a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Physics

Electrostatics: Electric Fields at a Point

Science and Mathematics
Education Research Group

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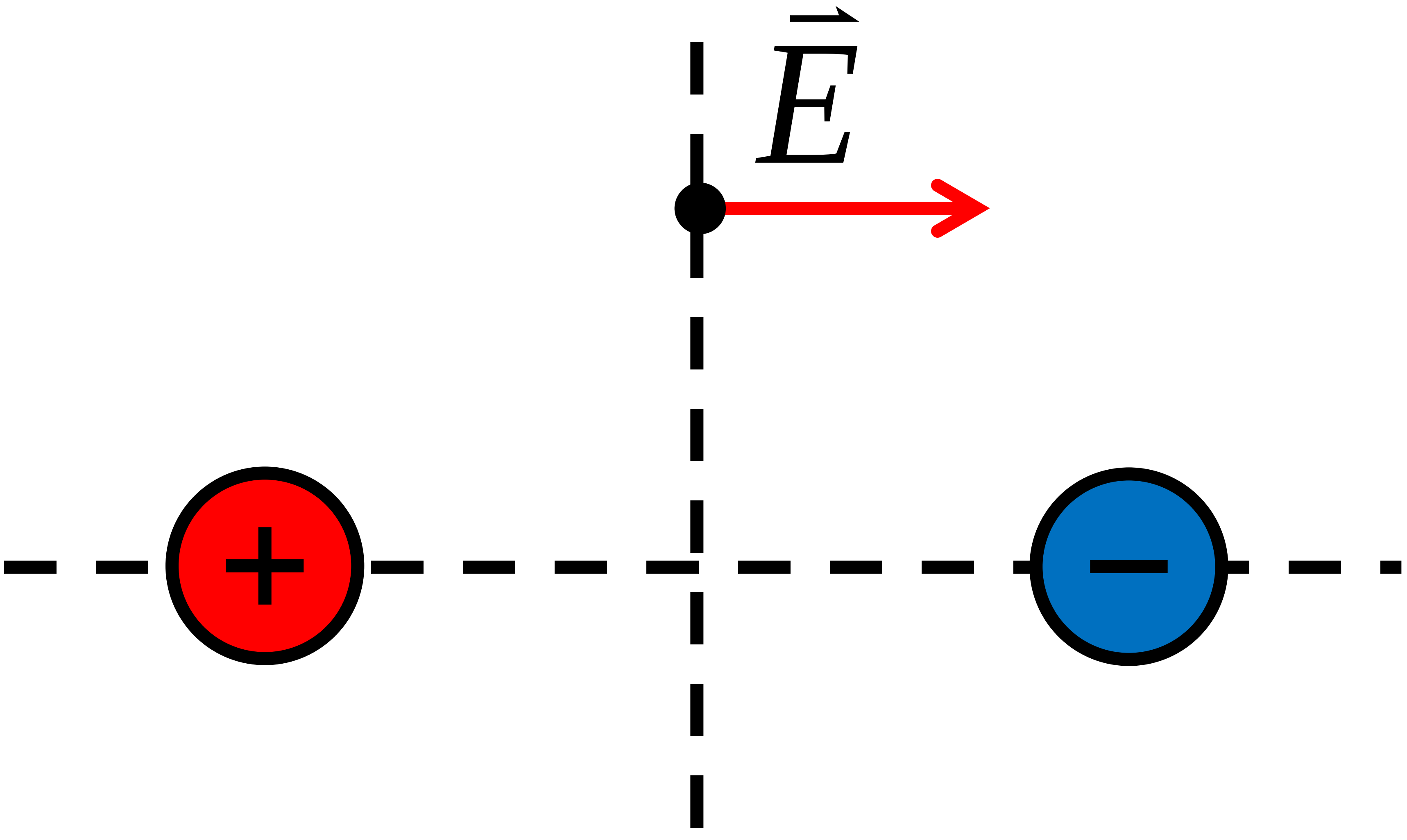
Electric Fields at a Point



