a place of mind

FACULTY OF EDUCATION

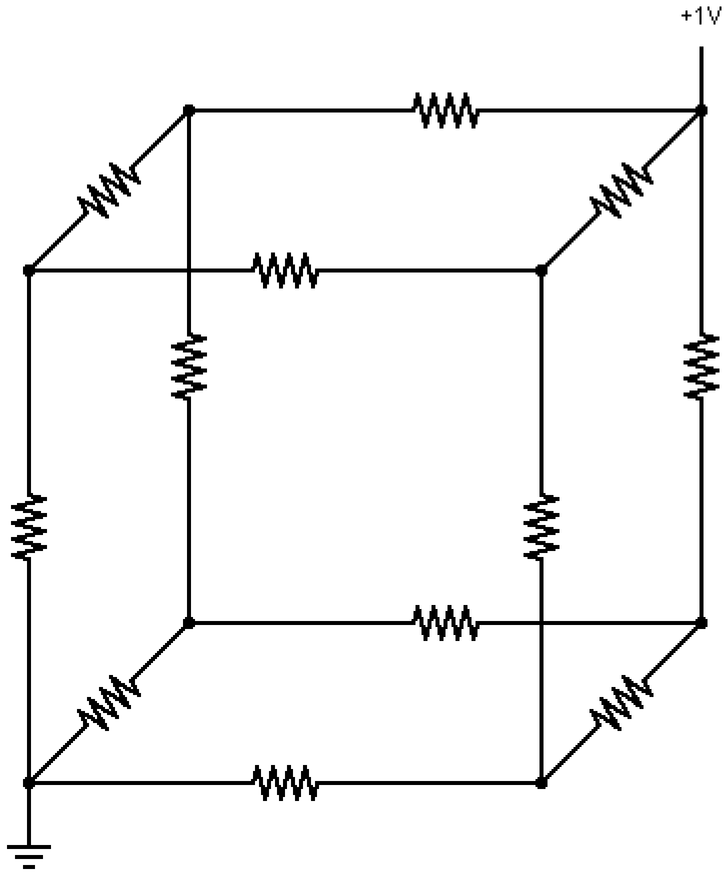
Department of
Curriculum and Pedagogy

Physics
Circuits

Science and Mathematics
Education Research Group

Supported by UBC Teaching and Learning Enhancement Fund 2012-2013

Resistor Shapes



Resistor Shapes I

Three identical 1 Ω resistors are connected in parallel as shown below. What is the total resistance of the circuit?

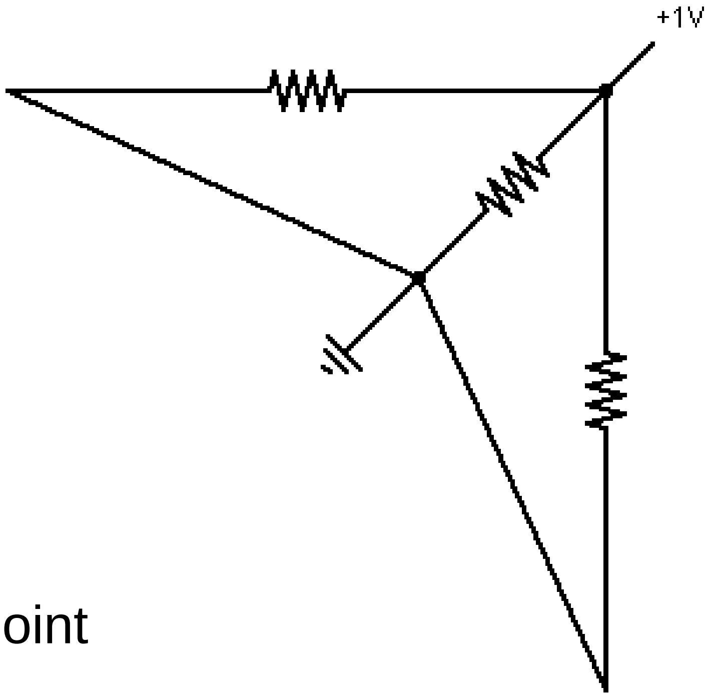
A. 3 Ω

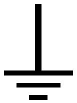
B. 1 Ω

C. 1/3 Ω

D. 3/2 Ω

E. 2/3 Ω





= Ground, or 0 V at that point

Circuit built on falstad.com/circuit

Solution

Answer: C

Justification: This is an example of three identical resistors connected in parallel. We know that the equation for parallel resistors is:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots = \sum_i \frac{1}{R_i} \Rightarrow \frac{1}{R_{total}} = \frac{1}{1\,\Omega} + \frac{1}{1\,\Omega} + \frac{1}{1\,\Omega} = \frac{3}{1\Omega}$$

