a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Physics

Dynamics: Forces

Science and Mathematics
Education Research Group

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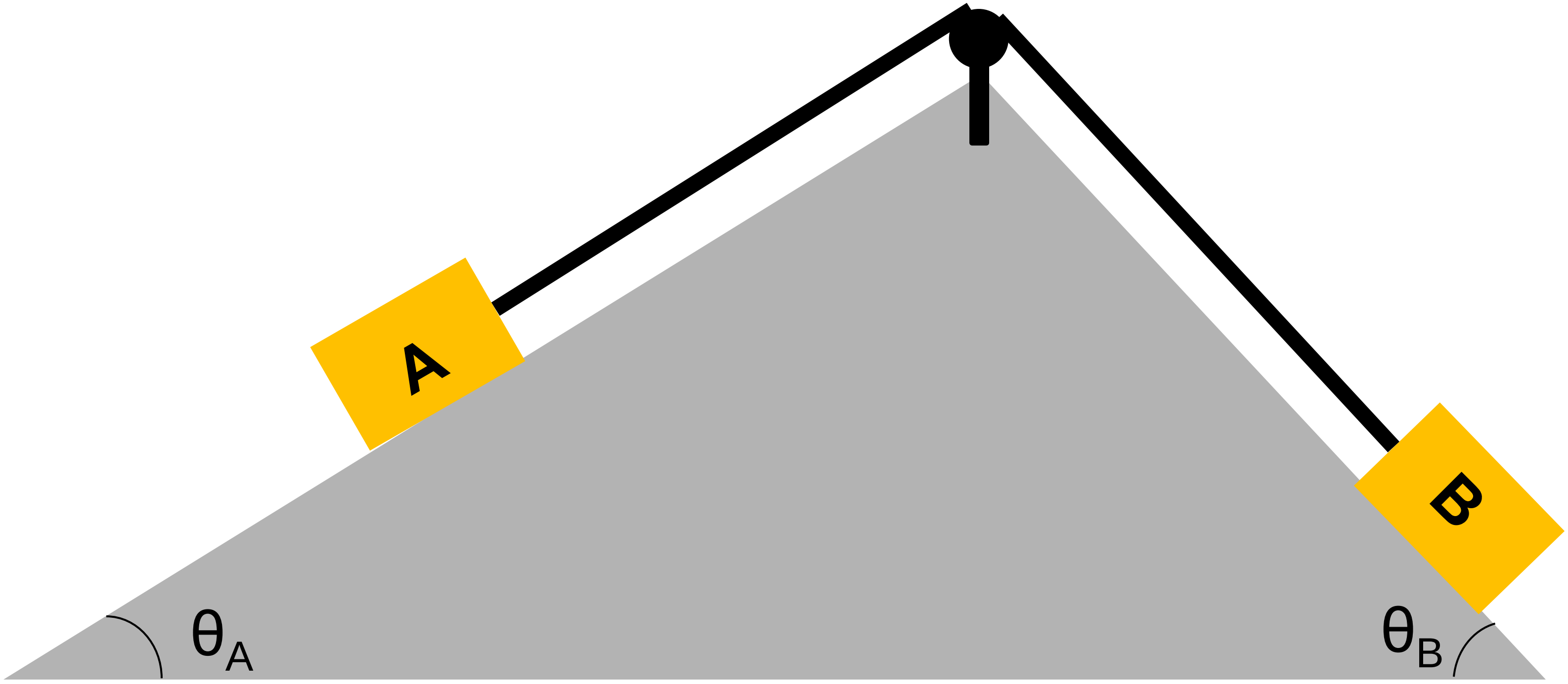
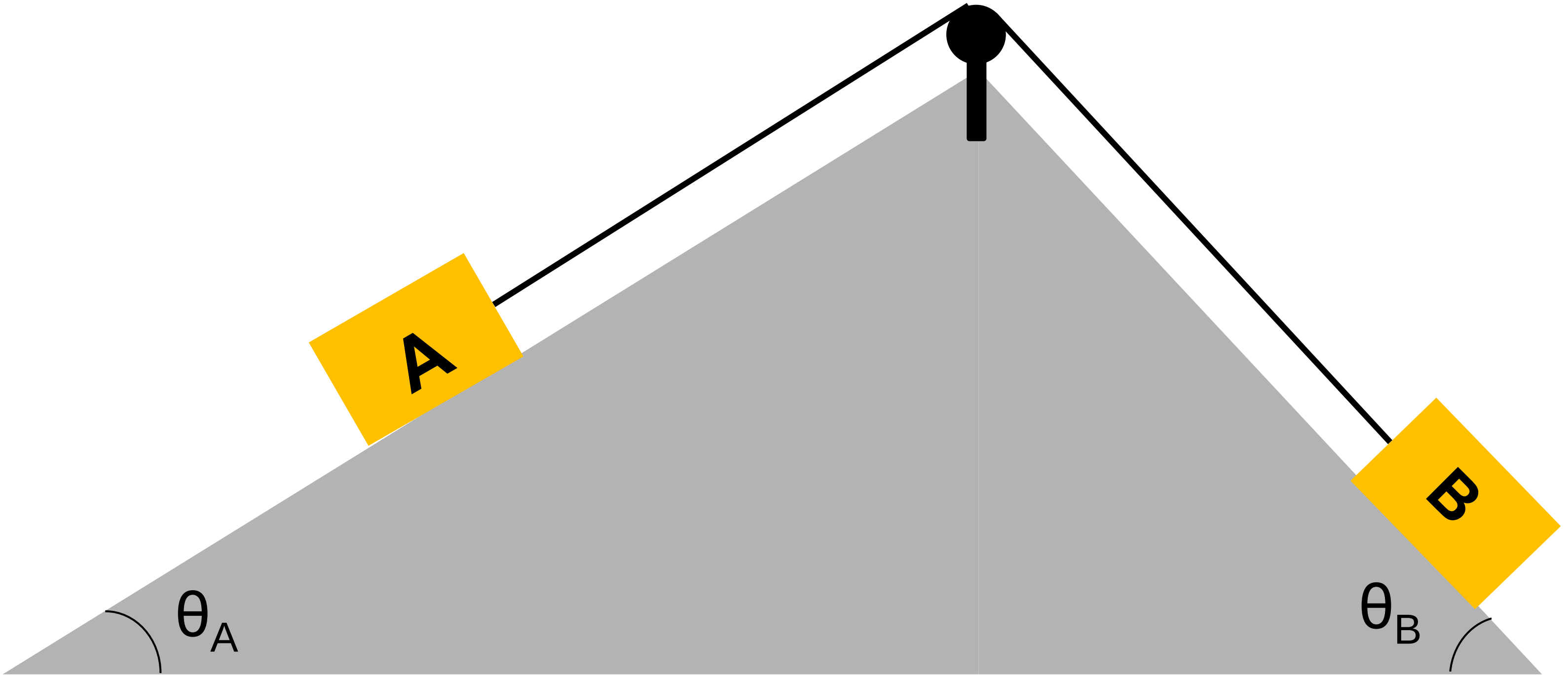
Two Blocks on a Pyramid