Electric Fields

The electric field is defined as the force per unit charge experienced by a positive test charge q:

$$\vec{E} = \frac{\vec{F}}{q}$$

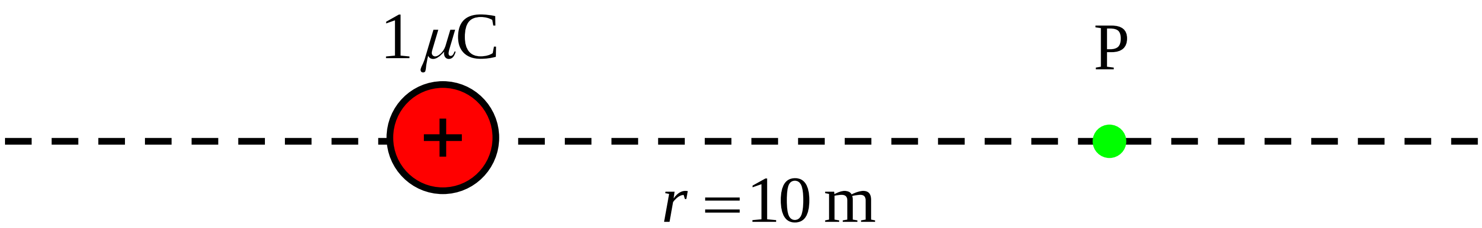
The electric field is a vector quantity with units newtons per coulomb (N/C). It is directed in the same direction as the force experienced by the small positive test charge inside the electric field.

The magnitude of the electric field from a single charge Q at a distance r is:

$$E = \frac{1}{q}\frac{kQq}{r^2} = \frac{kQ}{r^2}$$

Electric Field at a Point I

What is the magnitude and direction of the electric field at a point 10 m away from a +1 μC charge?



A. $E = 90\text{ N/C}$ $\longleftarrow$

B. $E = 90\text{ N/C}$ $\longrightarrow$

C. $E = 900\text{ N/C}$ $\longleftarrow$

D. $E = 900\text{ N/C}$ $\longrightarrow$

E. None of the above

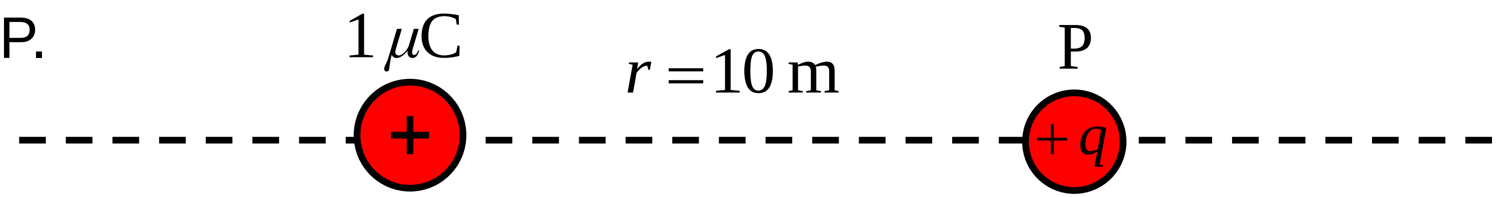
Coulomb's constant:

$$k = 9\times10^{9}\ \frac{\text{Nm}^2}{\text{C}^2}$$

Solution

Answer: B

Justification: Consider putting a small positive test charge at the point P.



The test charge will experience a repelling force to the right since two positive charges repel. The magnitude of the force can be calculated according to Coulomb's law:

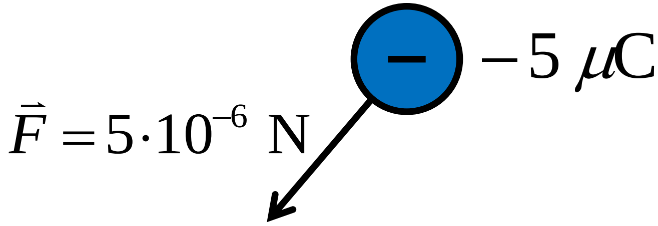
$$\vec{F} = k\frac{Qq}{r^2} = \frac{(9\cdot 10^9)(1\cdot 10^{-6})q}{10^2} = 90q\text{ N} \;\longrightarrow$$

The electric field at this point is therefore: $\vec{E} = \dfrac{\vec{F}}{q} = 90\,\dfrac{\text{N}}{C} \;\longrightarrow$

Notice that the electric field points in the same direction as the force experienced by a positive charge.

Electric Field at a Point II

To determine if there is an electric field at a point P, a small negative test charge is put at that point. It experiences a force as shown.



What is the electric field at that point?

