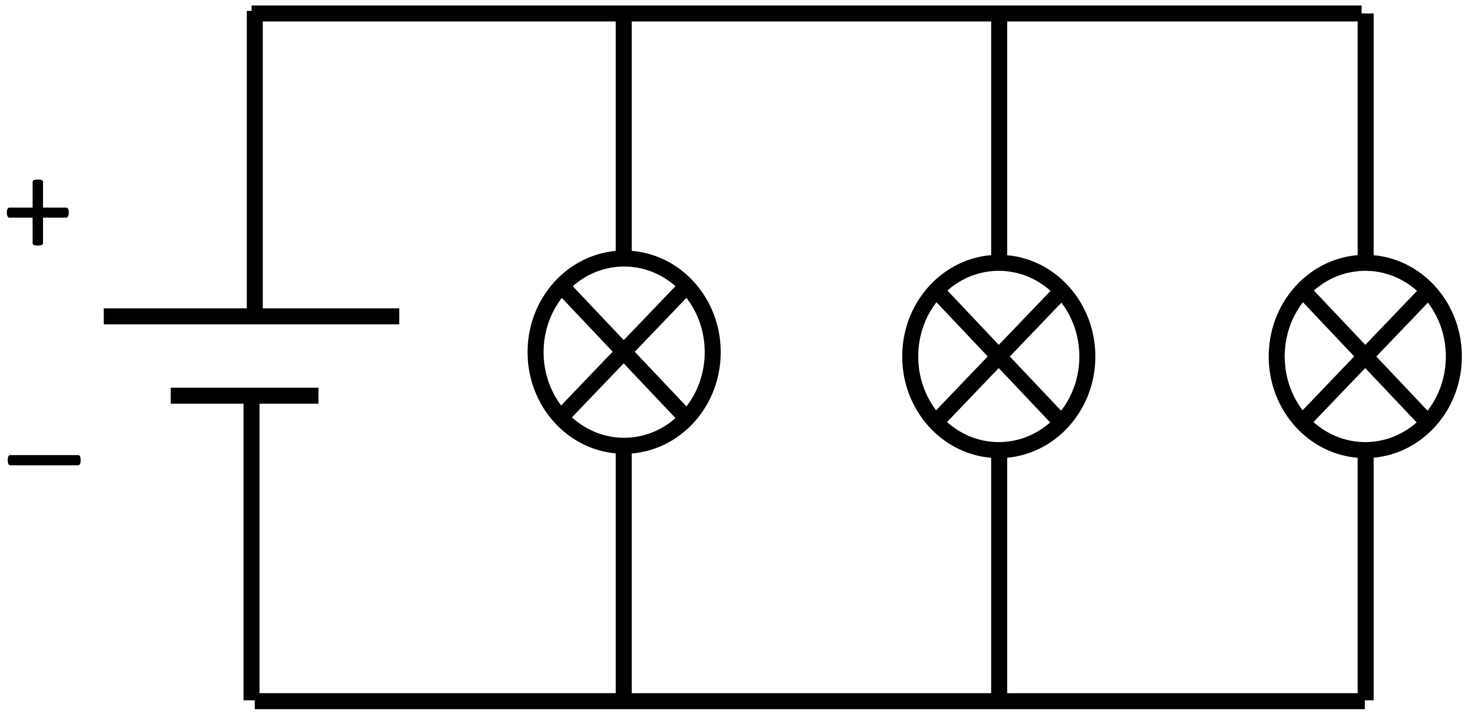
Department of
Curriculum and Pedagogy

Physics
Circuits: Parallel

Science and Mathematics
Education Research Group

Supported by UBC Teaching and Learning Enhancement Fund 2012-2013

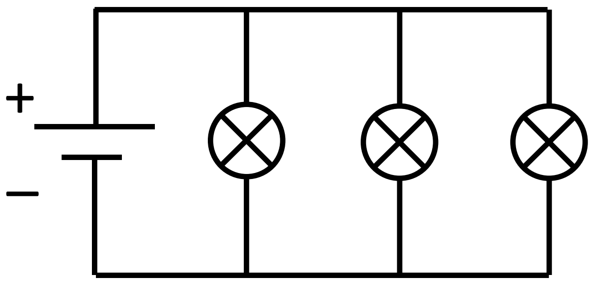
Parallel Circuits



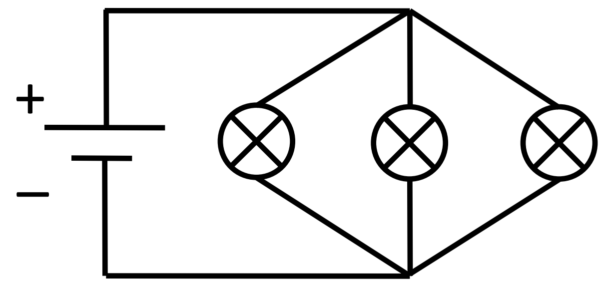
Parallel Circuits I

How many of these circuits demonstrate three light bulbs connected in parallel with each other?

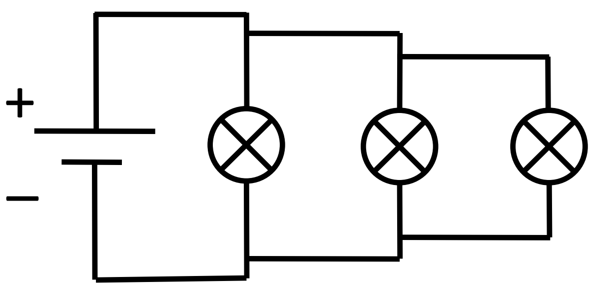
I.