Two Blocks on a Pyramid

In all the questions that follow the masses of the blocks are equal and will be denoted as m. However, the angles of the incline are different, thus: $m_A = m_B = m \text{ and } \theta_A < \theta_B$



Two Blocks on a Pyramid I

How many forces are acting on the block labelled A? The coefficient of static friction between the block and the pyramid is μ_s=0.01.

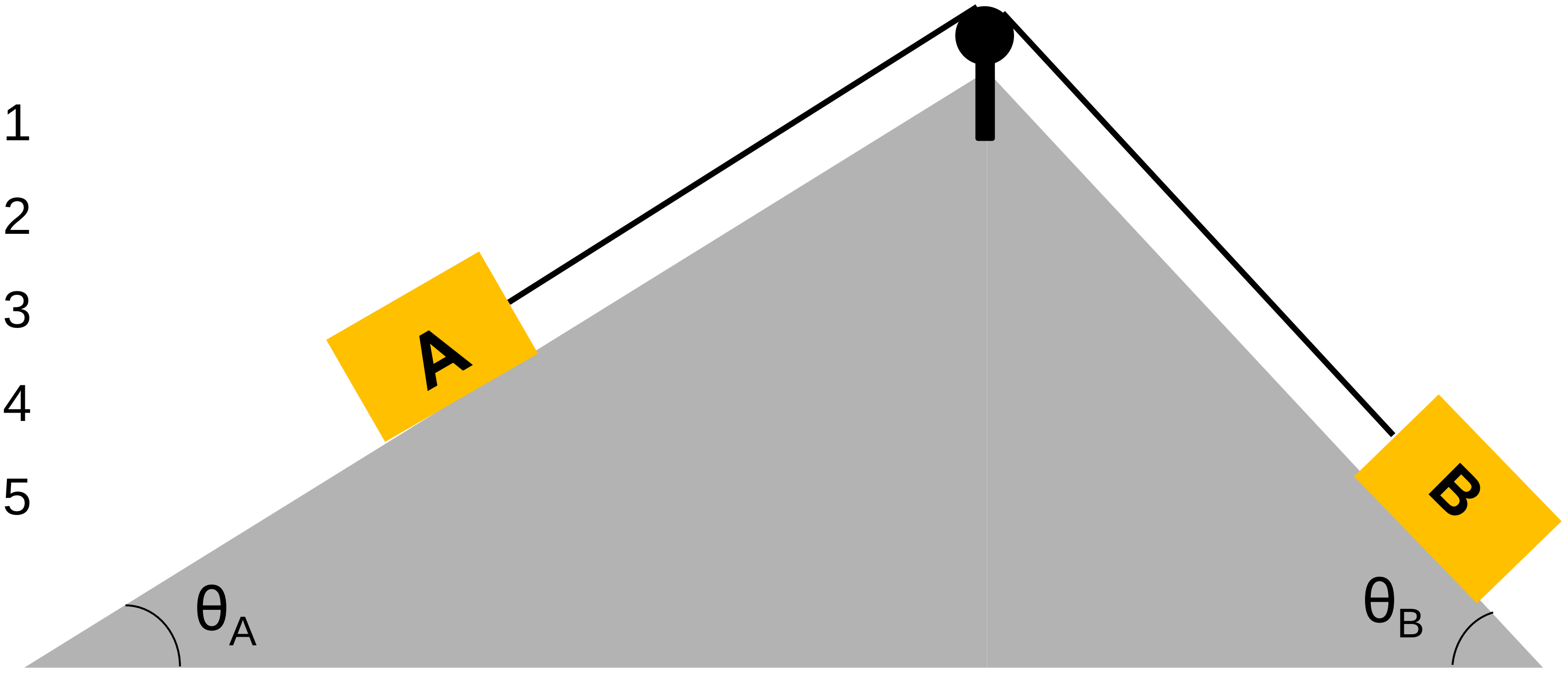
A. 1

B. 2

C. 3

D. 4

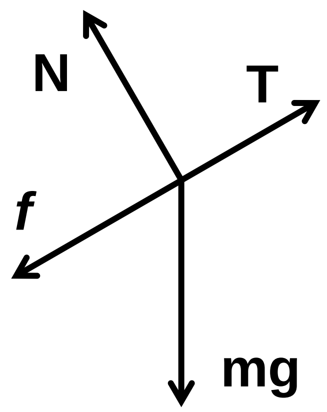
E. 5



Solution

Answer: D

Justification: To help our understanding, we can draw a free body diagram for block A.



As you can see, there are four forces acting on it: the tension provided by the string, the normal force perpendicular to the pyramid, the friction force from being in contact with the pyramid, and the gravitational force.

Two Blocks on a Pyramid II

Based on your intuition, what direction do you expect the blocks to move if they move at all? Think why you expect it.

Recall: $m_A = m_B = m$ and $\theta_A < \theta_B,\ \mu_s = 0.01$

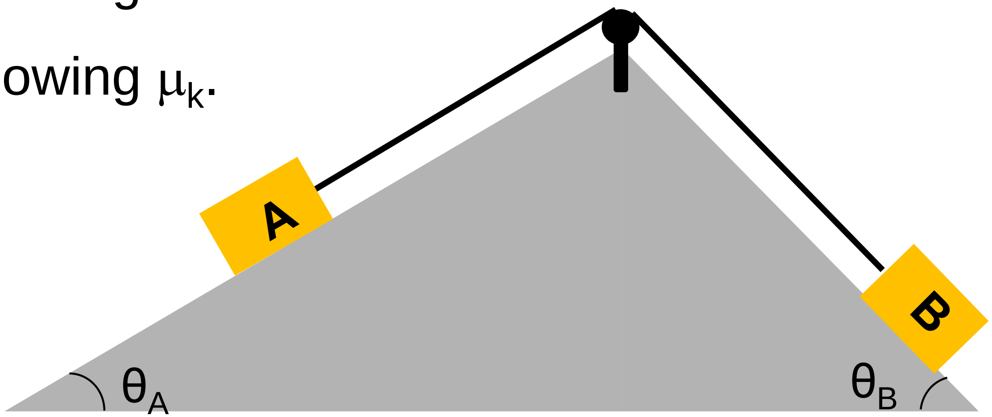
A. Left

B. Right

C. They will not move since they have the same mass

D. Cannot be determined without knowing their mass

E. Cannot be determined without knowing μ_k.



Solution

Answer: B

Justification: Since B rests on a much steeper slope than A, the component of gravity that is parallel to the slope is much higher for B than it is for block A. Therefore, the blocks will slide to the right.