A. $E = 1$ N/C ↗

B. $E = 1$ N/C ↙

C. There is no electric field

D. The distance from the electric field source must be known

E. A positive test charge must be used to determine the electric field

Solution

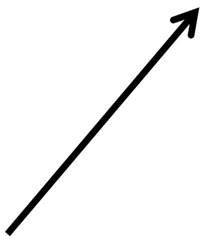
Answer: A

Justification: The test charge experiences a force, so the magnitude of the electric field is:

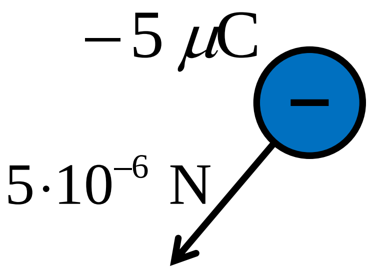
$$E = \frac{F}{q} = \frac{5 \cdot 10^{-6}\ \mathrm{N}}{5 \cdot 10^{-6}\ \mathrm{C}} = 1\,\frac{\mathrm{N}}{\mathrm{C}}$$

The direction of an electric field is defined to be in the same direction as the force experienced by a positive test charge. If a negative test charge is used instead, the direction of the electric field will be in the opposite direction.

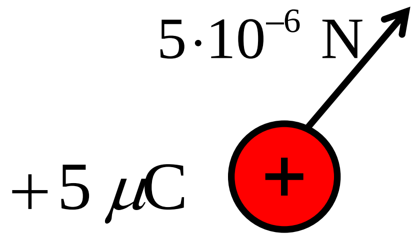
Direction of field:



Negative test charge:



Positive test charge:



Electric Field at a Point III

Suppose we put a positive test charge q at a point P and determine that there exists an electric field with magnitude 3 N/C.

If a positive test charge $2q$ is used instead, what will the electric field be at point P?

A. The electric field will be larger by a factor of 2

B. The electric field will be smaller by a factor of 2

C. The electric field will be the same

Solution

Answer: C

Justification: The test charge is twice as large, but this means the force experienced by the test charge will also be twice as large:

$$F_q = \frac{kQq}{r^2}, \quad \Rightarrow \quad F_{2q} = \frac{kQ(2q)}{r^2} = 2F_q$$

The electric field will therefore be the same as before:

$$E = \frac{2F_q}{2q} = \frac{F_q}{q}$$

This shows that the electric field does not depend on the test charge used to detect it. A small charge is normally used so that it does not disturb other charges or the electric field at other points.

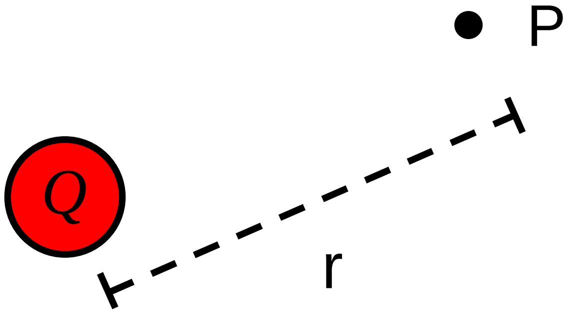
Electric Field at a Point IV

The magnitude of the electric field of a point charge Q, at a point a distance *r* away from the charge, is:

$$E = \frac{kQ}{r^2}$$

What happens to the magnitude of the field at the point P if the charge is halved?

A. Increases by a factor of 4
B. Increases by a factor of 2
C. Decreases by a factor of 4
D. Decreases by a factor of 2
E. Remains the same



Solution

Answer: D

Justification: The electric field is directly proportional to the amount of charge on the point charge. The charge Q appears in the numerator of the electric field equation, so changing Q will change E by the same factor.

$$E = \frac{kQ}{r^2}$$

If the charge decreases, the magnitude of the electric field will also decrease. The charge is halved, so the strength of the electric field will also be halved.

Initial Electric Field:

$$E_i = \frac{kQ}{r^2}$$

New Electric Field:

$$E_f = \frac{k(Q/2)}{r^2} = \frac{1}{2}\frac{k(Q)}{r^2} = \frac{1}{2}E_i$$

Electric Field at a Point V

The magnitude of the electric field of a point charge Q, at a point a distance r away from the charge, is:

$$E = \frac{kQ}{r^2}$$

What happens to the magnitude of the field at the point P if the distance is halved?

