a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

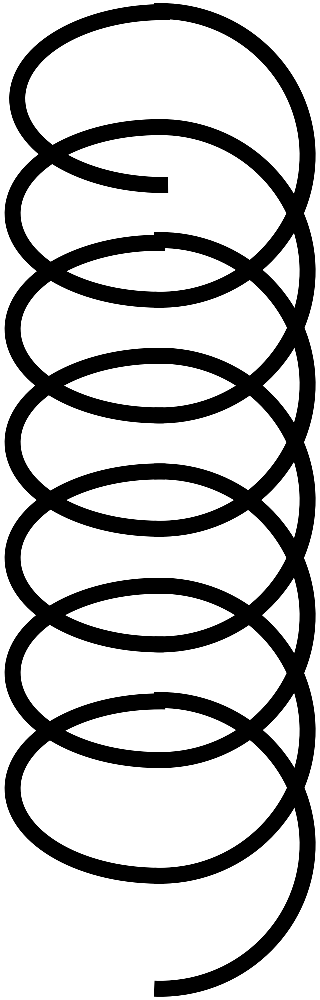
Physics

Dynamics: Springs

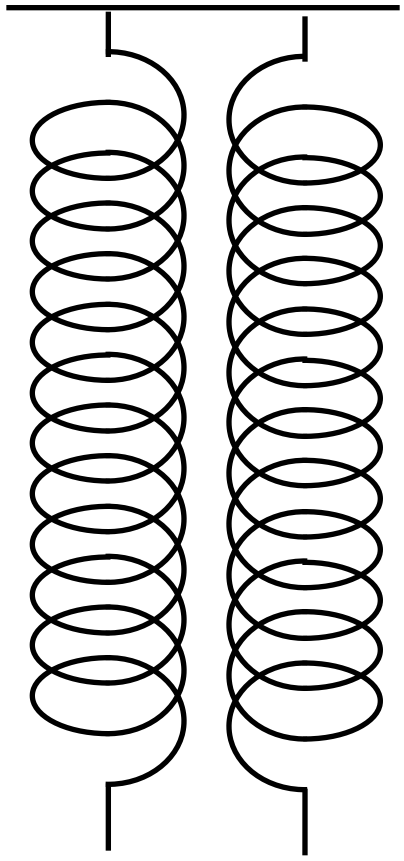
Science and Mathematics
Education Research Group

Supported by UBC Teaching and Learning Enhancement Fund 2012-2013

Springs in Series and Parallel



k_s



Vertical Springs I

A 0.50 m spring with spring constant 100 N/m hangs from the ceiling. A 2.0 kg block is tied to the spring. How much does the spring stretch? (Use $g = 10\ m/s^2$)

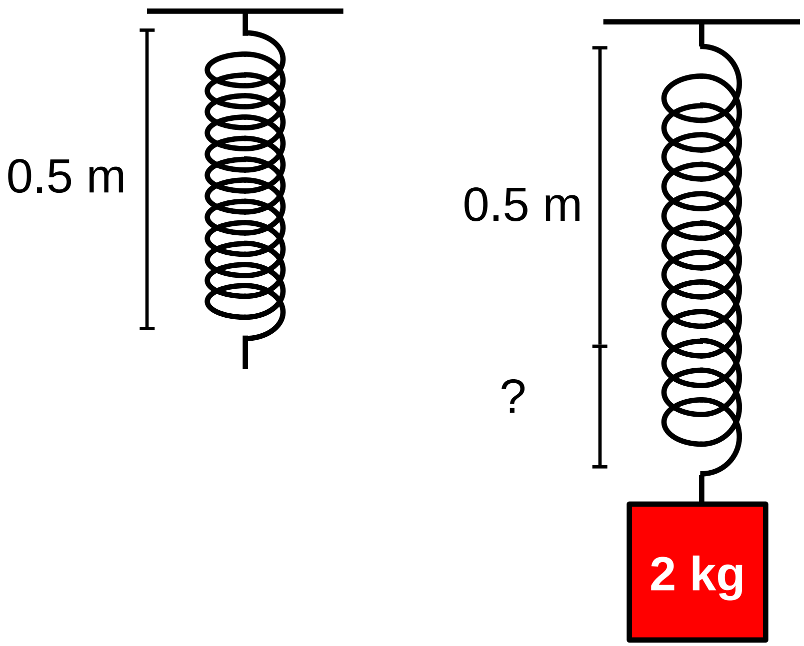
A. 2.0 m

B. 0.70 m

C. 0.52 m

D. 0.20 m

E. 0.020 m



Solution

Answer: D

Justification: The 2.0 kg mass applies a 20 N force downwards on the spring (this force is caused by gravity – the pull of the Earth). In order to support the 20 N downward force, the spring must apply a 20 N force upwards. Assume upwards is positive and downwards is negative.