$$R_{total} = \frac{1}{3}\ \Omega = \frac{R}{3}$$

This is a straightforward approach to solving this problem. However, we can also consider the symmetry of the problem and Ohm's law to take a different more elegant approach. It will be discussed in the next slide.

Solution - Continued

Answer: C

Justification: We can use the symmetry arguments to solve this problem. This argument will be especially useful for solving more difficult problems. In this case, the three resistors are equal, therefore, the branches have equal resistances and the current through each one of them must be equal:

$$I_{branch} = \frac{I}{3} \Rightarrow V = V_{branch} = I_{branch}R = \frac{I}{3}R$$

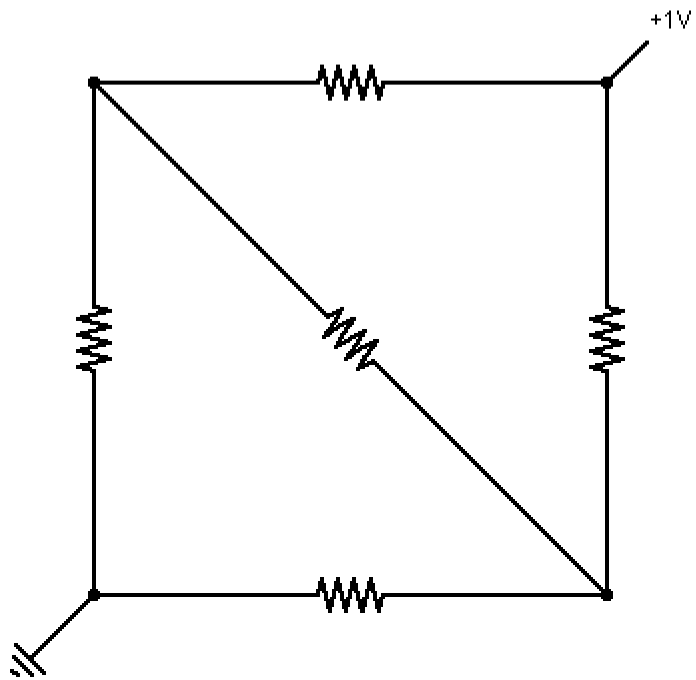
$$V = IR_{Total} \Rightarrow R_{Total} = \frac{V}{I} = \frac{IR}{3I} = \frac{R}{3} \Rightarrow R_{Total} = \frac{R}{N_{of \ \ braches}}$$

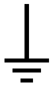
Notice, in a parallel circuit that has identical branches the total resistance is LESS than the resistance of each one of the branches. The current through each is one of them equals 1/3 of the total current – equal currents flow through each one of the identical parallel branches.

Resistor Shapes II

Identical 1 Ω resistors are arranged like the image below. What is the total resistance of the circuit?

A. 1 Ω
B. 2 Ω
C. 1/2 Ω
D. 1/3 Ω
E. 1/4 Ω




= Ground, or 0 V at that point

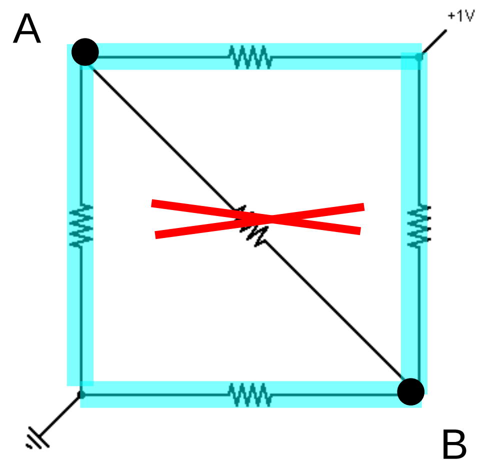
Circuit built on falstad.com/circuit

Solution

Answer: A

Justification: All the resistors are identical, so using the symmetry of the circuit we find that the current splits in half and passes through two resistors. The center resistor is ignored because the ends of the resistor have the same potential, as the circuit is symmetric. In other words, the potential of points A and B will be the same (no current flow from A to B or from B to A). Therefore, you can ignore segment AB. Then you have two identical 2-Ohm branches connected in parallel:

$$R_{Total} = \frac{R_{branch}}{2} = \frac{1\,\Omega + 1\,\Omega}{2} = 1\,\Omega$$



Additional Resources: A simulation of this circuit can be found at [goo.gl/m0yQm](goo.gl/m0yQm)

Resistor Shapes III

Identical 1 Ω resistors are arranged in a tetrahedron. What is the total resistance of the circuit?