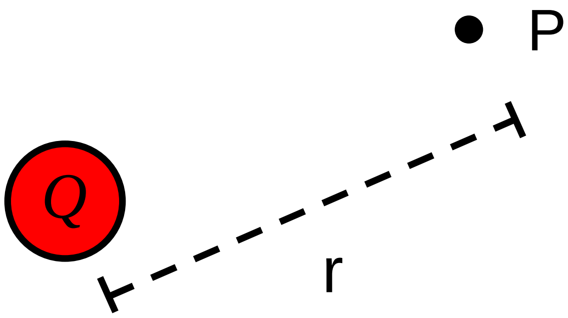
A. Increases by a factor of 4

B. Increases by a factor of 2

C. Decreases by a factor of 4

D. Decreases by a factor of 2

E. Remains the same



Solution

Answer: A

Justification: The electric field is inversely proportional to the square of the distance between a point and the source charge. This is because of the r^2 term in the denominator of the electric field equation.

When the distance increases by a factor of x, the electric field decreases by x^2. The distance is decreased by a factor of two in this question, so the electric field must increase by a factor of $2^2 = 4$.

Initial Electric Field:

$$E_i = \frac{kQ}{r^2}$$

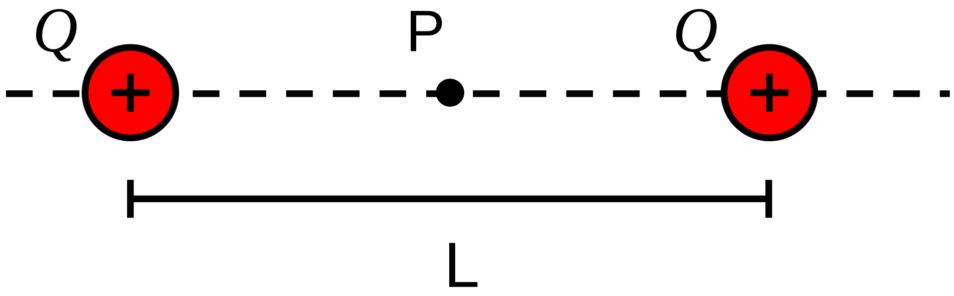
New Electric Field:

$$E_f = \frac{kQ}{(r/2)^2} = \frac{kQ}{r^2/4} = 4\frac{kQ}{r^2} = 4E_i$$

Electric Field at a Point VI

Two equal positive charges Q are L distance apart.

What is the electric field at the point P, the midpoint between the two charges?



A. $\dfrac{2kQ}{(L/2)^2}$

B. $\dfrac{kQ}{L^2}$

C. 0

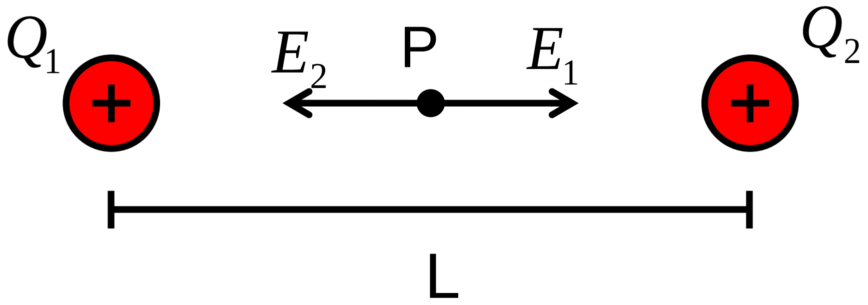
D. $-\dfrac{2kQ}{L^2}$

E. $-\dfrac{kQ}{(L/2)^2}$

Solution

Answer: C

Justification: Electric fields obey the law of **superposition**, meaning that the total electric field of a system is equal to the sum of all electric fields in the system.



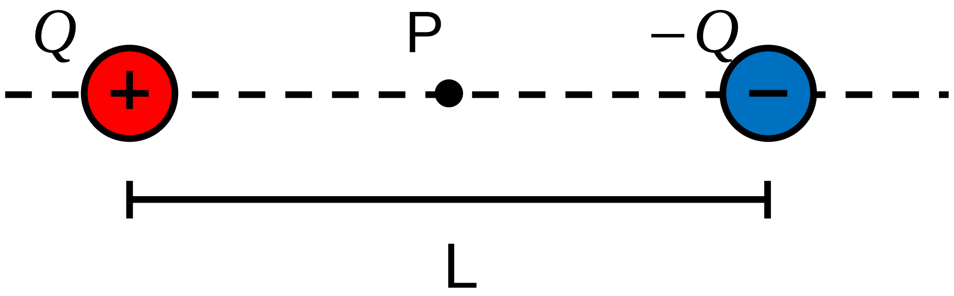
The electric field at point P caused by each charge is equal in magnitude, but opposite in direction. Adding them together results in no net electric field at the centre point.

