a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

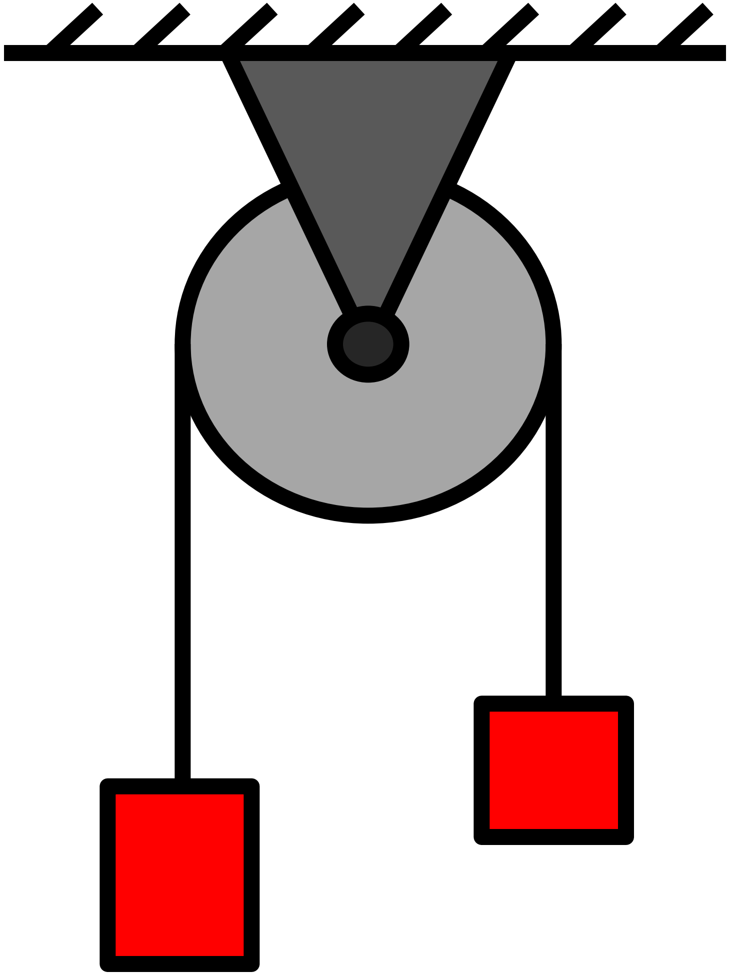
Physics

Dynamics: Atwood Machine

Science and Mathematics
Education Research Group

Supported by UBC Teaching and Learning Enhancement Fund 2012-2014

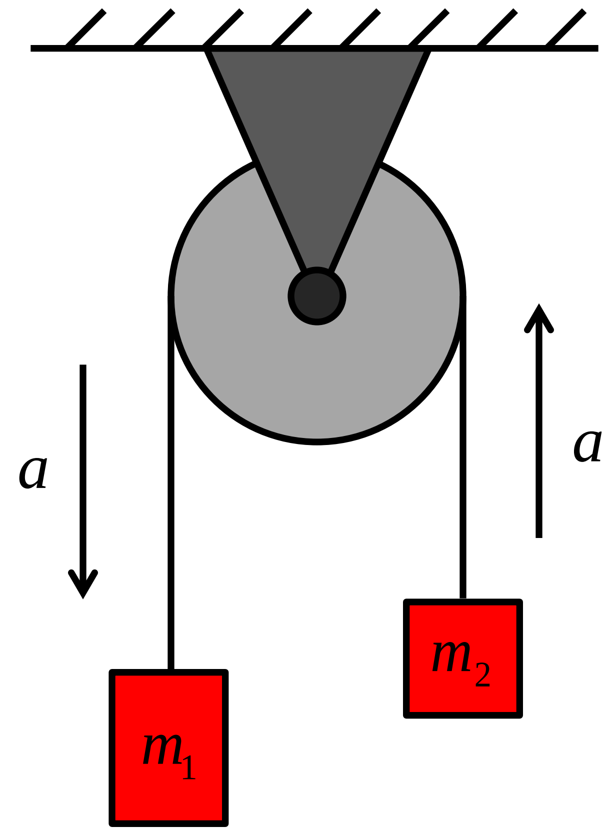
The Atwood Machine



The Atwood Machine

The Atwood Machine is a pulley system consisting of two weights connected by string. We will assume no friction and that both the string and pulley are massless.