$$\mathbf{F}_S = -k\mathbf{d}$$

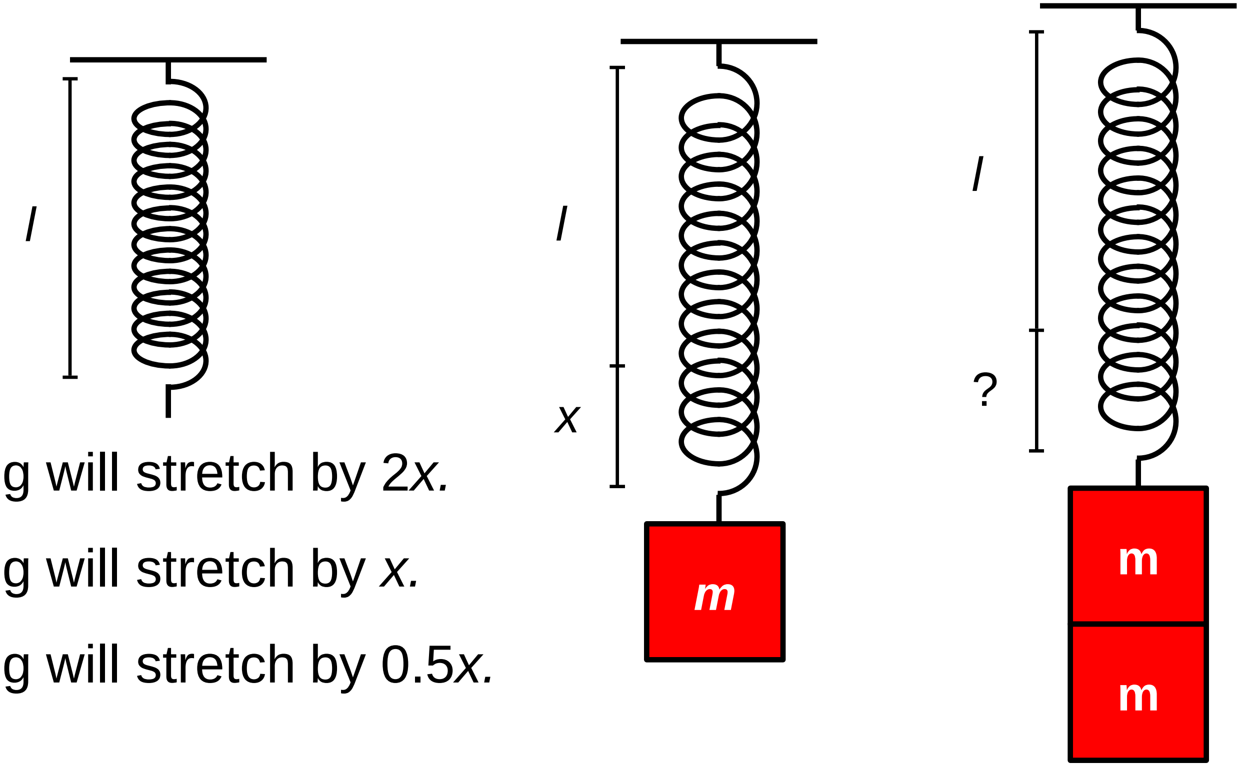
$$20\ \mathrm{N} = (-100\ \mathrm{N/m})\mathbf{d}$$

$$\mathbf{d} = -0.20\ \mathrm{m}$$

Therefore, the spring will stretch (extend downwards) by 0.20 m. The total length of the spring will be 0.70 m, but the stretch is only 20 cm.