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 = 0 \text{ N/C}$$

Electric Field at a Point VII

Two charges Q and -Q are a distance L apart.

What is the electric field at the point P, which is at the midpoint between the two charges?



A. $\dfrac{2kQ}{(L/2)^2}$

B. $\dfrac{kQ}{L^2}$

C. 0

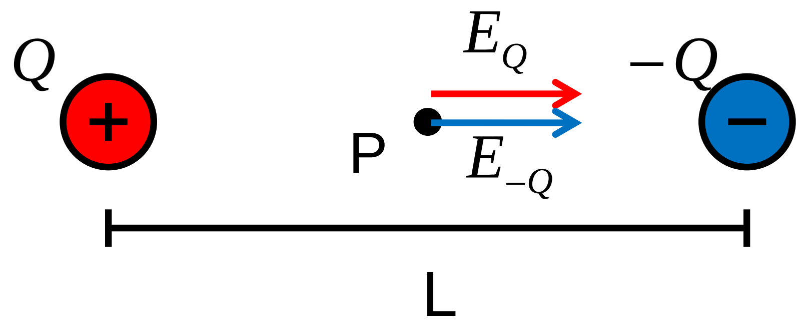
D. $-\dfrac{2kQ}{L^2}$

E. $-\dfrac{kQ}{(L/2)^2}$

Solution

Answer: A

Justification:



The electric field at point P caused by each charge is equal in magnitude, and both electric fields are in the same direction.

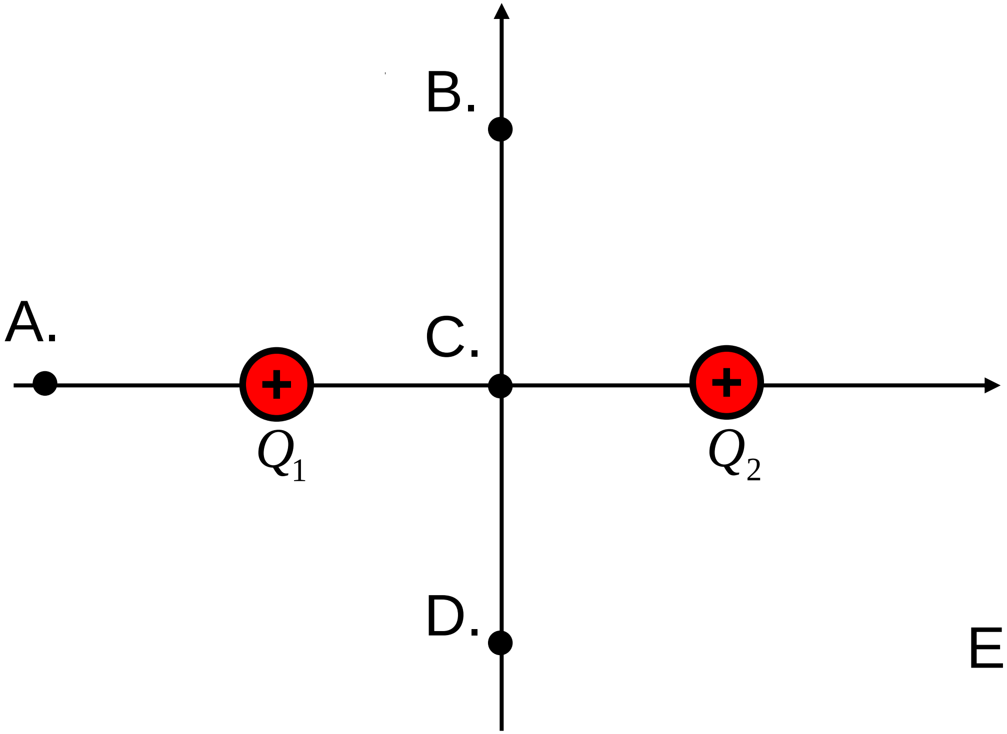
Adding the two electric fields together gives:

$$\vec{E}_{net} = \vec{E}_Q + \vec{E}_{-Q} = \frac{kQ}{(L/2)^2} + \frac{kQ}{(L/2)^2} = \frac{2kQ}{(L/2)^2}$$

Electric Field at a Point VIII

In the figure, Q_1 and Q_2 are equal in magnitude and the same distance from the origin.

Where would the electric field be vertically upward?