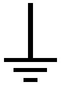
A. 6 Ω

B. 5 Ω

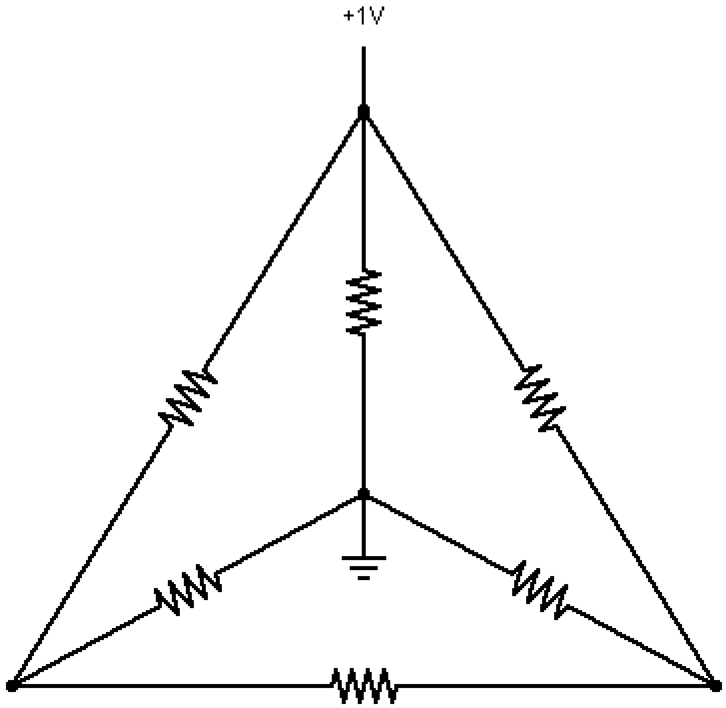
C. 2 Ω

D. 1 Ω

E. 1/2 Ω



= Ground, or 0 V at that point



Circuit built on falstad.com/circuit

Solution

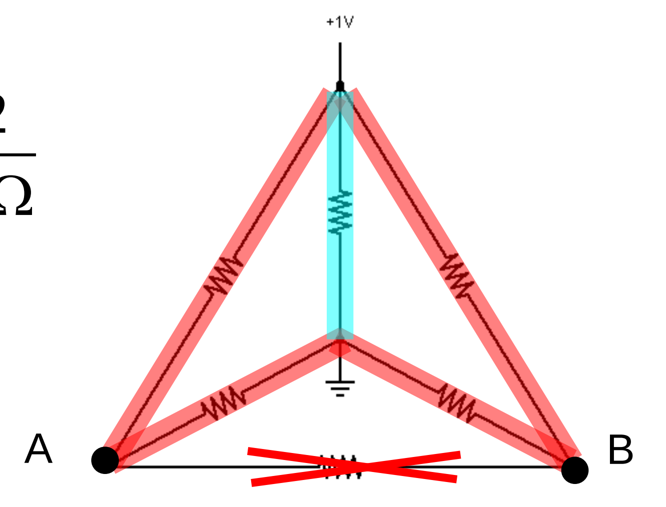
Answer: E

Justification: The left and right side of the circuit is symmetric, and therefore no current flows through the bottom resistor (AB) because the ends have the same potential. We can find R_{total} by considering the 2 resistors in series on the left and right, and the central resistor.

Additional Resources: A simulation of this circuit can be found at [goo.gl/Xx1It](goo.gl/Xx1It)

$$\frac{1}{R_{total}} = \frac{1}{(1+1)\ \Omega} + \frac{1}{(1+1)\ \Omega} + \frac{1}{1\ \Omega} = \frac{1}{R_{total}} = \frac{2}{1\ \Omega}$$

$$R_{total} = \frac{1}{2}\ \Omega; R_{total} < 1\ \Omega; R_{total} < 2\ \Omega$$



The total resistance is less than the resistance of each one of the branches.