Vertical Springs II

A spring with length *l* and spring constant k_s hangs from the ceiling. A mass *m* is placed on the spring and increases the length of the spring by *x*. By how much will a 2*m* mass stretch the spring from its original - un-stretched state?



A. The spring will stretch by 2*x.*

B. The spring will stretch by *x.*

C. The spring will stretch by 0.5*x.*

Solution

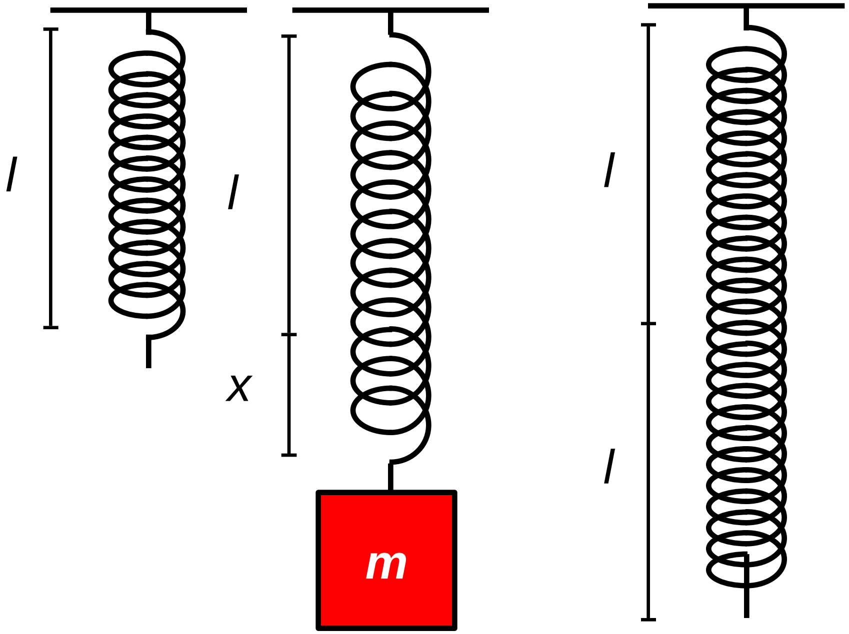
Answer: A

Justification: The tension force of a spring is directly proportional to the amount it is compressed or stretched from its rest position: $F = -kx$.

A 2m mass will exert a downward force twice as large as a 1m mass. Thus the spring must stretch twice as much in order to hold the 2m mass.

Vertical Springs III

A single spring is stretched by *x* when a mass m is attached. An <u>*identical*</u> spring is joined in *series* to the first spring. How much will the two springs stretch when a mass *m* is attached? (Assume the springs have negligible mass)



A. The spring will stretch by 2*x* since each spring stretches by x

B. The spring will stretch by *x* since the spring constant remains the same

C. The spring will stretch by 0.5x since there are 2 springs holding the mass

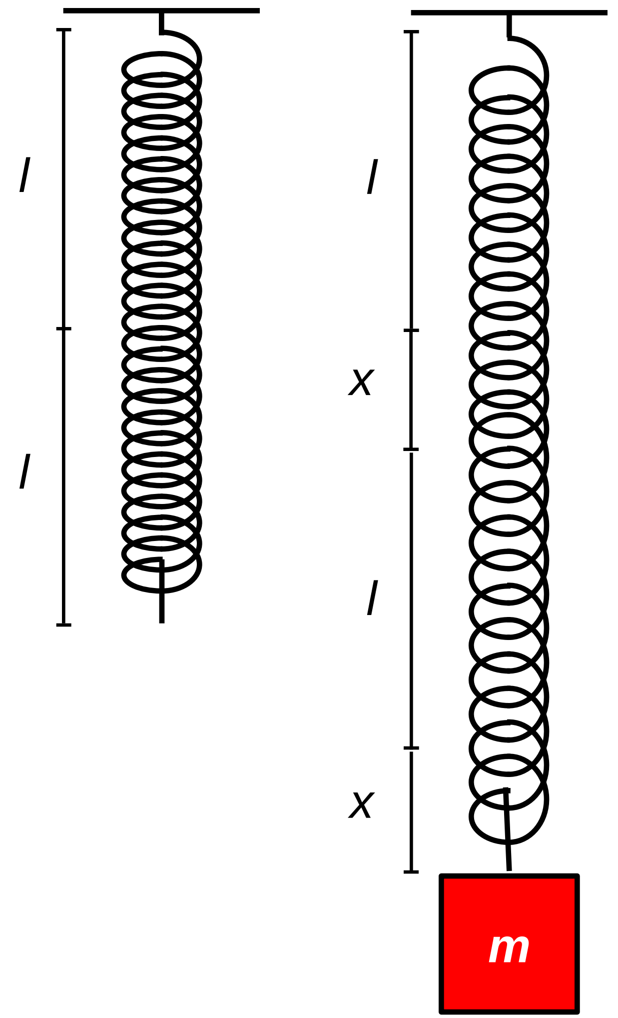
D. There is not enough information to answer