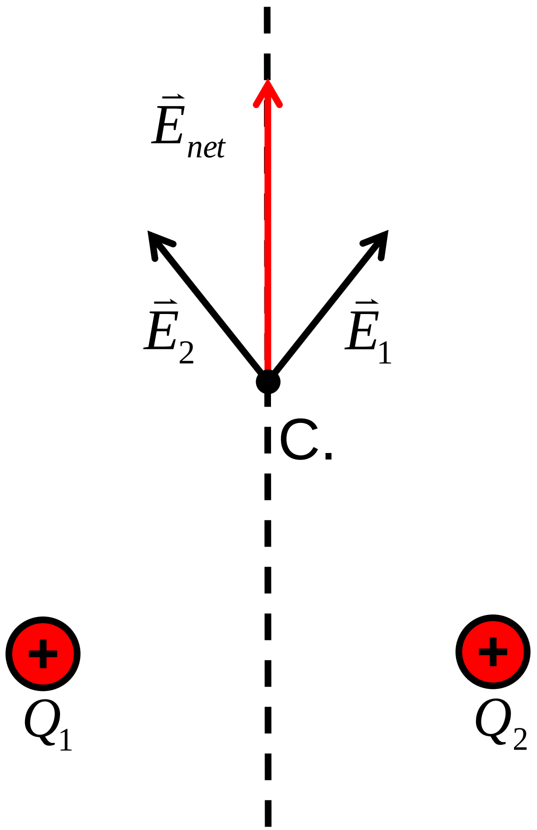
E. Not possible

Solution

Answer: B

Justification: The two charges are of equal distance away from the y-axis. They are also equal in size, and create equal electric fields. This means that the x-components of the electric fields will cancel at all points along the y-axis.

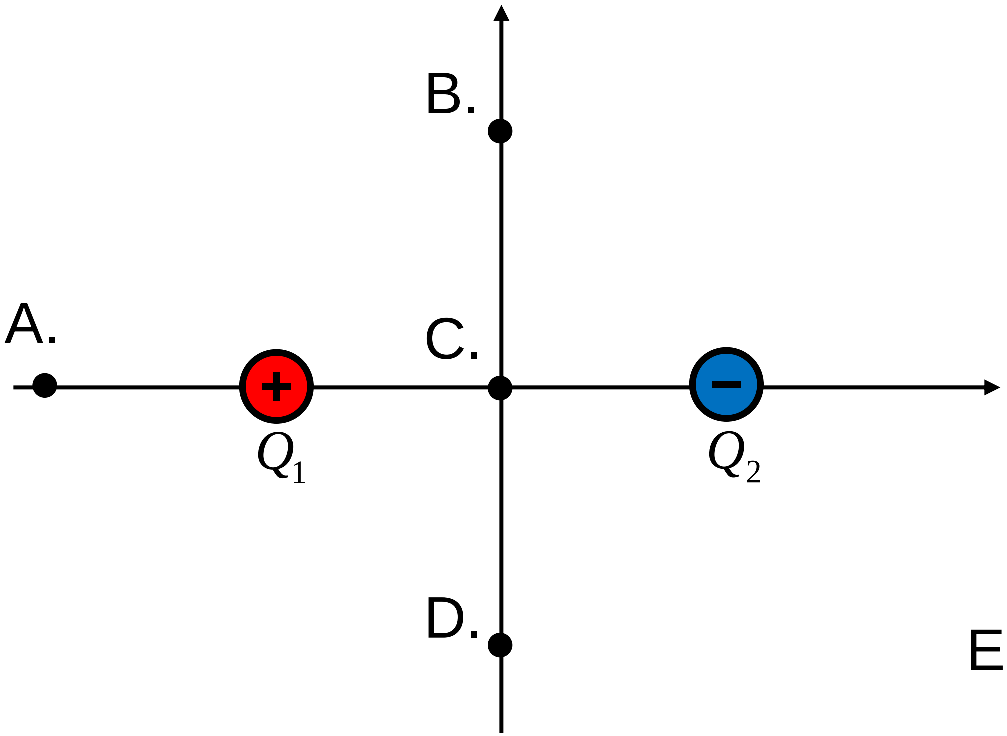
Electric field lines point away from positive charges. In order to have a net electric field pointing upwards, it is necessary to consider only those points on the positive y-axis.



Electric Field at a Point IX

In the figure, Q_1 and Q_2 are equal in magnitude but oppositely charged. They are the same distance from the origin.

Where would the electric field be vertically upward?