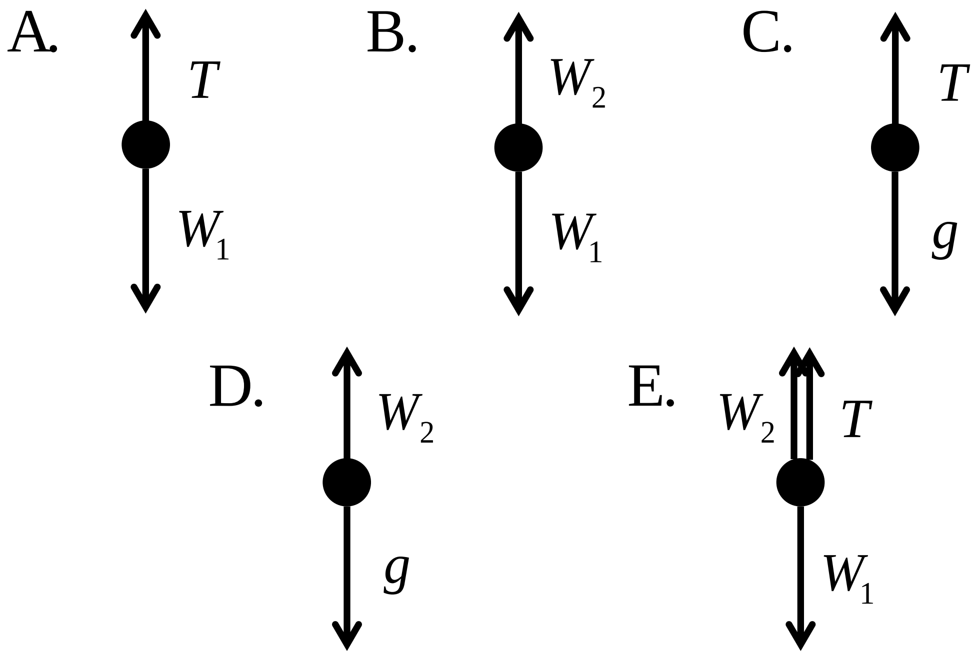
If the masses of the two weights are different, the weights will accelerate uniformly by a. Our axis is defined such that positive a indicates that m_1 accelerates downwards, while m_2 accelerates upwards.

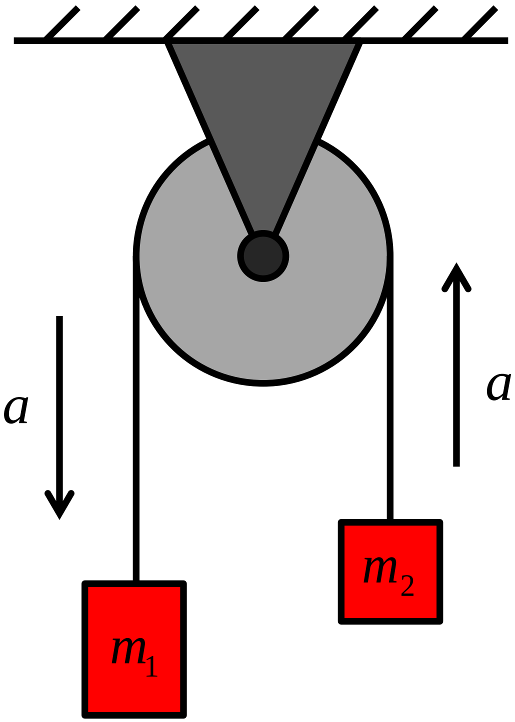


The Atwood Machine I

Let W_1 be the weight of m_1, and W_2 be the weight of m_2. Let the tension of the string be T. Assume that $m_1 > m_2$.

Which of the following is the correct free body diagram (force diagram) of m_1?

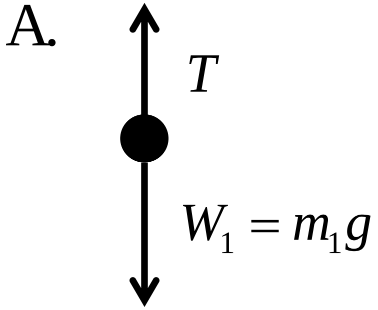




Solution

Answer: A

Justification: There are only two forces acting on m_1, the force of tension due to the string pulling it up and the force of its own weight pulling it down.



C and D are incorrect because g itself is not a force. B and E are incorrect because W_2 is not a force acting directly on m_1. Any force due to weight must point in the direction of g, which is downwards.

The Atwood Machine II

According to Newton's second law,

$$F_1 = m_1 a$$

which of the following expressions is true?

