a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Physics
Electrostatics: Electric Potential

Science and Mathematics
Education Research Group

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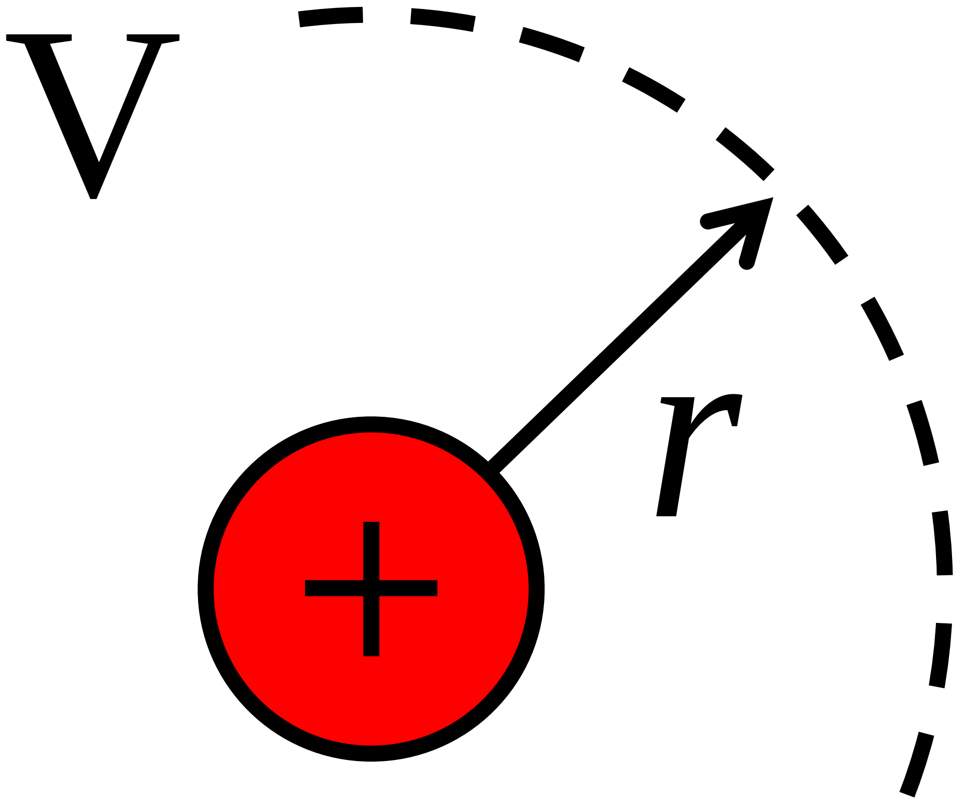
Electric Potential

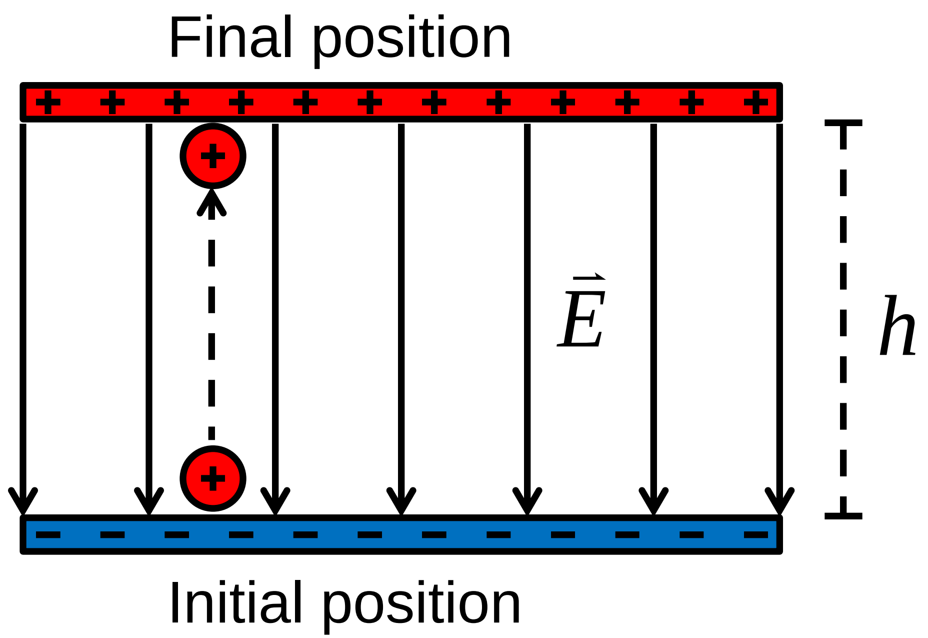


Electric Potential I

Work is done to move a positive charge from the negative end to the positive end of a parallel-plate capacitor.