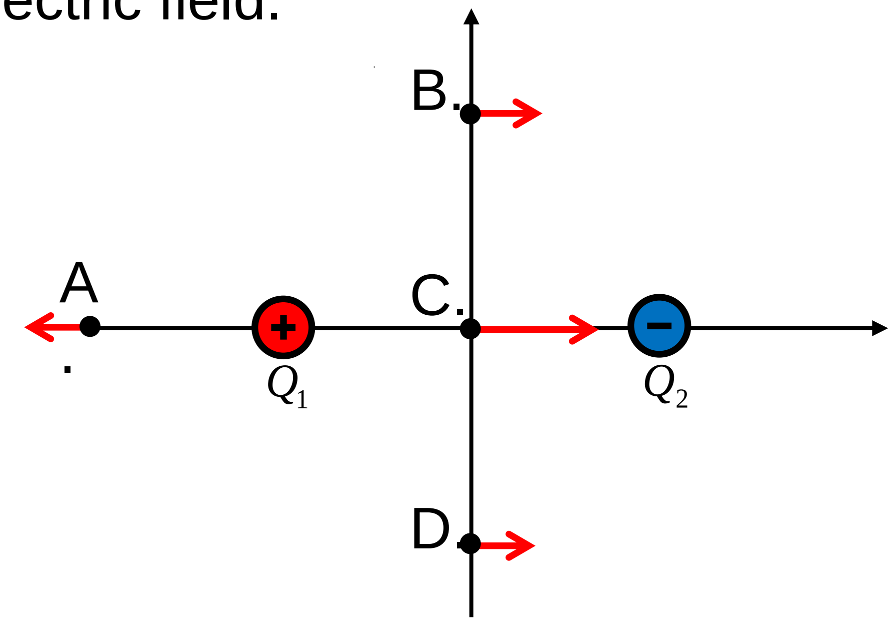


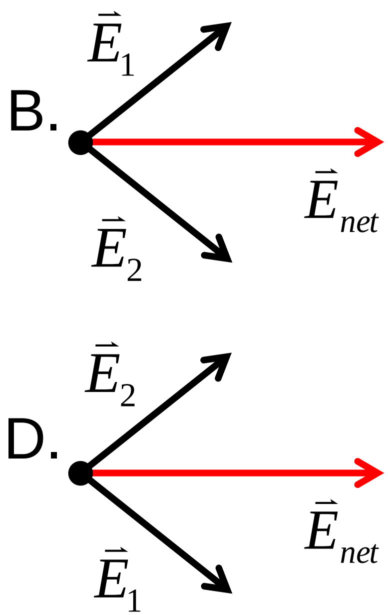
E. At none of these points

Solution

Answer: E

Justification: Determine the direction of the electric field at each point. Notice that the y-components of the electric field cancel at points B and D, so the electric fields are directed to the right. A and C are in line with the two charges, so there is no vertical component of electric field.