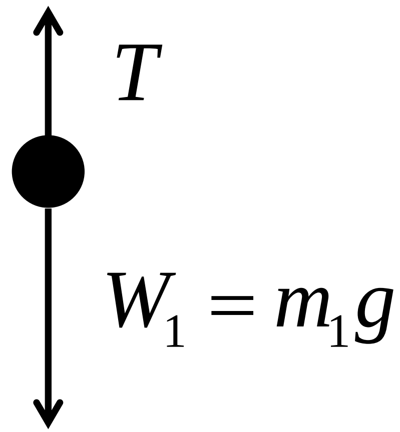
A. $T - m_1 g = m_1 a$

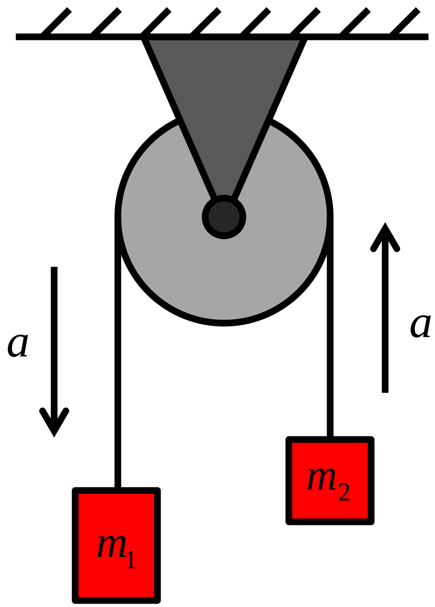
B. $m_1 g - T = m_1 a$

C. $m_1 a - T = m_1 g$

D. $T - m_1 a = m_1 g$

E. $T = m_1 g$

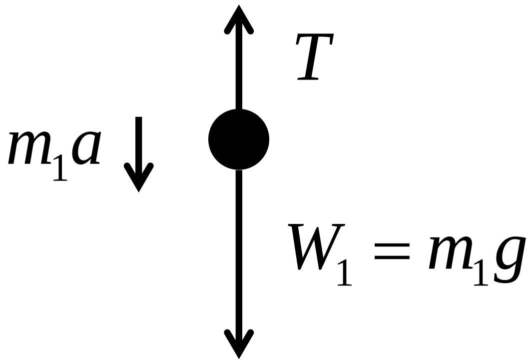




Solution

Answer: B

Justification: From the force diagram and taking downwards as positive acceleration for a,



$$F_1 = m_1 g - T$$

(Add forces in the same direction as a, subtract forces in the opposite direction of a)

Therefore, by Newton’s second law,

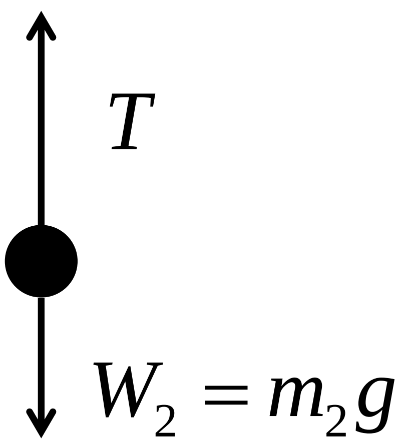
$$F_1 = m_1 a$$

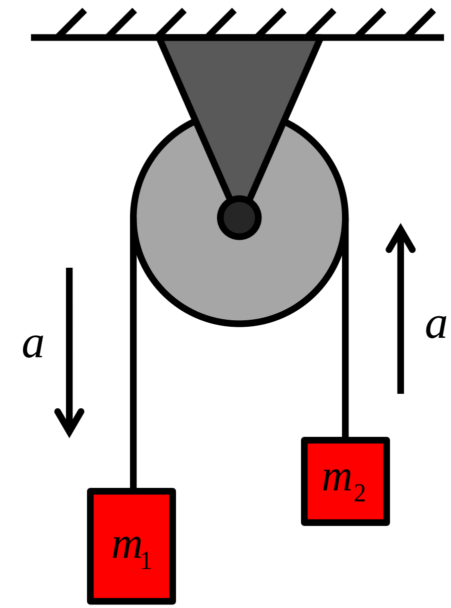
$$m_1 g - T = m_1 a$$

The Atwood Machine III

Which of the following is the correct expression obtained by applying Newton's second law for m_2?