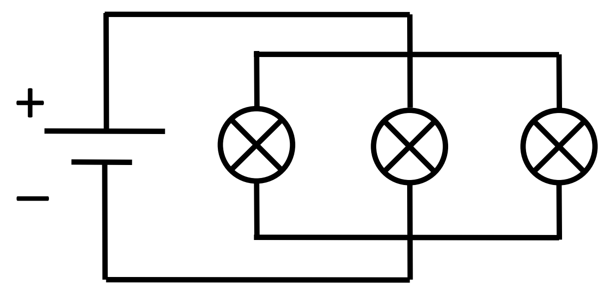
II.



III.



IV.



A. None

B. One

C. Two

D. Three

E. All Four

Solution

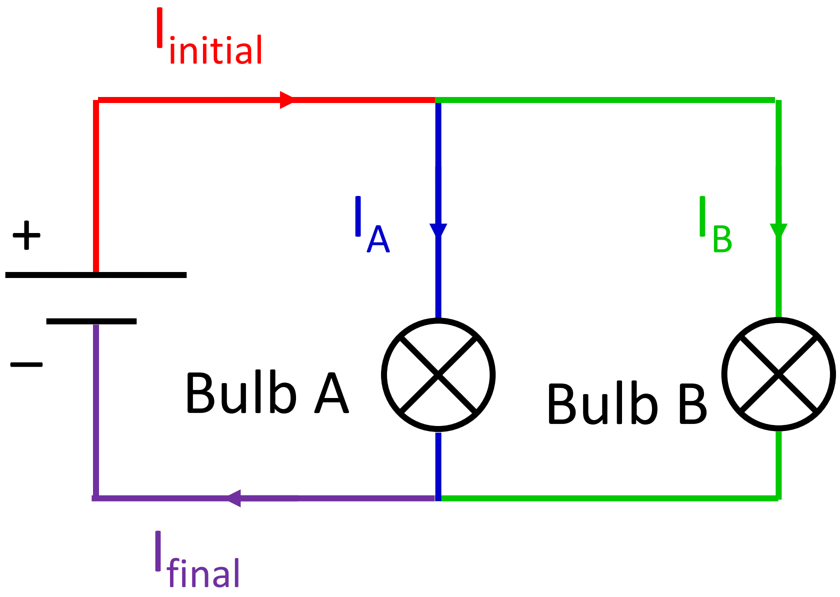
Answer: E

Justification: In each circuit, there is more than one path for the current to follow. Each circuit has three separate branches with a single light bulb in each branch. Therefore, all of the circuits have three light bulbs connected in parallel.

Moreover, even though the circuits might look differently, they are completely equivalent – they represent identical physical scenarios – the same currents flow through each one of the bulbs in every circuit.

Parallel Circuits II

Current $I_{initial}$ leaves the positive terminal of the battery. The current then splits into I_A and I_B as shown. Current I_{final} enters the negative terminal of the battery. Which of the following correctly describes the relationship between $I_{initial}$, I_A, I_B and I_{final}? (The light bulbs do not necessarily have the same resistance.)



A. $I_{initial} > I_A > I_B > I_{final}$

B. $I_{initial} = 2I_A = 2I_B = I_{final}$

C. $I_{initial} = I_A + I_B = I_{final}$

D. $I_{initial} = I_A - I_B = I_{final}$

E. Depends on the resistance of the bulbs

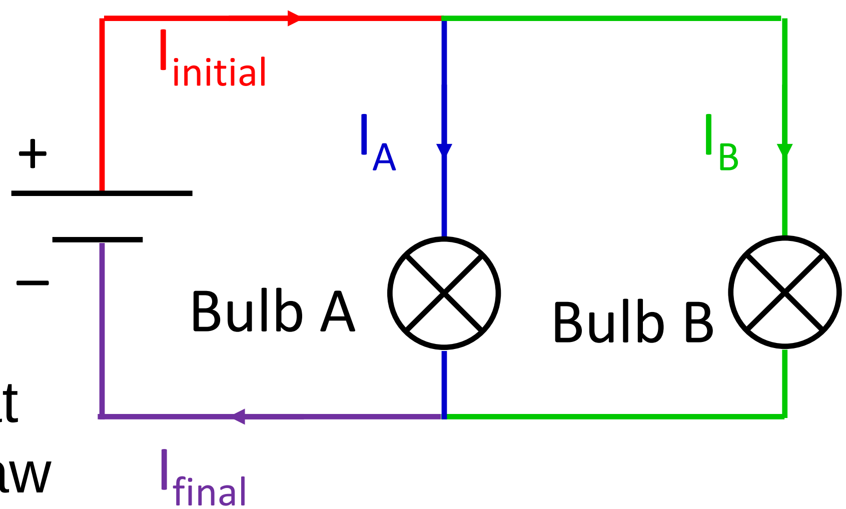
Solution

Answer: C

Justification: The total amount of current in a circuit is constant. When the circuit splits into two branches, the current $I_{initial}$ splits into I_A and I_B. Because no current is lost, the sum of I_A and I_B must equal $I_{initial}$. Likewise, when the branches of the circuit converge after the light bulbs, currents I_A and I_B converge to form I_{final}.