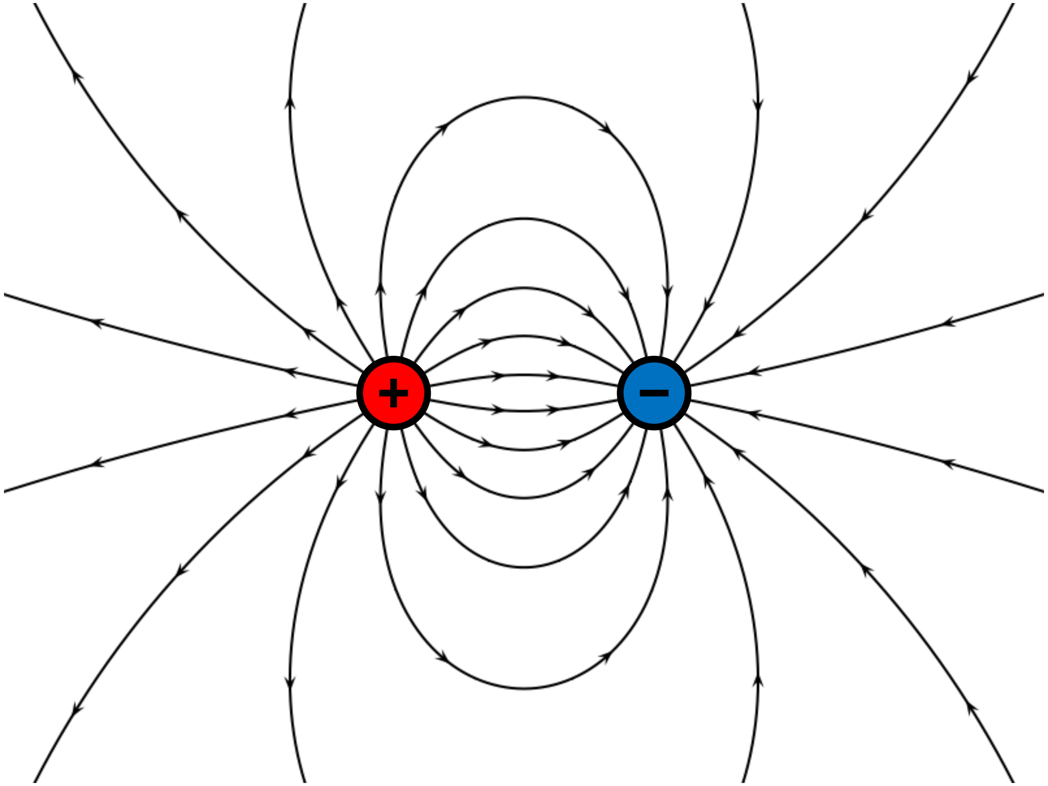




Solution Cont'd

Answer: E

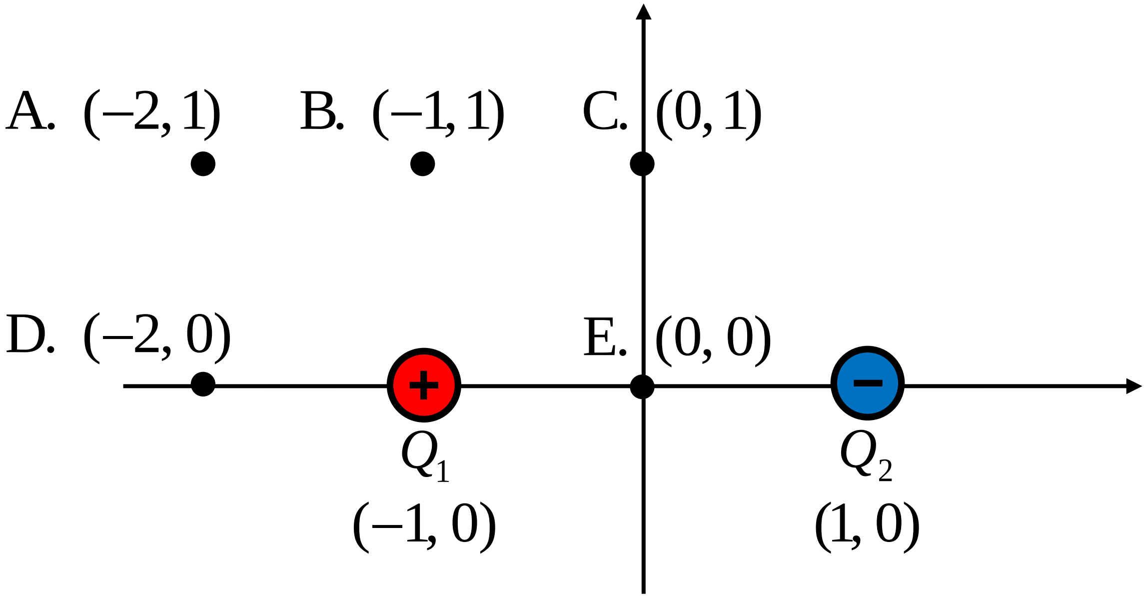
Justification: The electric field lines of a dipole are shown below.



Electric Field at a Point X

In the figure, Q_1 and Q_2 are equal in magnitude but oppositely charged. They are located at points (-1, 0) and (1,0) as shown in the diagram below.

At which point is the magnitude of the electric field the largest?



A. $(-2, 1)$

B. $(-1, 1)$

C. $(0, 1)$

D. $(-2, 0)$

E. $(0, 0)$

Solution

Answer: E

Justification: The point (0, 0) is at a distance 1 unit away from both Q_1 and Q_2. Recall from question 7 that the electric field components from Q_1 and Q_2 are added because both vectors point to the right. When the electric fields are added, no components cancel. The point (0, 0) is closest to both charges, so the field is strongest at (0, 0).

$$\vec{E}_{net} = \vec{E}_{Q_1} + \vec{E}_{Q_2} = \frac{kQ}{(1)^2} + \frac{kQ}{(1)^2} = 2kQ \longrightarrow$$

Try calculating the electric field at points (0, 1), and (–2, 0). What do you notice?