Solution

Answer: A

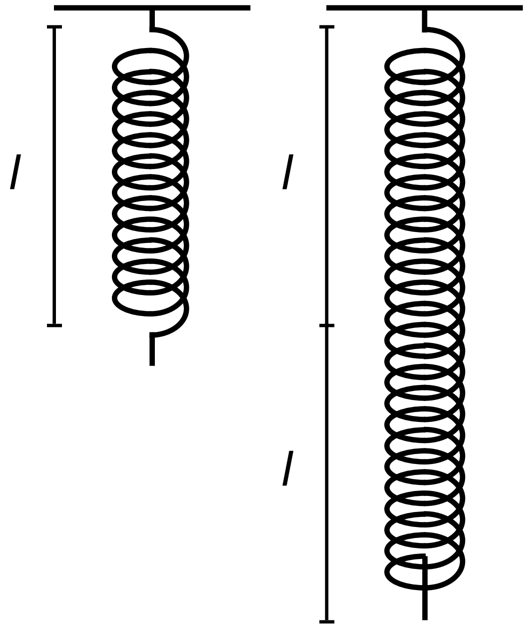
Justification: For springs with negligible mass, the tension along them has to be constant at all points. Since the tension of the spring holding the mass is equal to mg, the tension of the other spring is also mg. Each spring stretches by x, causing a total stretch of 2x.

This means that two identical springs connected in series will stretch twice as much as one spring would have stretched!



Vertical Springs IV

Two identical springs (each with spring constant k_s) are connected in series as shown. What is the spring constant of the two springs together (k_T)? (Assume the springs have negligible mass)



A. $k_T = 2k_s$

B. $k_T = k_s$

C. $k_T = 0.5k_s$

D. Cannot be determined

Solution

Answer: C

Justification: From question III, we learned that doubling the length of a spring will cause it to stretch twice as much.

The spring becomes "weaker," since a smaller force is required to stretch it by the same amount. The spring constant is therefore halved:

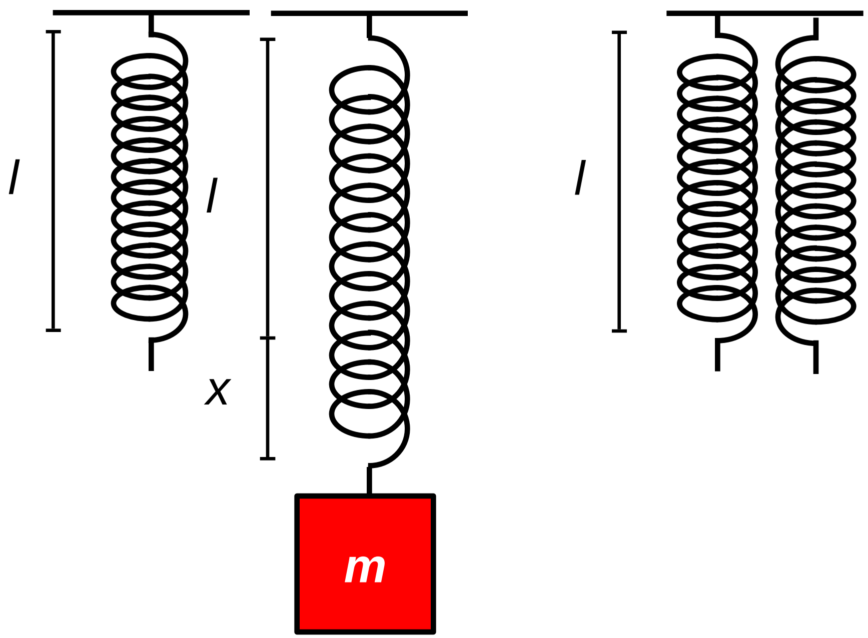
$$F = -k_T d$$

$$F = -k_T (2x)$$

$$k_T = \frac{F}{2x} = \frac{1}{2}\left(\frac{F}{x}\right) = \frac{1}{2} k_s$$

Vertical Springs V

A single spring is stretched by *x* when a mass m is attached. An *identical* spring is joined in *parallel* to the first spring. How much will the two springs stretch when a mass m is attached to both springs simultaneously? (Assume the springs have negligible mass)

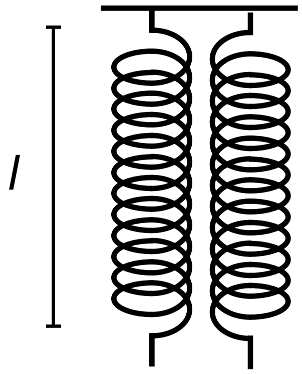


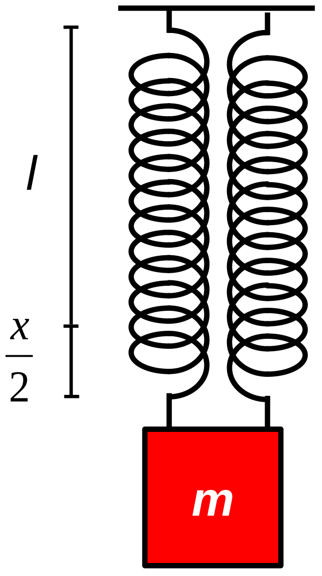
A. The spring will stretch by 2*x* since each spring stretches by *x*
B. The spring will stretch by *x* since the spring constant remains the same
C. The spring will stretch by 0.5*x* since there are 2 springs holding the mass m

Solution

Answer: C

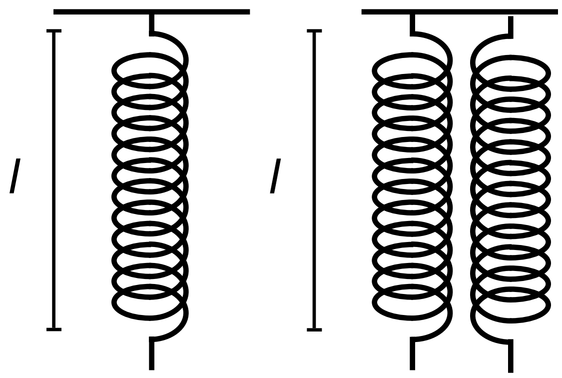
Justification: The downward force mg is now supported by 2 separate springs. Each spring must then exert an upward force equal to $\frac{mg}{2}$. Therefore, each spring will stretch half as much, or $\frac{x}{2}$.





Vertical Springs VI

Two identical springs (each with spring constant k_s) are connected in parallel as shown. What is the spring constant of the two springs together (k_T)? (Assume the springs have negligible mass)



A. $k_T = 2k_s$

B. $k_T = k_s$

C. $k_T = 0.5k_s$

D. Cannot be determined

Solution

Answer: A

Justification: We learned from question 5 that two springs in parallel will stretch half as much as a single spring with the same mass attached. It requires twice as much force to stretch the two springs.

The two springs are “stronger” and have twice the spring constant.