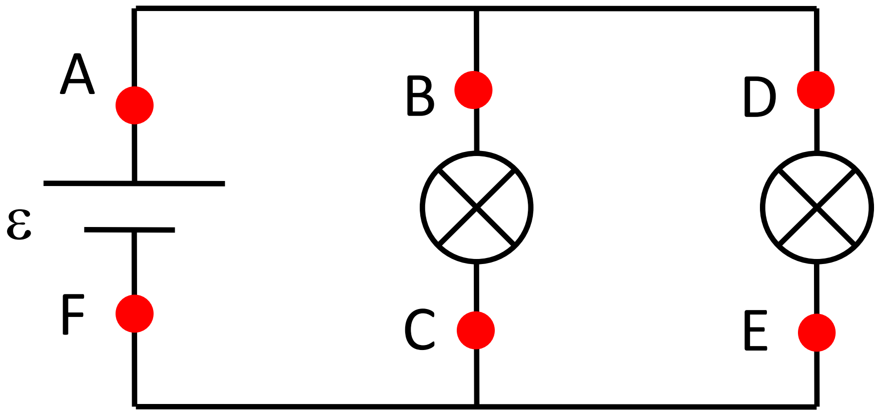
$I_{initial} = I_A + I_B = I_{final}$

The initial current will divide itself based on the resistance of the bulbs. Thus, we do not know how much current will flow through each one of the branches. We know, however, that the path with larger resistance will draw less current.



Parallel Circuits III

The potential difference across the battery is $\varepsilon = 9$ V. If the potential difference across the left bulb is $V_{BC} = -9$ V, what is the potential drop V_{DE} across the right light bulb? (The bulbs do not necessarily have the same resistance.)



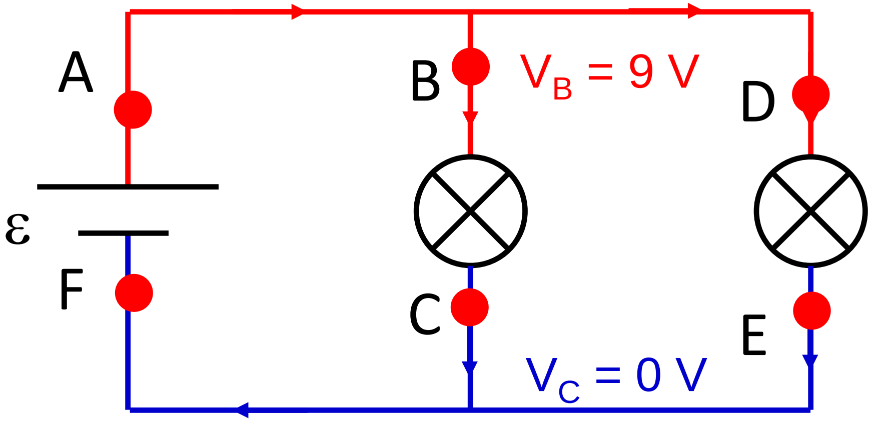
A. Exactly 9 V
B. Between 9 V and 4.5 V
C. Exactly 4.5 V
D. Between 4.5 V and 0 V
E. Exactly 0 V

Solution

Answer: A

Justification: Red wires have no resistance, so there is no potential difference along these wires, and all points along the red wire must have a potential of 9 V as compared to point F. Similarly, all points along the blue wires have the same potential (0 V as compared to point F). There are no voltage drawing elements between C and the battery. In order for there to be a potential difference of 9 V across the battery, points C and E must have a potential of 0 V.

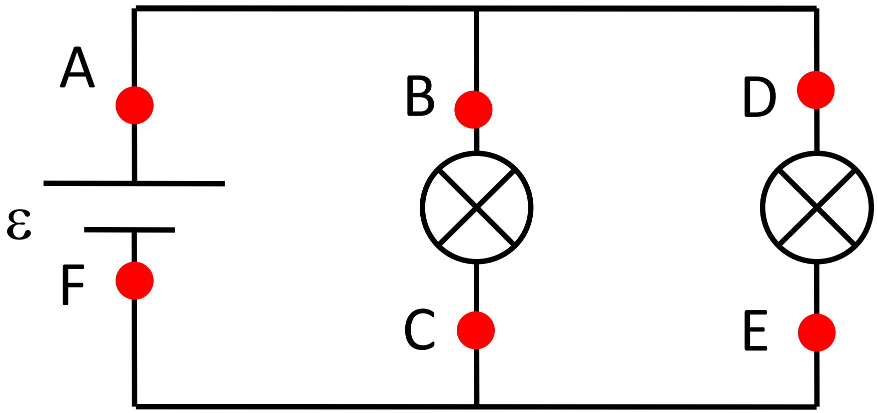
$$\Delta V_{DE} = V_E - V_D = -9\ \text{V}$$



Notice: the potential difference across each bulb is -9 V.

Parallel Circuits IV

The voltage (potential difference) across the battery is $\varepsilon = 9$ V. If the potential difference across the left bulb is $V_{BC} = -9$ V, what is the potential difference V_{CF} (point C to point F)? (The bulbs do not necessarily have the same resistance.)



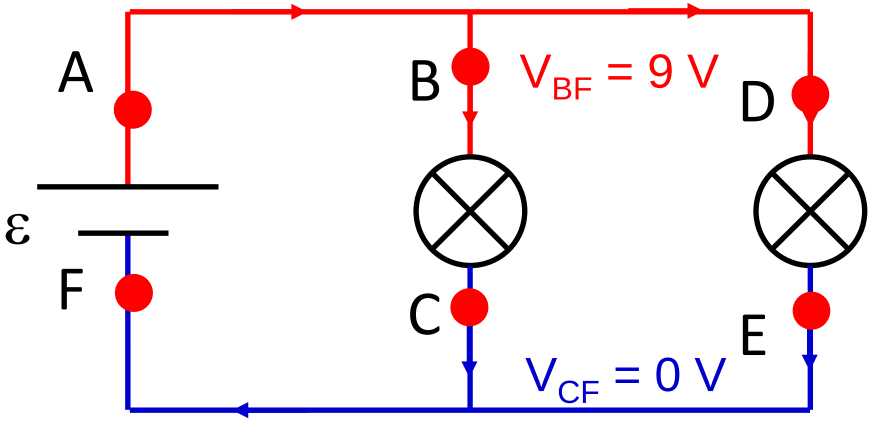
A. Exactly 9 V
B. Between 9 V and 4.5 V
C. Exactly 4.5 V
D. Between 4.5 V and 0 V
E. Exactly 0 V

Solution

Answer: E

Justification: Red wires have no resistance, so there is no potential difference along these wires, and all points along the red wire must have a potential of 9 V as compared to point F. Similarly, all points along the blue wires have the same potential (0 V as compared to point F). There are no voltage drawing elements between C and the battery. In order for there to be a potential difference of 9 V across the battery, points C and E must have a potential of 0 V.