The amount of friction from the pyramid is too small to make a difference as the coefficient of static friction is only 0.01 ($\mu_k < \mu_s$). The exact expression for the acceleration of the blocks will be developed later on. But you can see that you can think of the problem before doing the calculations. Notice, the masses of the blocks are equal and they cancel out; the coefficient of friction is the coefficient of kinetic friction:

$$a = g(\sin\theta_B - \sin\theta_A) - \mu_k g(\cos\theta_B + \cos\theta_A)$$

$$a = g\left[(\sin\theta_B - \sin\theta_A) - \mu_k(\cos\theta_B + \cos\theta_A)\right]$$

Two Blocks on a Pyramid III

What is component of the gravitational force along the incline acting on block A? Recall: $m_A = m_B = m$ and $\theta_A < \theta_B$

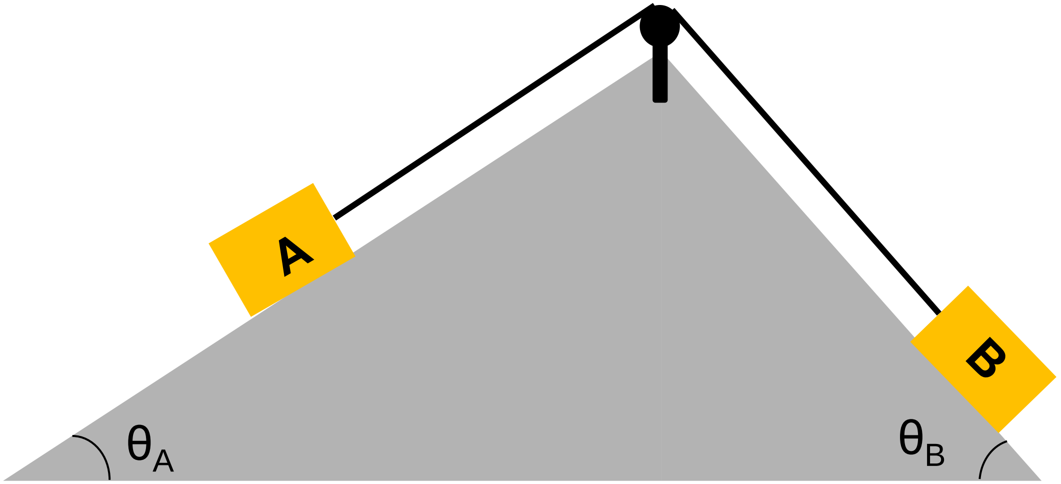
A. $mg\cos\theta_A$

B. $mg\sin\theta_A$

C. $mg\tan\theta_A$

D. $\dfrac{mg}{\sin\theta_A}$

E. $mg\sin\theta_B$

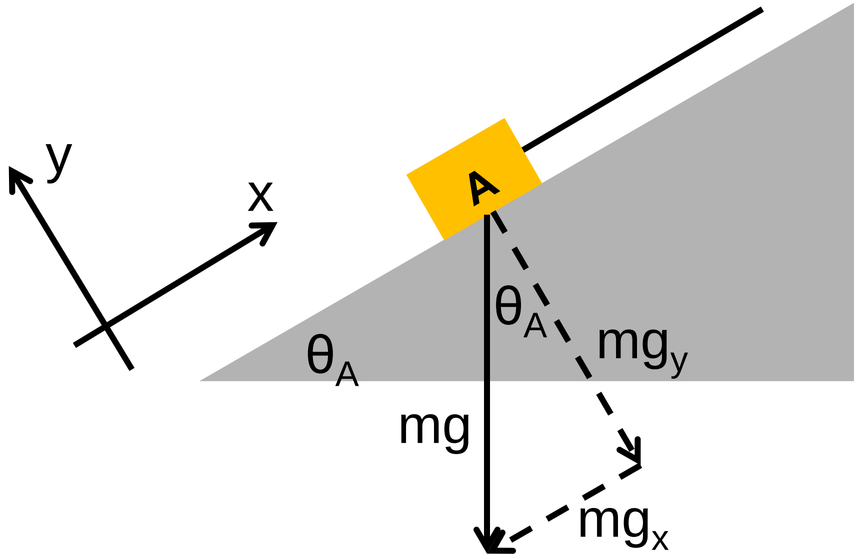


Solution

Answer: B

Justification: The component of the gravitational force on block A in the direction of the surface of the incline is shown in the figure. Notice what angles in the triangle correspond to the angles of the incline.

Also pay attention at the choice of coordinate axes!



The other options make mistakes in trigonometry.

Two Blocks on a Pyramid IV

What is the correct expression for the force of friction acting on block A? Recall: $m_A = m_B = m$ and $\theta_A < \theta_B$