Press for hint





A. $T - m_2 g = m_2 a$

B. $m_2 g - T = m_2 a$

C. $m_2 a - T = m_2 g$

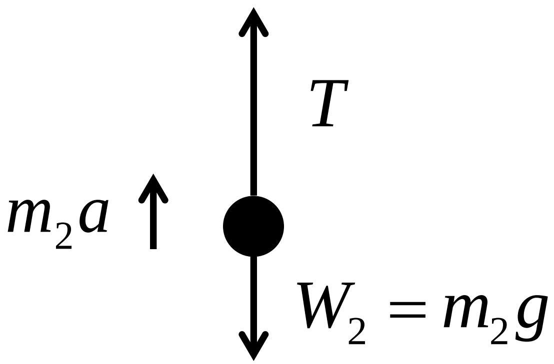
D. $T - m_2 a = m_2 g$

E. $T = m_2 g$

Solution

Answer: A

Justification: This question is solved in the same way we as the previous question, for m_1. It is a good idea to start by drawing the free body diagram for m_2.



$$F_2 = T - m_2 g = m_2 a$$

(Add forces in the same direction as a, subtract forces in the opposite direction of a)

Notice that the magnitude of T is the same for both m_1 and m_2. However, in the case of m_2, it is acting in the same direction as the acceleration rather than opposing it.

The Atwood Machine IV

What is the acceleration of the two weights?

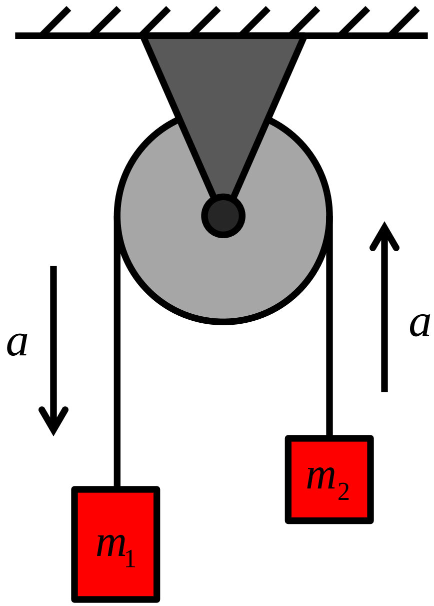
A. $a = g(m_1 - m_2)$

B. $a = g\dfrac{m_1 - m_2}{m_1}$

C. $a = g\dfrac{m_1 - m_2}{m_2}$

D. $a = g\dfrac{m_1 + m_2}{m_1 - m_2}$

E. $a = g\dfrac{m_1 - m_2}{m_1 + m_2}$



$$m_1 g - T = m_1 a$$

$$T - m_2 g = m_2 a$$

Solution

Answer: E

Justification: We have already found two expressions for the acceleration, although they contain the unknown quantity T.

$$m_1 g - T = m_1 a$$

$$T - m_2 g = m_2 a$$

Since T is equal along all points on the string, it is the same in both equations. Adding the two equations will eliminate T:

$$m_1 g - m_2 g = m_1 a + m_2 a$$

$$g(m_1 - m_2) = a(m_1 + m_2)$$

$$a = g\,\frac{m_1 - m_2}{m_1 + m_2}$$

The Atwood Machine V

Now that we have found an equation for the acceleration, we can analyze some how the Atwood Machine behaves.

Under what conditions will a be negative?