Resistor Shapes IV

Identical 1 Ω resistors are arranged in a cube. What is the total resistance of the circuit?

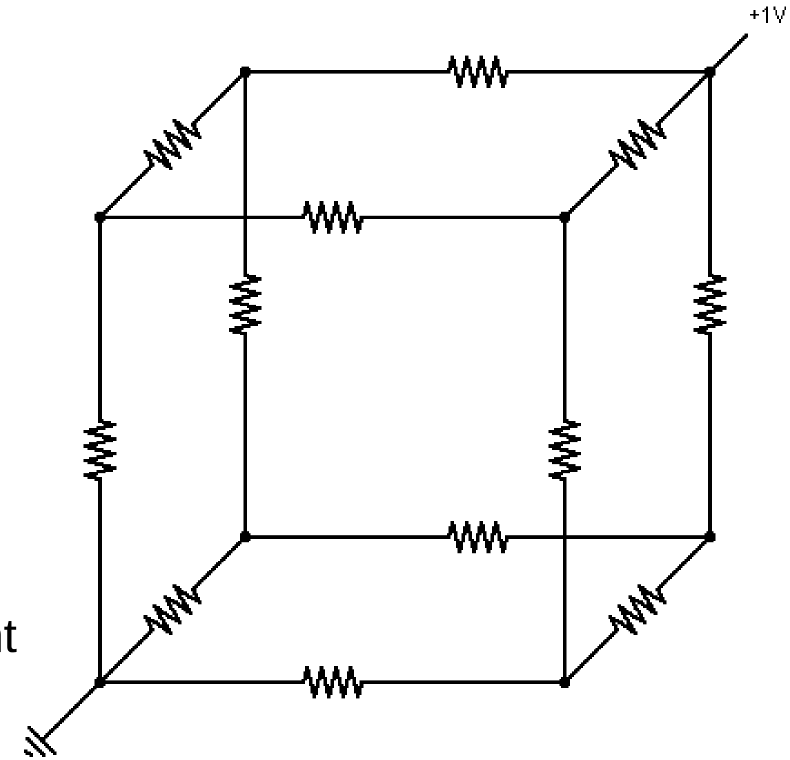
A. 6/5 Ω

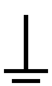
B. 1 Ω

C. 5/6 Ω

D. 2/3 Ω

E. 1/12 Ω



 = Ground, or 0 V at that point

Circuit built on falstad.com/circuit

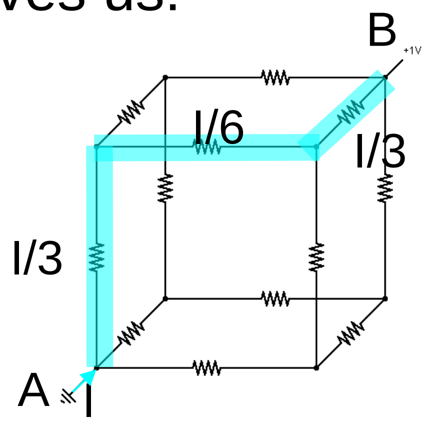
Solution

Answer: C

Justification: The cube here is highly symmetric. We chose any possible path for the current to go from point A to B. If we start at the current source (A), we see that the current divides into thirds, then into sixths, and the combines back into thirds. This gives us:

$$V = V_{\text{any parallel branch}} = \frac{I}{3}R + \frac{I}{6}R + \frac{I}{3}R = \frac{5}{6}RI$$

$$V = IR_{Total} \Rightarrow R_{Total} = \frac{V}{I} = \frac{5RI}{6I} = \frac{5}{6}R \Rightarrow R_{Total} = \frac{5}{6}R = \frac{5}{6}\ \Omega$$