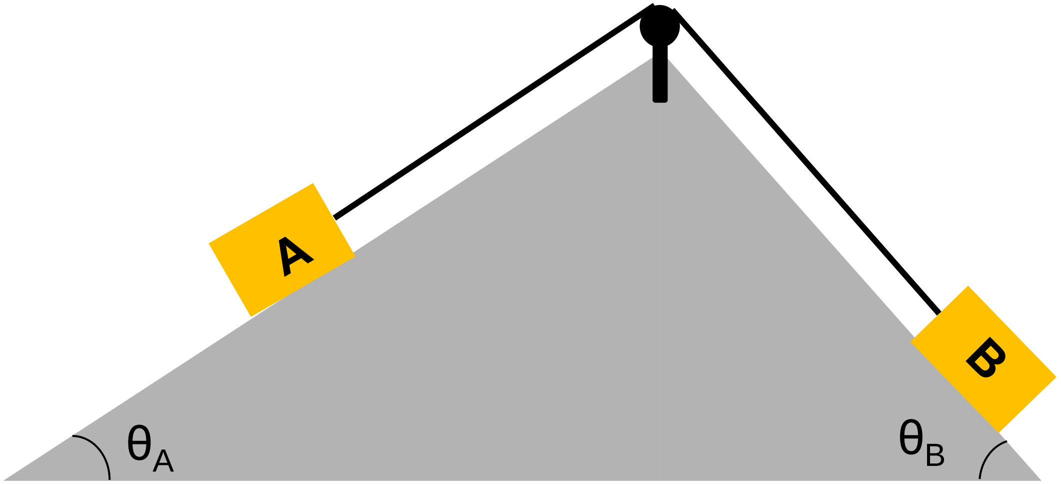
A. $\dfrac{\mu mg}{\sin\theta_A}$

B. $\dfrac{\mu mg}{\cos\theta_A}$

C. $\mu mg\sin\theta_A$

D. $\mu mg\cos\theta_A$

E. $\mu g\cos\theta_A$



Solution

Answer: D

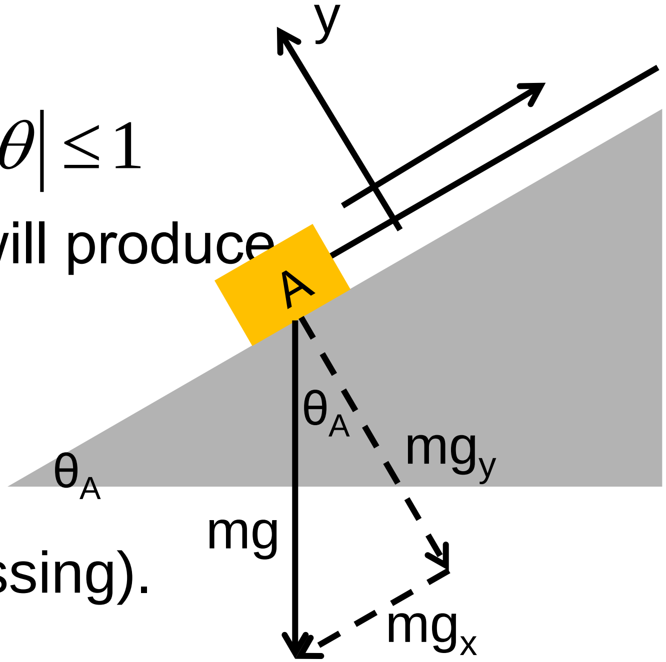
Justification: The normal force on A is equal to the vertical component of the force of gravity, $mg\cos\theta_A$. $F_{fric} = \mu N$. The friction is therefore $\mu mg\cos\theta_A$. Depending on the state of motion of the block you chose either static or kinetic friction.

$$|\sin\theta| \leq 1 \text{ and } |\cos\theta| \leq 1$$

Notice, A and B don't make sense as they will produce a force larger than mg:

C is wrong as it uses the wrong component of mg (trigonometric mistake).

E is incorrect as it has wrong units (m is missing).



Two Blocks on a Pyramid V

What is component of the gravitational force along the incline acting on block B? Recall: $m_A = m_B = m$ and $\theta_A < \theta_B$