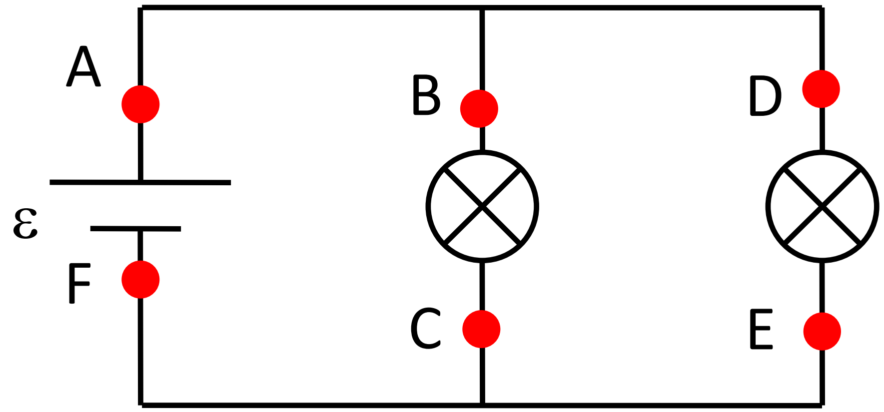
$\Delta V_{CF} = 0\text{ V}$



Notice: the potential difference across each bulb is 9 V.

Parallel Circuits V

Rank the potential differences across different elements of this electric circuit (the resistance of the left bulb is larger than the resistance of the right bulb).



a. $V_{AF} > V_{BC} = V_{DE} > V_{BD} = V_{EC} = V_{CF} = V_{AD} = V_{EF}$

b. $V_{AF} > V_{BC} > V_{DE} > V_{BD} = V_{EC} = V_{CF} = V_{AD} = V_{EF}$

c. $V_{AF} = V_{BC} = V_{AB} > V_{BD} = V_{EC} = V_{CF} > V_{AD} = V_{EF}$

d. $V_{AF} < V_{BC} < V_{AB} > V_{BD} = V_{EC} = V_{CF} = V_{AD} = V_{EF}$

e. $V_{AF} = V_{BC} = V_{DE} > V_{BD} = V_{EC} = V_{CF} = V_{AD} = V_{EF}$

Solution

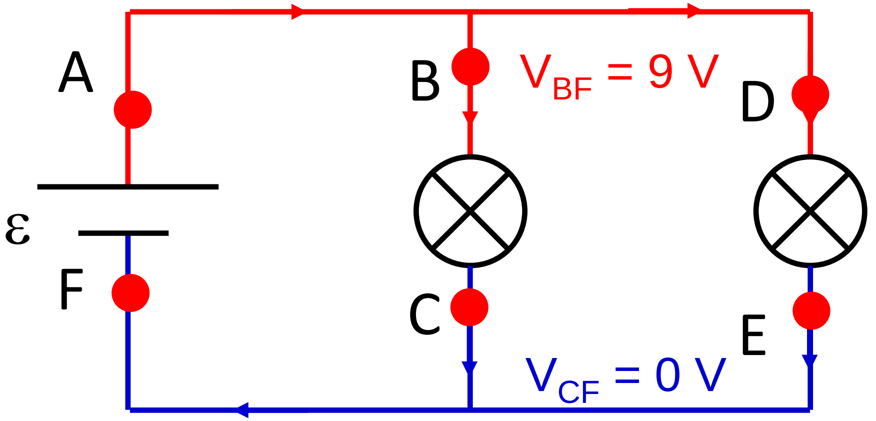
Answer: E

Justification: See the explanations to the previous questions:

As each bulb is connected in parallel to the battery, each has the same potential difference across it as the voltage across the battery terminals. All other potential differences are zero, as all the wires have zero resistance.

$$V_{AF} = V_{BC} = V_{DE} = 9V$$

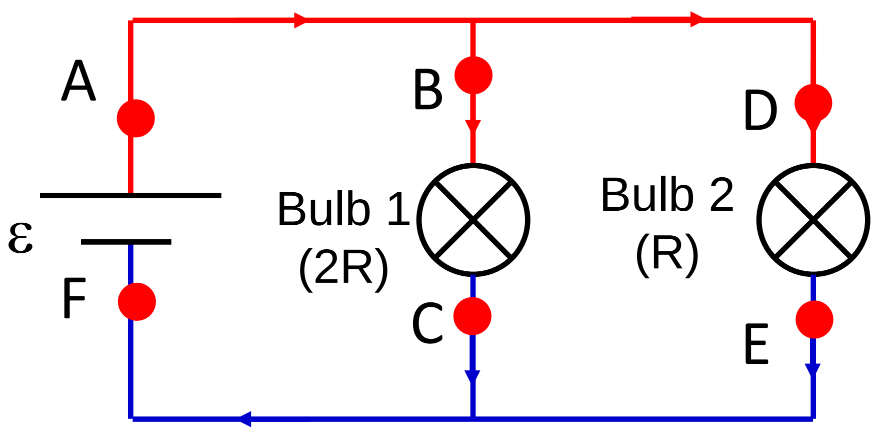
$$V_{BD} = V_{EC} = V_{CF} = V_{AD} = V_{EF} = 0V$$



Parallel Circuits VI

An ideal 9 V battery is connected to two light bulbs as shown in the diagram. Bulb 1 has a resistance of 2R while Bulb 2 has a resistance of R.