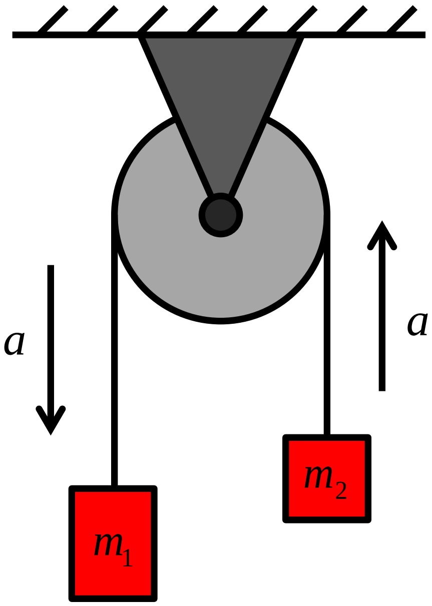
A. $m_1 > m_2$

B. $m_1 \geq m_2$

C. $m_1 = m_2$

D. $m_1 \leq m_2$

E. $m_1 < m_2$



$$a = g\,\frac{m_1 - m_2}{m_1 + m_2}$$

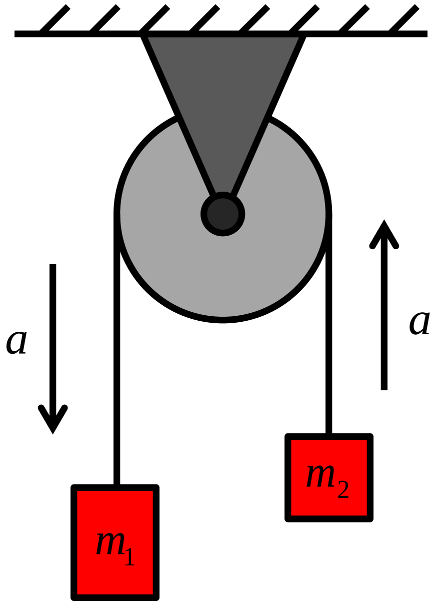
Solution

Answer: E

Justification: Recall that positive a implies that m_1 accelerates downwards, while m_2 accelerates upwards. If a is negative, this means that m_2 will accelerates downwards.

We know intuitively that m_2 will accelerate downwards if $m_2 > m_1$. This is also reflected in the formula we derived.

$$a = g\frac{m_1 - m_2}{m_1 + m_2} < 0 \quad \text{since} \quad m_1 - m_2 < 0$$



The Atwood Machine VI

How should the masses of the weights be chosen such that the neither weight accelerates?

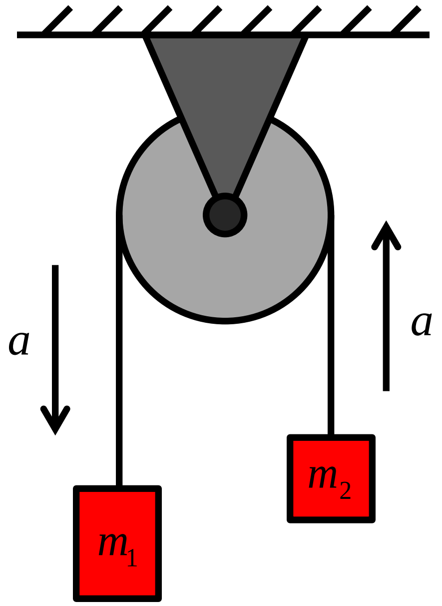
A. $m_1 > m_2$

B. $m_1 \geq m_2$

C. $m_1 = m_2$

D. $m_1 \leq m_2$

E. $m_1 < m_2$



$$a = g\frac{m_1 - m_2}{m_1 + m_2}$$

Solution

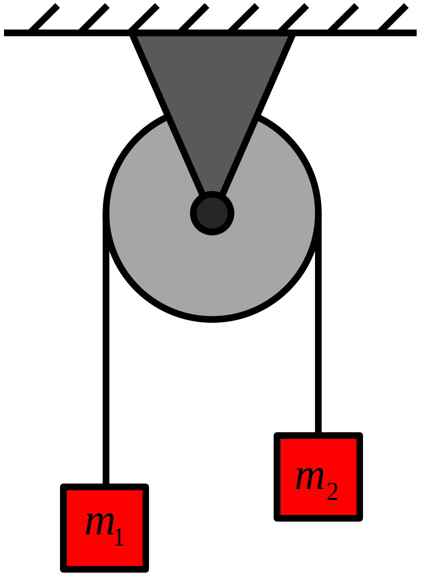
Answer: C

Justification: If neither weight accelerates, then $a = 0$. This occurs when $m_1 = m_2$. No matter where the two weights are positioned, the weights will not move.

$$a = g\frac{m_1 - m_2}{m_1 + m_2}$$

$$= g\frac{0}{m_1 + m_1} \quad \text{since} \quad m_1 = m_2$$

$$= 0$$



The Atwood Machine VII

Suppose that $m_1 >> m_2$ (one of the weights is much heavier than the other).

How can the acceleration be approximated in this case?