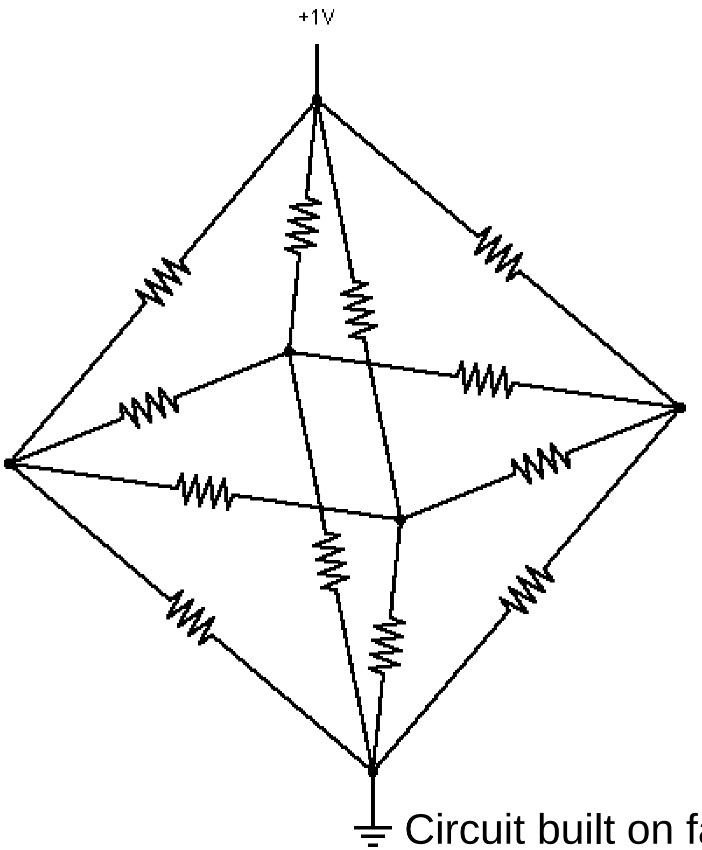


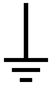
Additional Resources: A simulation of this circuit can be found at [goo.gl/mX5Bo](goo.gl/mX5Bo)

Resistor Shapes V

Identical 1 Ω resistors are arranged in an octahedron. What is the total resistance of the circuit?

A. 2 Ω
B. 1 Ω
C. 1/2 Ω
D. 1/8 Ω
E. 1/12 Ω



 = Ground, or 0 V at that point

Circuit built on falstad.com/circuit

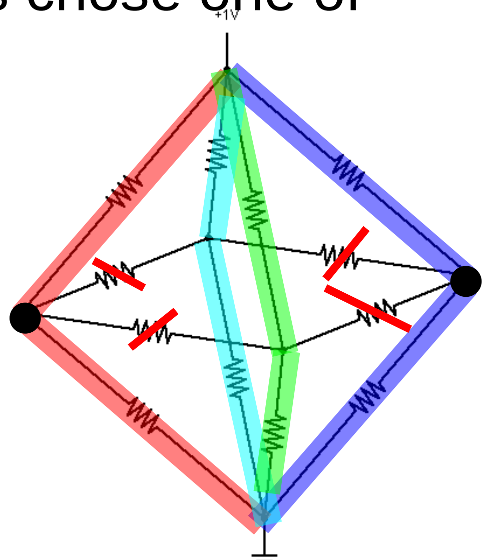
Solution

Answer: C

Justification: Considering the symmetry, we find that all of the resistors in the central ring have no current because they have the same potential all around them. Notice, the current splits into four equal parts and passes through two resistors. Let us chose one of the possible paths (for example the green one):

$$V = V_{\text{any parallel branch}} = \frac{I}{4} 2R = \frac{RI}{2}$$

$$V = IR_{Total} \Rightarrow R_{Total} = \frac{V}{I} = \frac{RI}{2I} = \frac{R}{2} \Rightarrow R_{Total} = \frac{R}{2} = 0.5\ \Omega$$