What are the potential differences across Bulb 1 (ΔV_1) and Bulb 2 (ΔV_2)? (In this question, we consider the magnitude of ΔV).



A. $\Delta V_1 = 9.0$ V, $\Delta V_2 = 9.0$ V

B. $\Delta V_1 = 9.0$ V, $\Delta V_2 = 4.5$ V

C. $\Delta V_1 = 6.0$ V, $\Delta V_2 = 3.0$ V

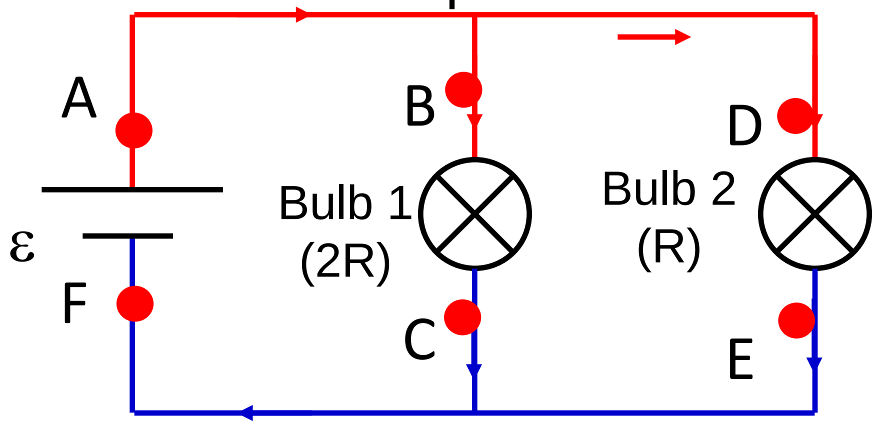
D. $\Delta V_1 = 9.0$ V, $\Delta V_2 = 4.5$ V

E. $\Delta V_1 = 3.0$ V, $\Delta V_2 = 6.0$ V

Solution

Answer: A

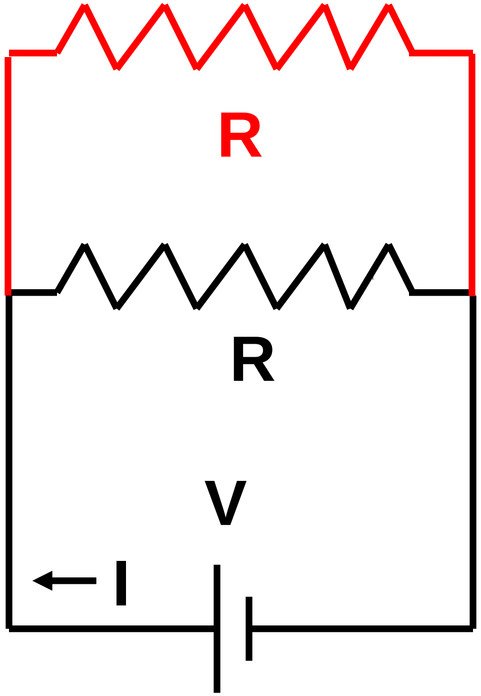
Justification: From the previous questions, we know the bulbs are connected in parallel to each other and to the battery. Therefore potential differences across them equal 9 V.



The sign of potential difference: Let us consider bulb 1. Point B has a higher potential than point C, therefore if we measure potential difference across the bulb as $V_{BC} = V_C - V_B = -9$ V in the direction of the current (from B to C). Negative potential difference indicates that a light bulb consumes energy.

Parallel Circuits VII

Consider a circuit consisting of a battery and a resistor. We add a second identical resistor in parallel with the first one, as shown below in red. The current I that is drawn from the battery:



A. Increases

B. Decreases

C. Stays the same

Solution

Answer: A

Justification: Originally, the amount of current was $I_{orig} = \dfrac{V}{R}$

In a parallel circuit, the potential difference across each branch is the same. The total resistance in the circuit has dropped, because the two resistors are connected in parallel. Since the voltage provided by the battery has not changed, the current must then double.

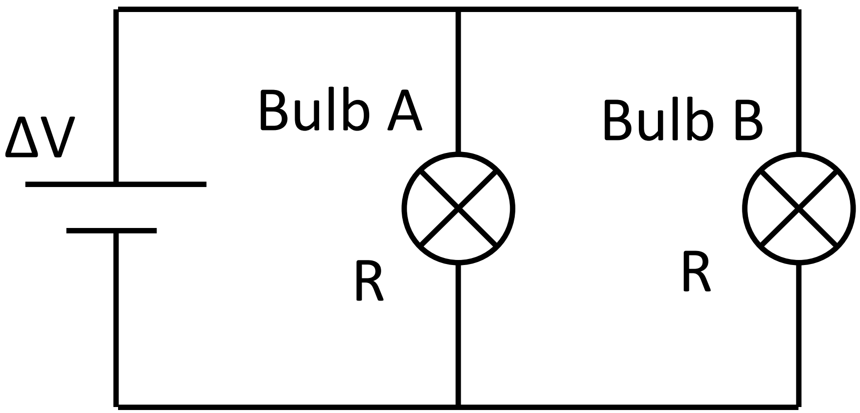
$$V = I_{new} R_{total} = i_1 R_1 = i_2 R_2$$

$$R_{total} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} = \frac{R_1 R_2}{R_1 + R_2} = \frac{R^2}{2R} = \frac{R}{2}$$

$$V = I_{new} R_{total} = I_{new} \frac{R}{2} \Rightarrow I_{new} = \frac{2V}{R} = 2 I_{orig}$$

Parallel Circuits VIII

Two *identical* light bulbs are connected in a parallel circuit as shown in the diagram. How will the bulbs' brightness compare with one another?