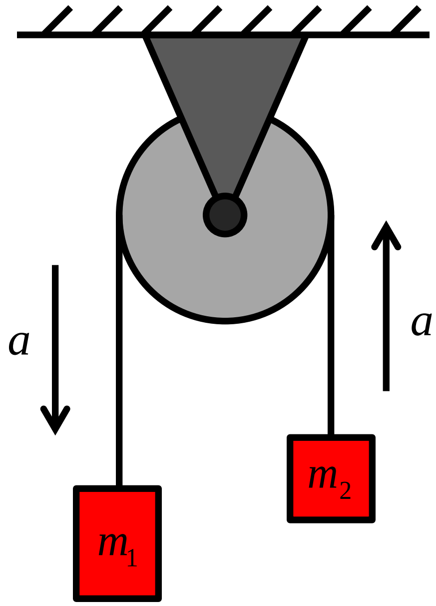
A. $a \approx (m_1 + m_2)g$

B. $a \approx m_1 g$

C. $a \approx 2g$

D. $a \approx g$

E. $a \approx 0$



$$a = g\frac{m_1 - m_2}{m_1 + m_2}$$

Solution

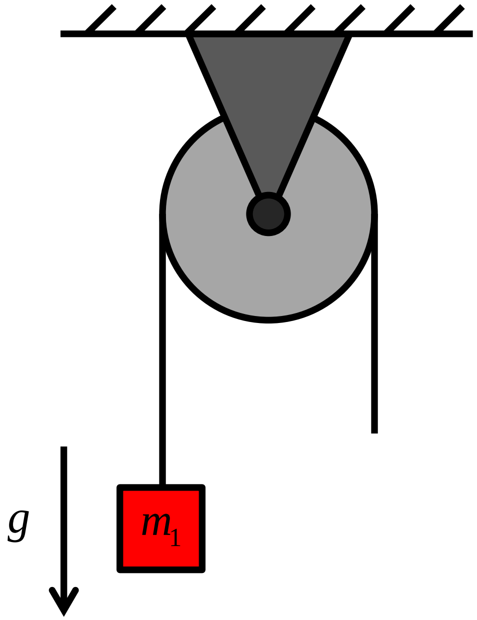
Answer: D

Justification: If $m_1 >> m_2$, then we can make the following approximations:

$$m_1 - m_2 \approx m_1$$

$$m_1 + m_2 \approx m_1$$

$$a \approx g \frac{m_1}{m_1} = g$$



Compared to m_1, we can approximate that $m_2 = 0$. We can treat m_1 as if it were in freefall.

The Atwood Machine VIII

Depending on the values chosen for the mass for m_1 and m_2, we can achieve a variety of values for a.

What are all the possible values of a that can be obtained by varying the masses?

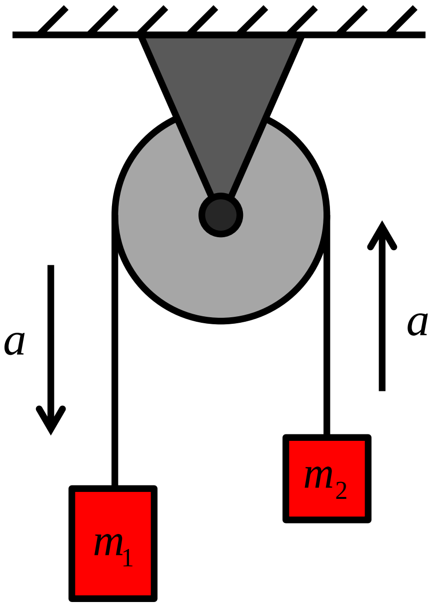
A. $-\infty < a < \infty$

B. $0 < a < \infty$

C. $0 < a < g$

D. $-g < a < g$

E. $a < g$



$$a = g\,\frac{m_1 - m_2}{m_1 + m_2}$$

Solution

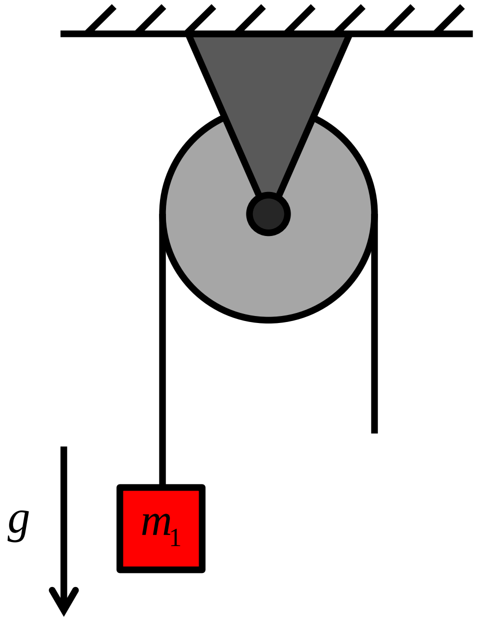
Answer: D

Justification: The largest possible value for a is obtained when $m_2 = 0$.

$$a = g\,\frac{m_1 - 0}{m_1 + 0} = g$$

The smallest value of a is obtained when $m_1 = 0$.

$$a = g\,\frac{0 - m_2}{0 + m_2} = -g$$



No matter how large we choose m_1, the weights cannot accelerate upwards or downwards faster than g. What would happen if this were not true?