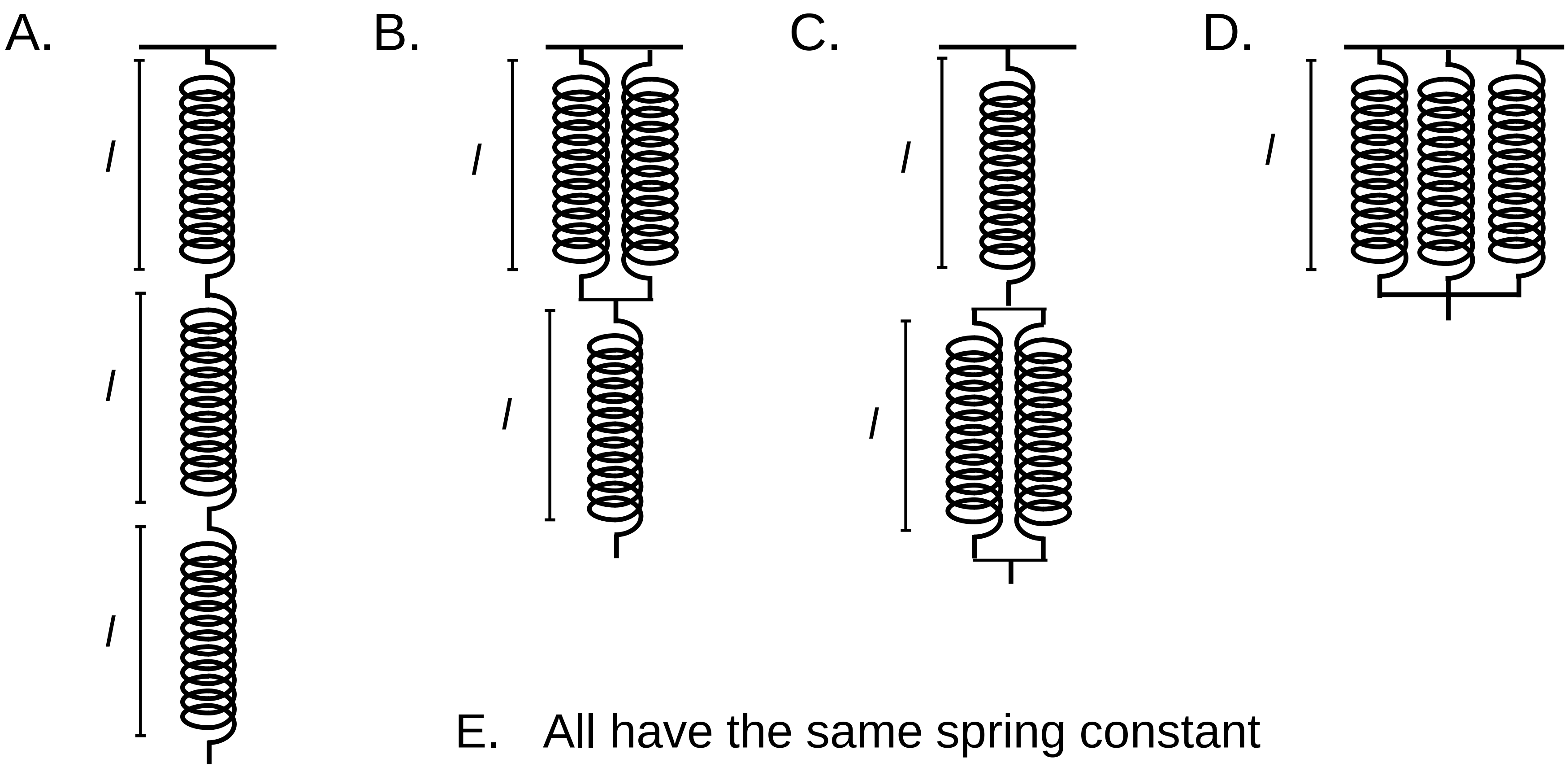
$$F = -k_T d$$

$$F = -k_T \left(\frac{x}{2}\right)$$

$$k_T = \frac{2F}{x} = 2\left(\frac{F}{x}\right) = 2k_s$$

Vertical Springs VII

Which collection of springs has the largest spring constant?



E. All have the same spring constant

Solution

Answer: D

Justification: Spring A has 3 springs in series, so the spring constant is $\frac{k_s}{3}$.

Spring B and Spring C have springs connected in parallel and in series. The springs in parallel stretch 0.5*x*, and the single spring stretches *x*. The total stretch is 1.5*x*, giving a spring constant of $\frac{2k_s}{3}$

Spring D has 3 springs in parallel, so the spring constant is $3k_s$. For this case:

$$F = -k_T d$$

$$F = -k_T (\frac{x}{3})$$

$$k_T = \frac{3F}{x} = 3\left(\frac{F}{x}\right) = 3k_s$$