A. Bulb A is brighter than Bulb B
B. Bulb B is brighter than Bulb A
C. The bulbs are equally bright
D. Bulb A is lit but Bulb B is not
E. Bulb B is lit but Bulb A is not

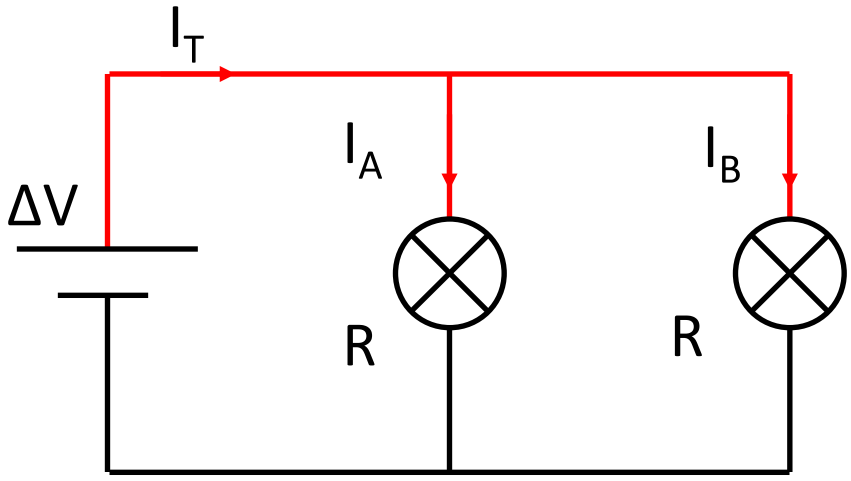
Solution

Answer: C

Justification: We know from the previous question that the potential difference across both bulbs is ΔV.

Since the potential difference is the same across both bulbs, the current I_A and I_B depends only on the resistance through each path. Since the bulbs have the same resistance, the current through the bulbs is the same.

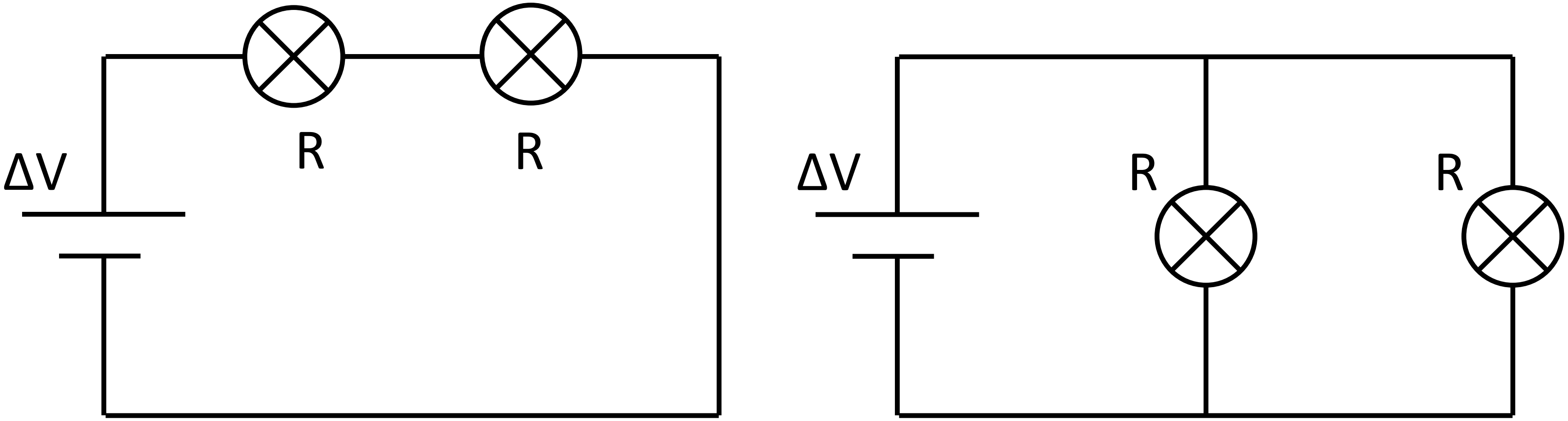
$$I_A = \frac{\Delta V}{R}, \quad I_B = \frac{\Delta V}{R}$$



Since voltage, current, and resistance are the same across each bulb, they must dissipate the same amount of energy and are, therefore, equally bright.

Series and Parallel Circuits IX

Two ***identical*** light bulbs are connected first in a series circuit and then in a parallel circuit with the same battery. In which circuit will the bulbs be brighter?



A. The bulbs will be brighter in the series circuit

B. The bulbs will be brighter in the parallel circuit

C. The bulbs will be equally bright in both circuits

Solution

Answer: B

Justification:

We know from previous questions that the potential difference across the bulbs in parallel is ΔV. The potential difference across the bulbs in series must be 0.5ΔV since the bulbs have the same resistance. The power dissipated by a bulb is:

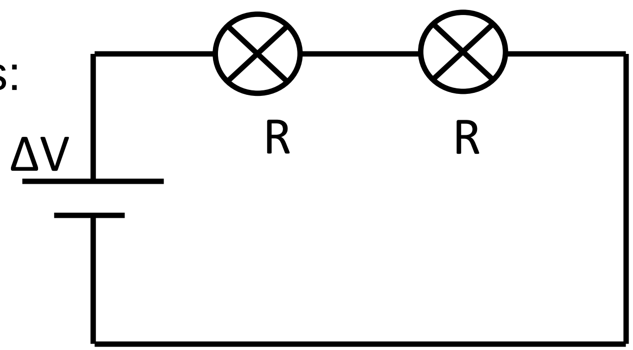
$$P = \frac{\Delta V^2}{R},$$

So the bulbs with the highest potential difference will be the brightest.