The Atwood Machine IX

What is the tension on the string?

A. $T = g\dfrac{m_1}{m_1 + m_2}$

B. $T = 2g\dfrac{m_1 m_2}{m_1 + m_2}$

C. $T = g\dfrac{m_1 m_2}{m_1 - m_2}$

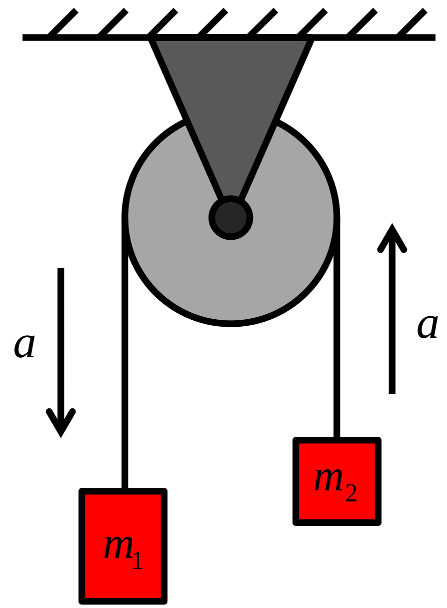
D. $T = 2g\dfrac{m_1 m_2}{m_1 - m_2}$

E. $T = m_1 g + m_2 g$

Press for hint

$$m_1 g - T = m_1 a$$

$$T - m_2 g = m_2 a$$



$$a = g\frac{m_1 - m_2}{m_1 + m_2}$$

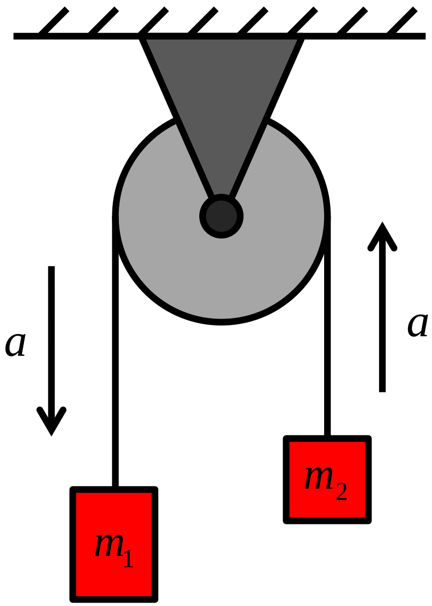
Solution

Answer: B

Justification: We can substitute the solution for a into either one of the force equations:

$$m_1 g - T = m_1 a$$

$$T - m_2 g = m_2 a$$



For example, using the top equation gives:

$$T = m_1 g - m_1 g \frac{m_1 - m_2}{m_1 + m_2}$$

$$= \frac{m_1 g(m_1 + m_2) - m_1 g(m_1 - m_2)}{m_1 + m_2}$$

$$= \frac{2 m_1 m_2 g}{m_1 + m_2}$$

Solution Continued

Answer: B

Justification: Many answers can be ruled out by doing a few quick calculations.

Answer A is incorrect because it does not have units of force.

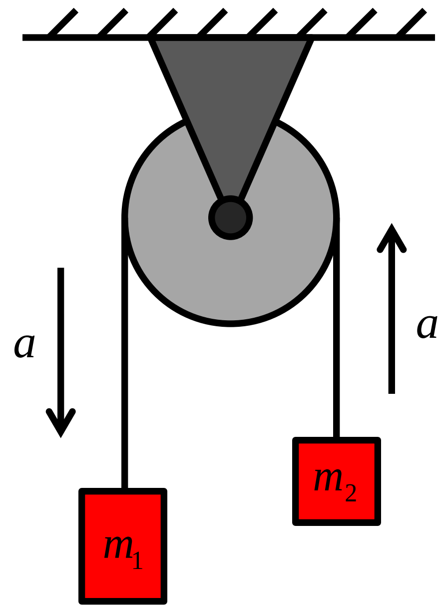
Answer C and D are incorrect because it is possible to obtain negative values for tension.

Suppose we let $m_1 = 0$. We should expect our formula to return $T = 0$, since m_2 is in freefall. This is not true for answer E.

The Atwood Machine X

Suppose m_1 is fixed, although we are free to choose any value for m_2.

How should m_2 be chosen (in terms of m_1) such that $T = m_1 g$?