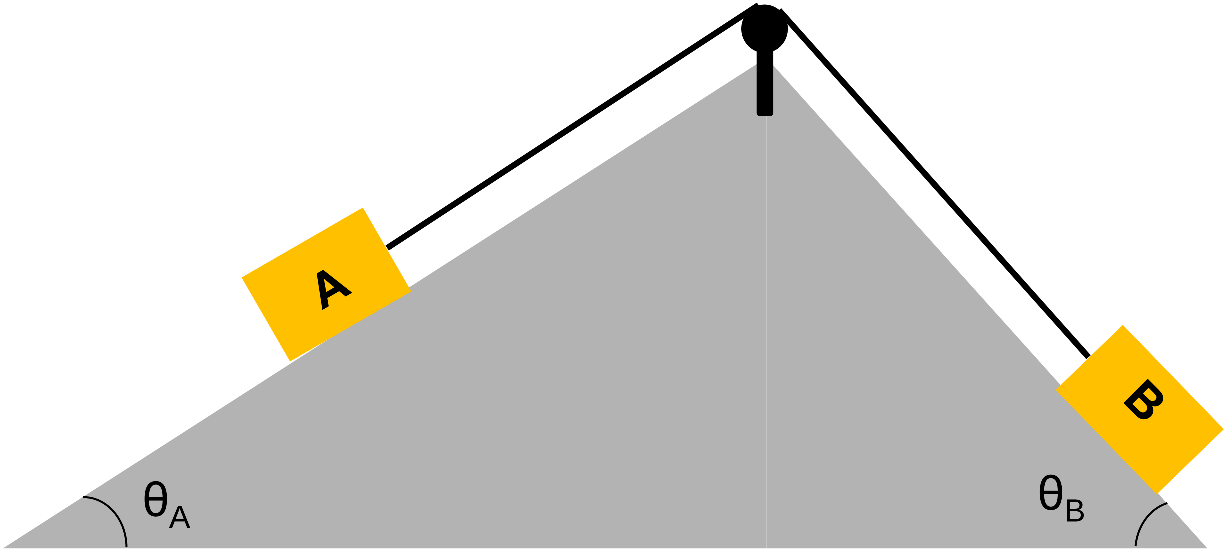
A. $mg\cos\theta_B$

B. $mg\sin\theta_B$

C. $mg\tan\theta_B$

D. $\dfrac{mg}{\sin\theta_B}$

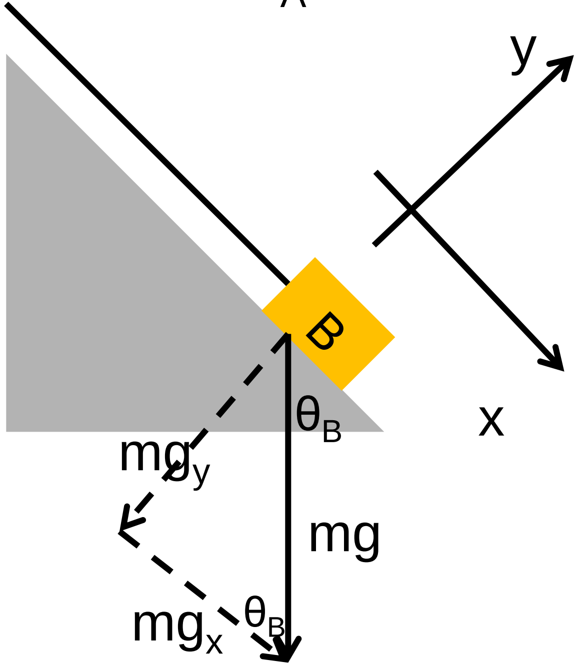
E. $\mu mg\sin\theta_A$



Solution

Answer: B

Justification: Only the angle changed from what is otherwise the same situation as for mass A (see question III). Therefore, it is the same answer as earlier, except with θ_B instead of θ_A.



Once again, we chose a coordinate system such as one axis is along the incline and another axis is directed perpendicular to it.

Two Blocks on a Pyramid VI

Recalling that blocks A and B have the same mass. What is the correct expression for the acceleration of block A?

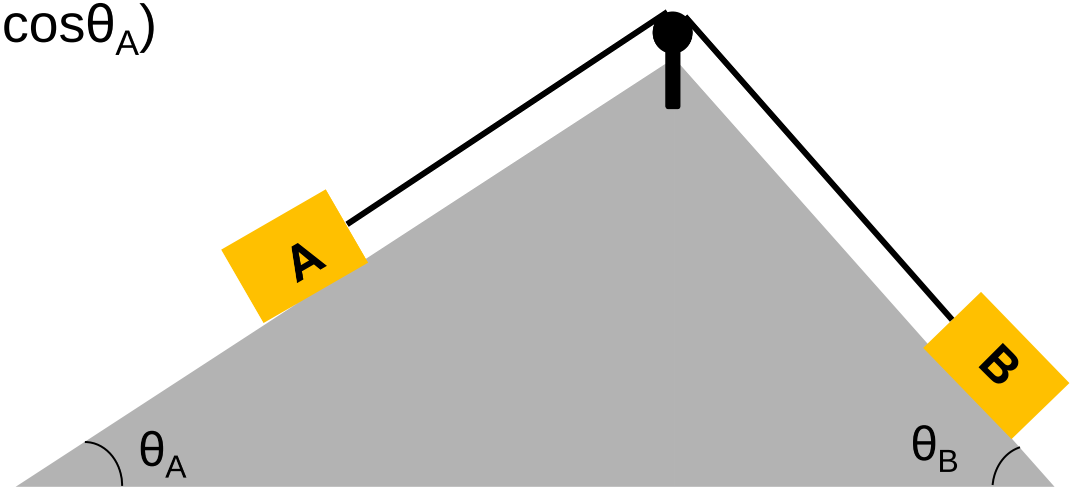
A. $g(\sin\theta_B - \sin\theta_A) - \mu g(\cos\theta_B - \cos\theta_A)$