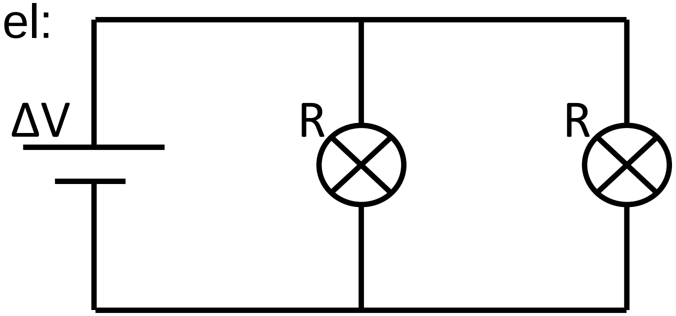
Series and Parallel Circuits X

Two *identical* light bulbs are connected first in a series circuit and then in a parallel circuit with the same battery. Which of the follow equations best describes the relative amount of power dissipated in each circuit?

Series:



Parallel:



A. $P_{series} = \dfrac{P_{parallel}}{4}$

B. $P_{series} = \dfrac{P_{parallel}}{2}$

C. $P_{series} = P_{parallel}$

D. $P_{series} = 2P_{parallel}$

E. $P_{series} = 4P_{parallel}$

Solution Part I

Answer: A

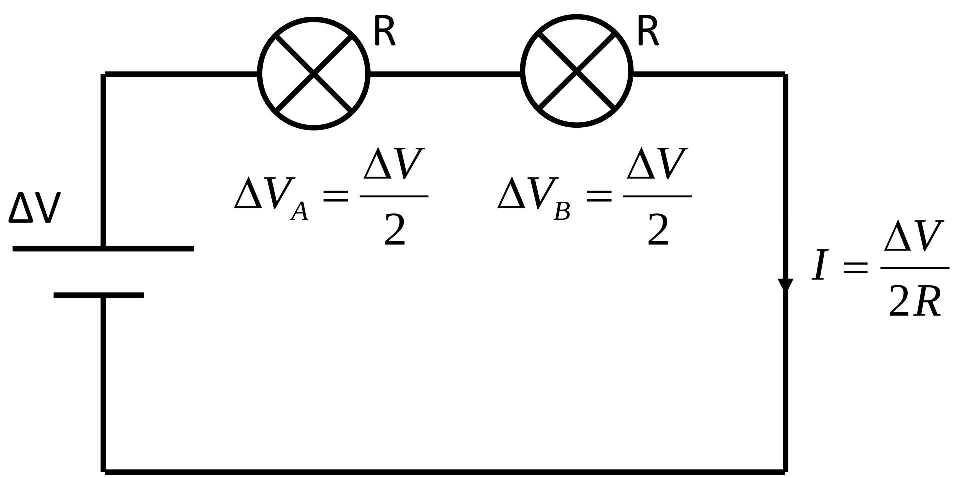
Justification: The current across the 2 bulbs is $I = \frac{\Delta V}{R_T} = \frac{\Delta V}{2R}$.

The potential difference across each bulb is $\Delta V_A = \Delta V_B = IR = \left(\frac{\Delta V}{2R}\right)R = \frac{\Delta V}{2}$

Using any formula for power gives: $P_A = P_B = \left(\frac{\Delta V_A^2}{R}\right) = \frac{\Delta V^2}{4R}$. Since

there are 2 bulbs, the total power dissipated in the circuit is:

.



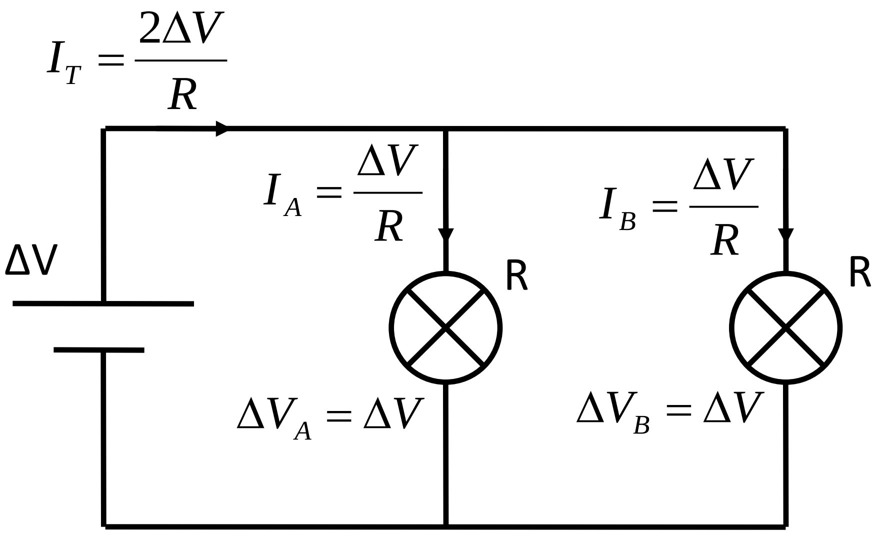
$$P_T = P_A + P_B = \frac{\Delta V^2}{2R}$$

Solution Part II

Answer: A

Justification: In a parallel circuit, the potential difference across each bulb must be ΔV and the resistance of each bulb is R. The power dissipated in each bulb is $P_A = P_B = \dfrac{\Delta V^2}{R}$.