A. $m_2 = 0$

B. $m_2 >> m_1$

C. $m_2 = m_1$

D. $m_2 = 2m_1$

E. $m_2 = \frac{1}{2} m_1$

$$T = 2g \frac{m_1 m_2}{m_1 + m_2}$$

Solution

Answer: C

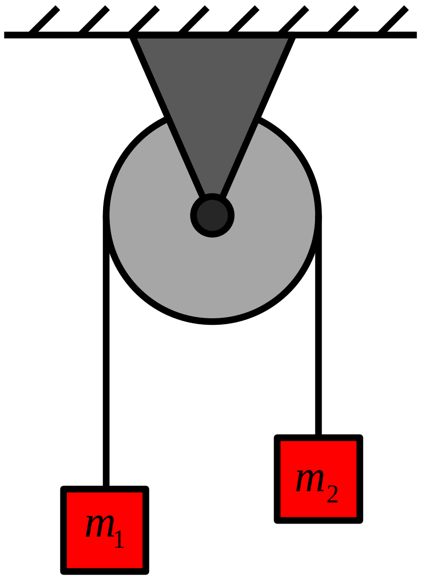
Justification: If the tension of the string is equal to the weight of m_1, then m_1 is not accelerating. Recall that there is no acceleration when $m_1 = m_2$. From the formula:

$$T = 2g\frac{m_1 m_2}{m_1 + m_2}$$

$$= \frac{2gm^2}{2m} \qquad m_1 = m_2 = m$$

$$= mg$$

$$= m_1 g = m_2 g$$



The Atwood Machine XI

What is the force on the pulley required to hold it up along the ceiling?

(Recall that we are assuming a massless pulley and string)

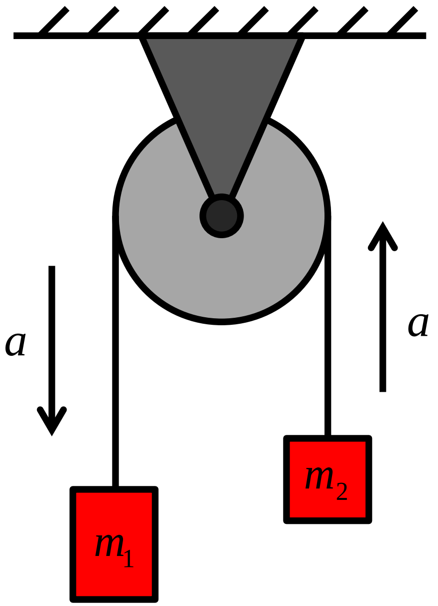
A. $F_{\text{pulley}} = g\dfrac{m_1 m_2}{m_1 + m_2} = 0.5T$

B. $F_{\text{pulley}} = 2g\dfrac{m_1 m_2}{m_1 + m_2} = T$

C. $F_{\text{pulley}} = 4g\dfrac{m_1 m_2}{m_1 + m_2} = 2T$

D. $F_{\text{pulley}} = m_2 g + m_1 g$

E. $F_{\text{pulley}} = 2(m_1 + m_2)g$

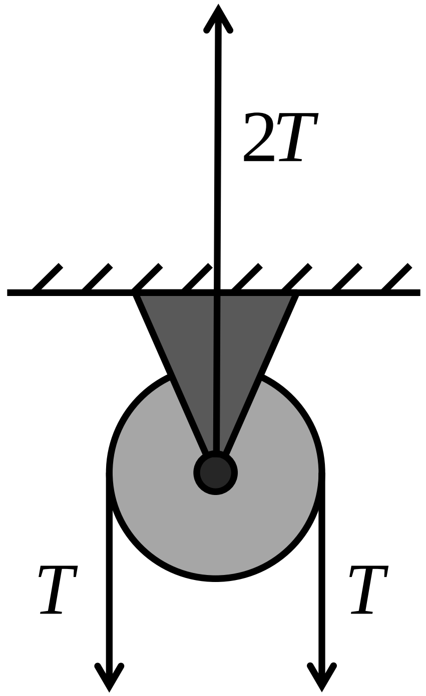


Solution

Answer: C

Justification: The pulley must be held up by twice the tension on the string.

$$F_{\text{pulley}} = 2T = 4g\,\frac{m_1 m_2}{m_1 + m_2}$$



Notice that in the formula above, $F = 0$ if either m_1 or m_2 is zero. Since the masses will be in freefall, no force is required to hold the massless pulley up.

Also notice that $F_{\text{pulley}} = m_1 g + m_2 g$, the sum of the weight of the two masses, only when the weights are not accelerating ($m_1 = m_2$).