B. $g(\sin\theta_B + \sin\theta_A) - \mu g(\cos\theta_B + \cos\theta_A)$

C. $g(\sin\theta_B - \sin\theta_A) - \mu g(\cos\theta_B + \cos\theta_A)$

D. $g(\sin\theta_B + \sin\theta_A) - \mu g(\cos\theta_B - \cos\theta_A)$

E. None of the above



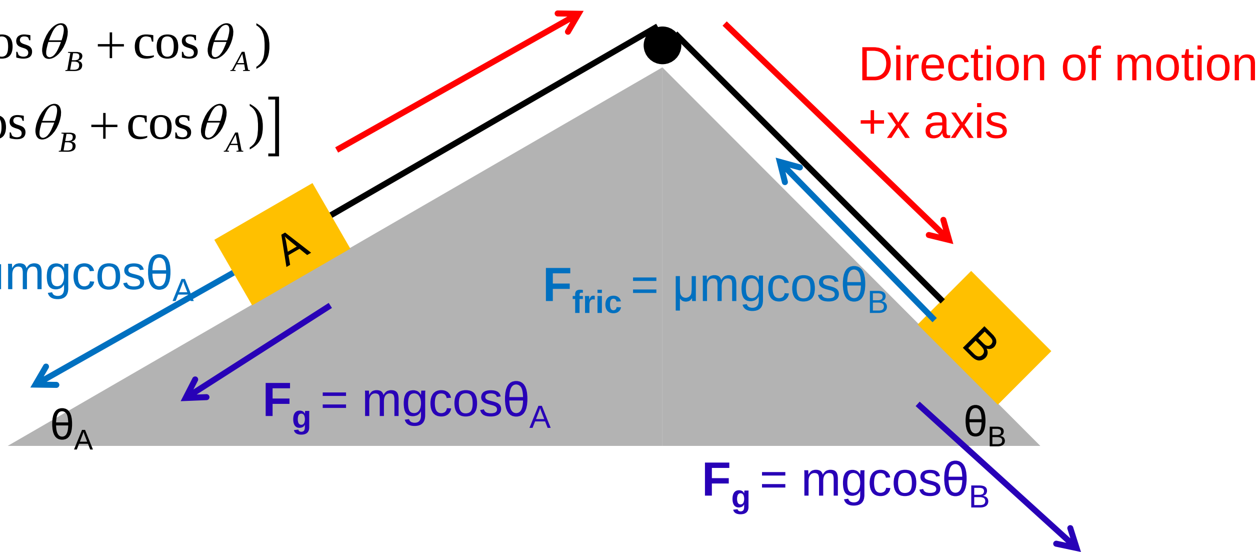
Solution

Answer: C

Justification: Assume that the blocks move to the right, driven by the gravitational force on block B ($mg\sin\theta_B$). Both blocks experience a friction force in the opposite direction of this motion. Additionally, block A "feels" a gravitational force in the opposite direction. Adding these forces together, we get $mg\sin\theta_B$ - $\mu mg\cos\theta_B$ - $mg\sin\theta_A$ - $\mu mg\cos\theta_A$. To get the acceleration we divide by m, and simplify, to get the answer

$$a = g(\sin\theta_B - \sin\theta_A) - \mu_k g(\cos\theta_B + \cos\theta_A)$$

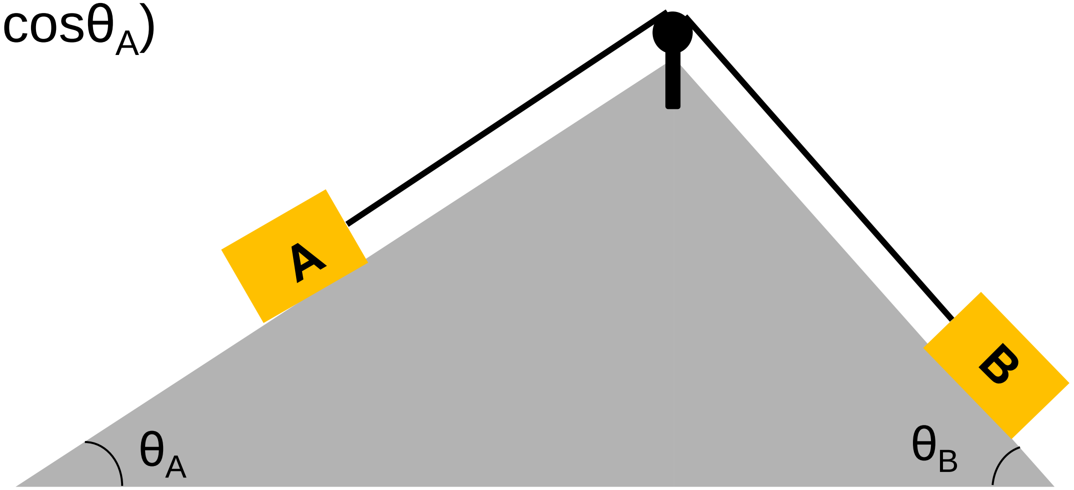
$$a = g\left[(\sin\theta_B - \sin\theta_A) - \mu_k(\cos\theta_B + \cos\theta_A)\right]$$



Two Blocks on a Pyramid VII

Recalling that blocks A and B have the same mass. What is the correct expression for the acceleration of block B?