What type of energy has the work been converted to?

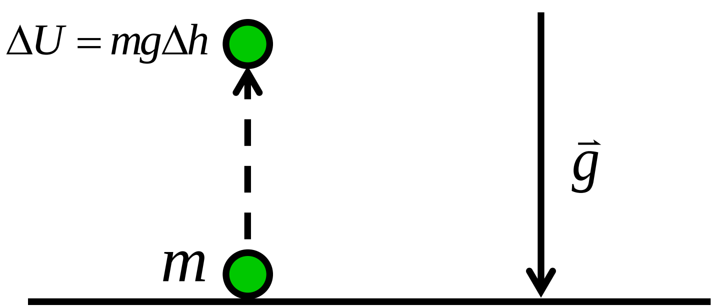
A. Kinetic energy
B. Chemical potential energy
C. Gravitational potential energy
D. Electric potential energy
E. Potential difference

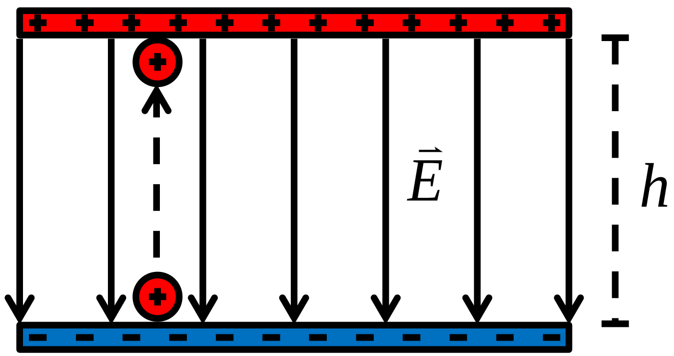


Solution

Answer: D

Justification: Moving a ball to a height h above the ground increases its gravitational potential energy by $\Delta U = mg\Delta h$. If the ball is released, it will accelerate downwards due to the downward pull of the gravitational field, g.





Similarly, if we do work on a proton to move it against an electric field, we say its electric potential energy will increase. Releasing the proton will cause it to accelerate back to the negative plate.

Solution Continued

It is important to understand why the other types of energy did not increase:

A. The kinetic energy of the proton does not increase because it begins and ends with zero velocity.

B. The proton does gain gravitational potential energy equal to:

$$\Delta U_{grav} = mg\Delta h$$

However, the mass of the proton is so small (10^{-27} kg) that we almost always ignore gravitational forces when dealing with electric charges.

C. No chemical changes are involved in moving a proton.

E. Potential difference is not a measure of energy, but the difference in electric potential between two points.

Electric Potential II

A proton (with charge q) initially at rest on a negatively charged plate is moved upwards by h towards a positively charged plate.

What is the formula for the change in electric potential energy of the proton?