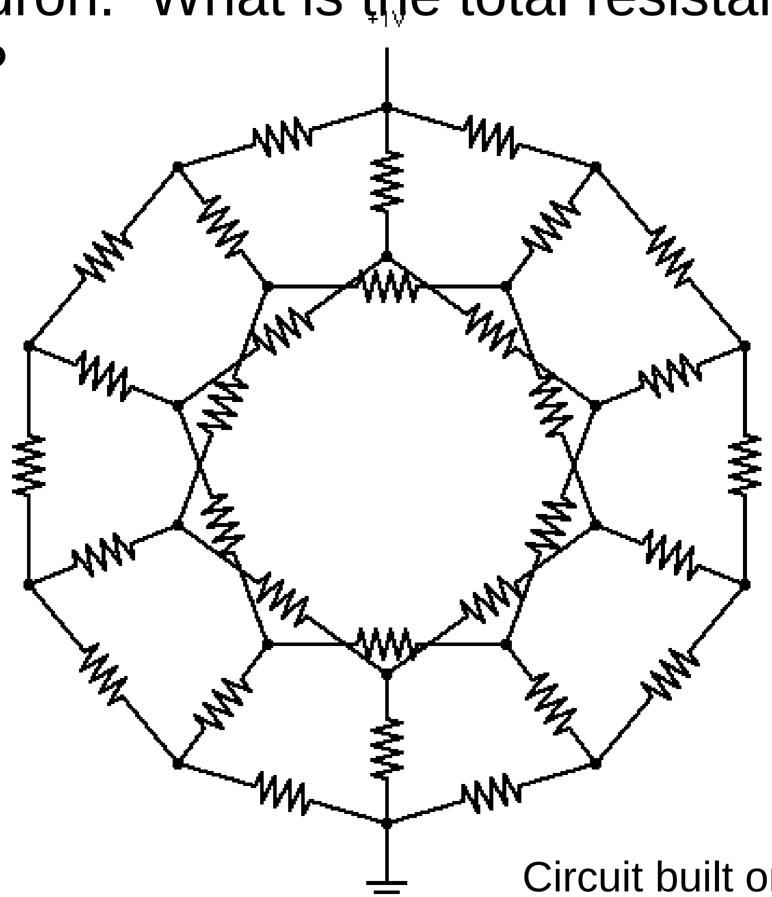
Additional Resources: A simulation of this circuit can be found at goo.gl/DjqfD

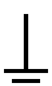
In the diagram, Red, Green, Blue, Cyan are equivalent

Resistor Shapes VI

Identical 1 Ω resistors are arranged in an dodecahedron. What is the total resistance of the circuit?

A. 7/6 Ω
B. 1 Ω
C. 6/7 Ω
D. 5/6 Ω
E. 3/5 Ω



 = Ground, or 0 V at that point

Circuit built on falstad.com/circuit

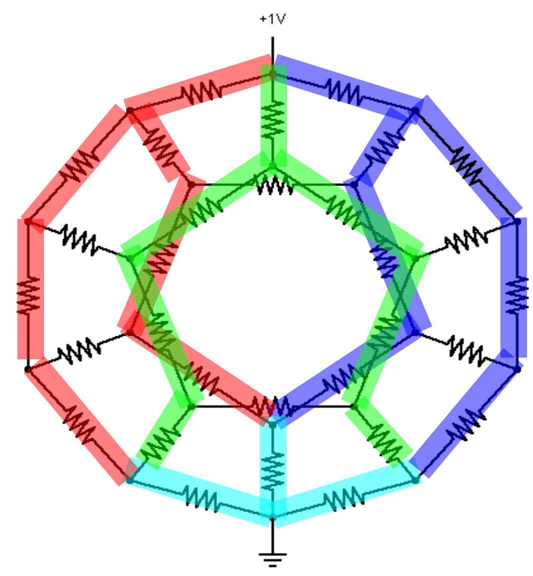
Solution

Answer: A

Justification: This diagram is a bit crowded but one can see that the current divides into thirds, then divides into sixths, where it passes through 3 resistors before fusing back into thirds and passing through one resistor:

$$V = V_{\text{any parallel branch}} = \frac{I}{3}R + \frac{I}{6}3R + \frac{I}{3}R = \frac{7RI}{6}$$

$$V = IR_{Total} \Rightarrow R_{Total} = \frac{V}{I} = \frac{7RI}{6I} = \frac{7R}{6} \Rightarrow R_{Total} = \frac{7R}{6} = \frac{7}{6}\ \Omega$$