A. $g(\sin\theta_B - \sin\theta_A) - \mu g(\cos\theta_B - \cos\theta_A)$
B. $g(\sin\theta_B + \sin\theta_A) - \mu g(\cos\theta_B + \cos\theta_A)$
C. $g(\sin\theta_B - \sin\theta_A) - \mu g(\cos\theta_B + \cos\theta_A)$
D. $g(\sin\theta_B + \sin\theta_A) - \mu g(\cos\theta_B - \cos\theta_A)$
E. None of the above



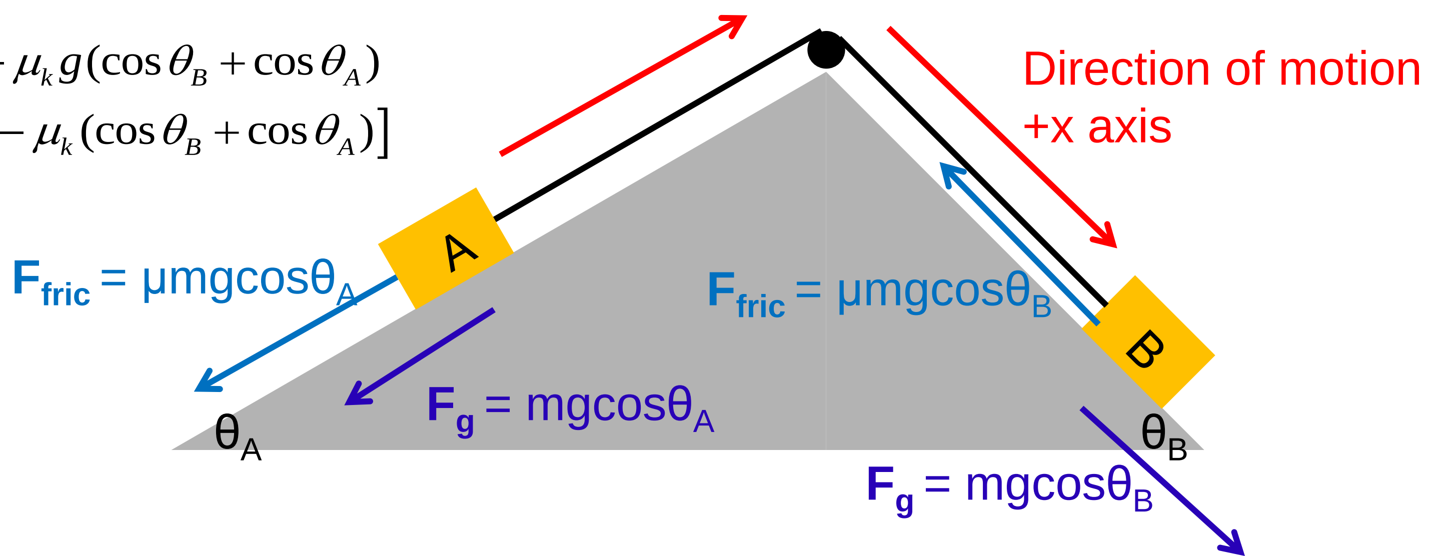
Solution

Answer: C

Justification: Assume that the blocks move to the right, driven by the gravitational force on block B ($mg\sin\theta_B$). Both blocks feel a friction force in the opposite direction of this motion. Additionally, block A feels a gravitational force in the opposite direction. Adding these forces together, we get $mg\sin\theta_B - \mu mg\cos\theta_B - mg\sin\theta_A - \mu mg\cos\theta_A$. To get the acceleration we divide by m, and simplify, to get the answer:

$$a = g(\sin\theta_B - \sin\theta_A) - \mu_k g(\cos\theta_B + \cos\theta_A)$$

$$a = g\left[(\sin\theta_B - \sin\theta_A) - \mu_k(\cos\theta_B + \cos\theta_A)\right]$$



Solution

It is important to realize that since blocks A and B constitute the same system, their accelerations must be the same. Therefore, instead of talking about the acceleration of block A or acceleration of block B we can talk about the acceleration of the system. Another important point is the choice of coordinate axes. It is useful to chose the axes, such as the acceleration of the system is directed along one of them – for example, the x-axis.

$$a = g(\sin\theta_B - \sin\theta_A) - \mu_k g(\cos\theta_B + \cos\theta_A)$$

$$a = g\left[(\sin\theta_B - \sin\theta_A) - \mu_k(\cos\theta_B + \cos\theta_A)\right]$$