The total power dissipated in the circuit is: $P_T = P_A + P_B = \dfrac{2\Delta V^2}{R}$.



Compared to the series circuit:

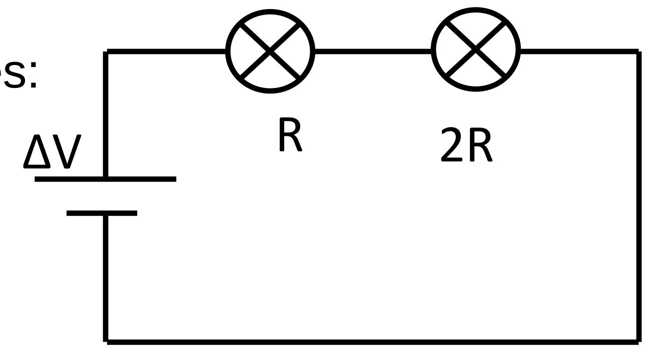
$$\frac{P_{parallel}}{P_{series}} = \frac{\dfrac{2\Delta V^2}{R}}{\dfrac{\Delta V^2}{2R}} = 4$$

The bulbs in parallel dissipate 4 times as much power and are therefore brighter.

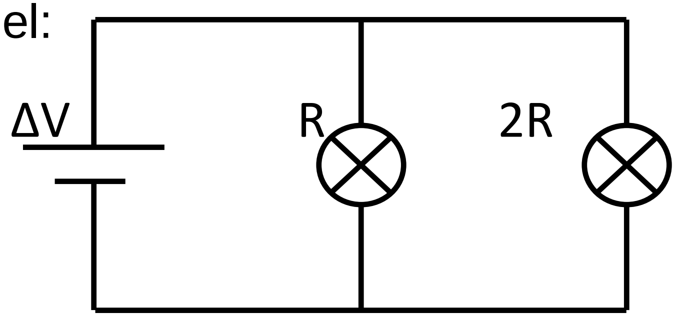
Series and Parallel Circuits XI

Two *<u>different</u>* light bulbs are connected first in a series circuit and then in a parallel circuit with the same battery. What is true about the relative brightness of the light bulbs?

Series:



Parallel:



A. Light bulb R is always brighter
B. Light bulb 2R is always brighter
C. Light bulb R is brighter in series and is dimmer in parallel than light bulb 2R
D. Light bulb 2R is brighter in series and is dimmer in parallel than light bulb R

Solution

Answer: D

Justification: Let us first consider a series circuit. Since the same current flows through each one of the light bulbs, the bulb that has more resistance will be brighter:

$$P = I^2 R \Rightarrow P_1 = I^2 R < P_2 = I^2 (2R) = 2I^2 R$$

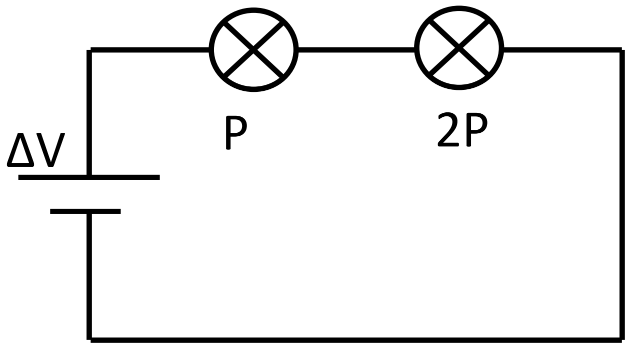
Now we are ready to consider a parallel circuit. In that circuit the potential differences across the light bulbs are the same:

$$P = \frac{(\Delta V)^2}{R} \Rightarrow P_1 = \frac{(\Delta V)^2}{R} > P_2 = \frac{(\Delta V)^2}{2R} = \frac{(\Delta V)^2}{2R}$$

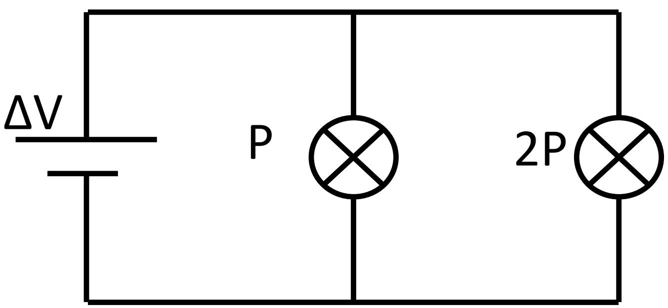
Series and Parallel Circuits XII

Two *different* light bulbs are connected first in a series circuit and then in a parallel circuit with the same battery. The bulbs are ranked as P and 2P in terms of their nominal power. What is true about the relative brightness of the light bulbs?

Series:



Parallel:



A. Light bulb P is always brighter
B. Light bulb 2P is always brighter
C. Light bulb P is brighter in series and is dimmer in parallel than light bulb 2P
D. Light bulb 2P is brighter in series and is dimmer in parallel than light bulb R

Solution

Answer: C

Justification: The key to understanding this question is the concept of “NOMINAL POWER”. When the light bulbs are manufactured, their nominal power is calculated considering the light bulbs are connected in parallel to a standard outlet (110 V in North America). Thus the resistance of a 100 W light bulb is lower than the resistance of the 200 W light bulb. Knowing the resistances of the light bulbs use the previous questions to answer this current one:

$$P = \frac{(\Delta V)^2}{R} \Rightarrow R_{100\ W} = \frac{(\Delta V)^2}{P} = \frac{(110\ V)^2}{100\ W} = 121\ \Omega$$

$$R_{200\ W} = \frac{(\Delta V)^2}{P} = \frac{(110\ V)^2}{200\ W} = 60.5\ \Omega$$