Red, Green, Blue are equivalent

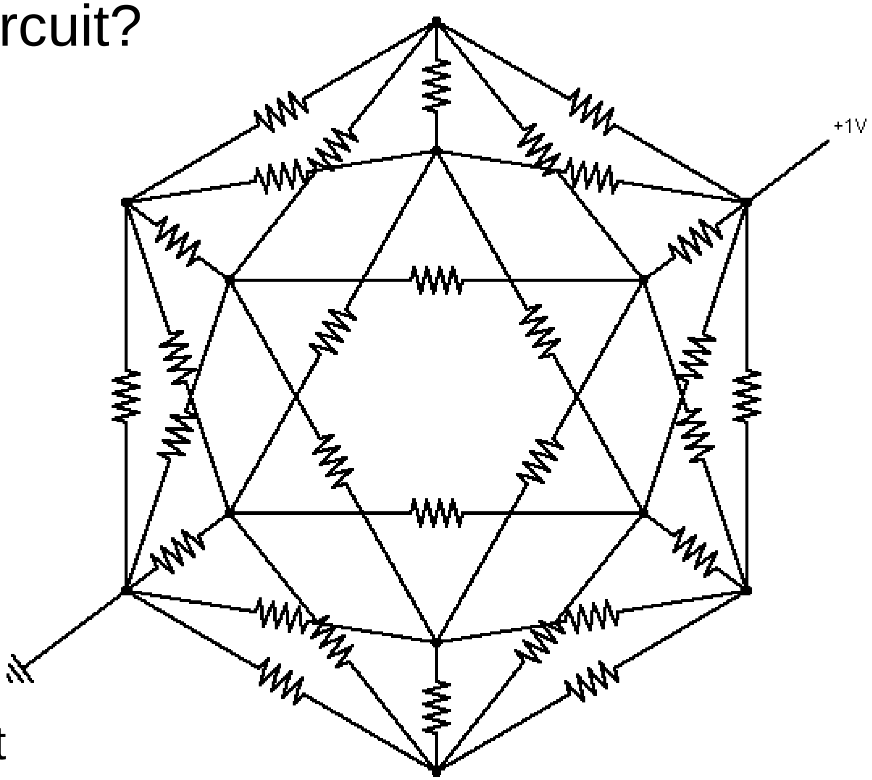
Additional Resources: A simulation of this circuit can be found at goo.gl/QQUDa, goo.gl/DvJ10

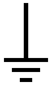
Comment: There is no current through the black branches of the circuit.

Resistor Shapes VII

Identical 1 Ω resistors are arranged in an icosahedron. What is the total resistance of the circuit?

A. 10 Ω
B. 5 Ω
C. 1 Ω
D. 1/2 Ω
E. 2/5 Ω




= Ground, or 0 V at that point

Circuit built on falstad.com/circuit

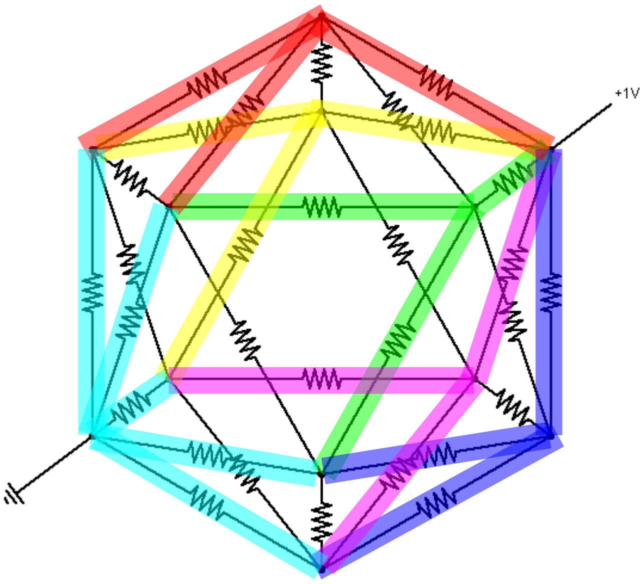
Solution

Answer: D

Justification: We will apply the same strategy as before. Following the voltage source, we see that the current splits into fifths (blue), then into tenth (red and green), and then fuses into fifths.

$$V = V_{\text{any parallel branch}} = \frac{I}{5}R + \frac{I}{10}R + \frac{I}{5}R = \frac{5RI}{10} = \frac{RI}{2}$$

$$V = IR_{Total} \Rightarrow R_{Total} = \frac{V}{I} = \frac{RI}{2I} = \frac{R}{2} \Rightarrow R_{Total} = \frac{R}{2} = \frac{1}{2}\ \Omega$$



Red, Green, Blue, Yellow, Magenta are equivalent

Additional Resources: A simulation of this circuit can be found at goo.gl/cDsPJ

Comment: There is no current through the black branches of the circuit.