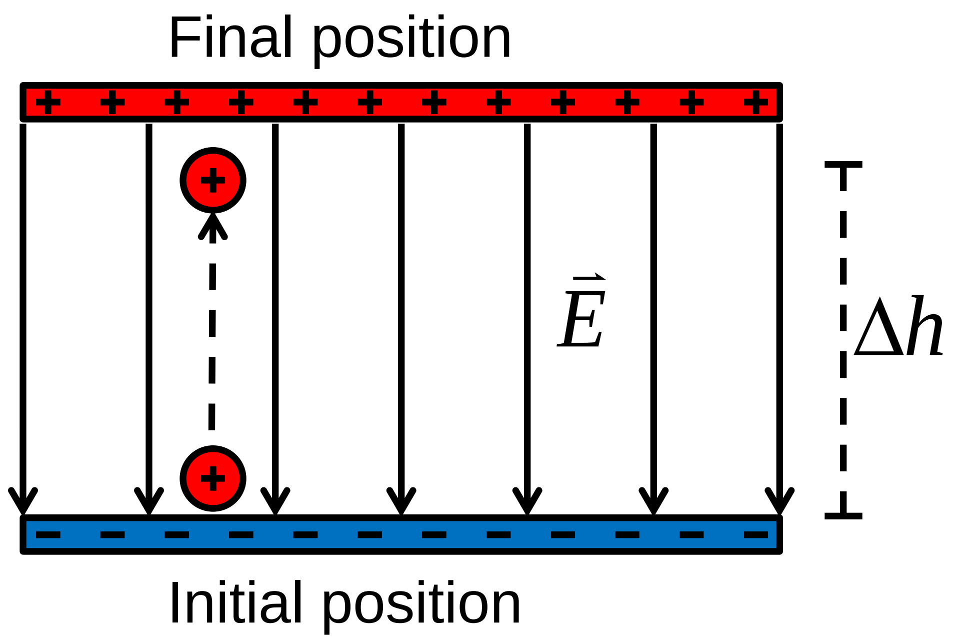
A. $\Delta U = mg\Delta h$

B. $\Delta U = qE\Delta h$

C. $\Delta U = \frac{E}{q}\Delta h$

D. $\Delta U = \frac{qE}{\Delta h}$

E. $\Delta U = 0$



Solution

Answer: B

Justification: The force exerted on the proton inside the electric field is: $\vec{E} = \dfrac{\vec{F}}{q} \quad \Rightarrow \quad \vec{F} = q\vec{E}$ $\downarrow$

The work done moving an object an distance Δh through a force F is given by:

$$W = Fd\cos(\theta) = (qE)(\Delta h)\cos(180°) = -qE\Delta h$$

Here, the proton is moving against the direction of force, so the angle between the direction of motion and the direction of force is 180°.

This work is converted to electric potential energy. Notice that the energy of the capacitor and proton system is conserved.

$$\Delta U + W = 0$$

$$\Delta U = -W = qE\Delta h$$

Solution Continued

Notice the similarities between the electric potential energy and gravitational potential energy.

Electric Potential Energy	Gravitational Potential Energy
$\Delta U = qE\Delta h$	$\Delta U = mg\Delta h$
q = charge	m = mass
E = electric field	g = gravitational field
Δh = change in height	Δh = change in height

Note: The formula for electric potential energy only applies to the case where the electric field is uniform, such as in between two charged plates. Most questions will deal with uniform electric fields.

Electric Potential III

We move a proton horizontally by a distance d inside a parallel-plate capacitor.

What is the change in the electric potential energy of the proton?

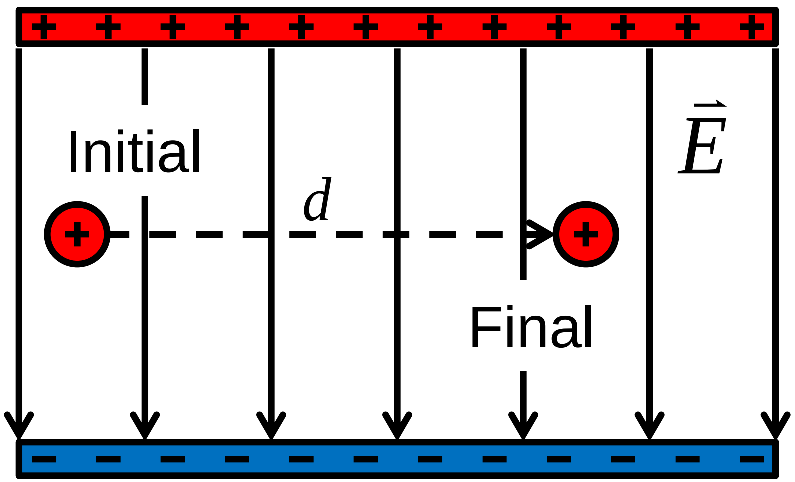
A. $\Delta U = mgd$

B. $\Delta U = qEd$

C. $\Delta U = \dfrac{E}{q}d$

D. $\Delta U = \dfrac{qE}{d}$

E. $\Delta U = 0$



Solution

Answer: E

Justification: Imagine moving a ball along the surface of a table. Its gravitational potential energy does not change because it does not change in height.

$$\Delta U = mg\Delta h = 0 \text{ J}$$

Likewise, when moving a charge perpendicular to an electric field, the electric potential energy of the charge does not change.

This can be seen by calculating the work required to move the charge a distance d, when the angle between the direction of force and the displacement is 90°.

$$W = Fd\cos(\theta) = (qE)d\cos(90°) = 0 \text{ J}, \qquad \cos(90°) = 0$$

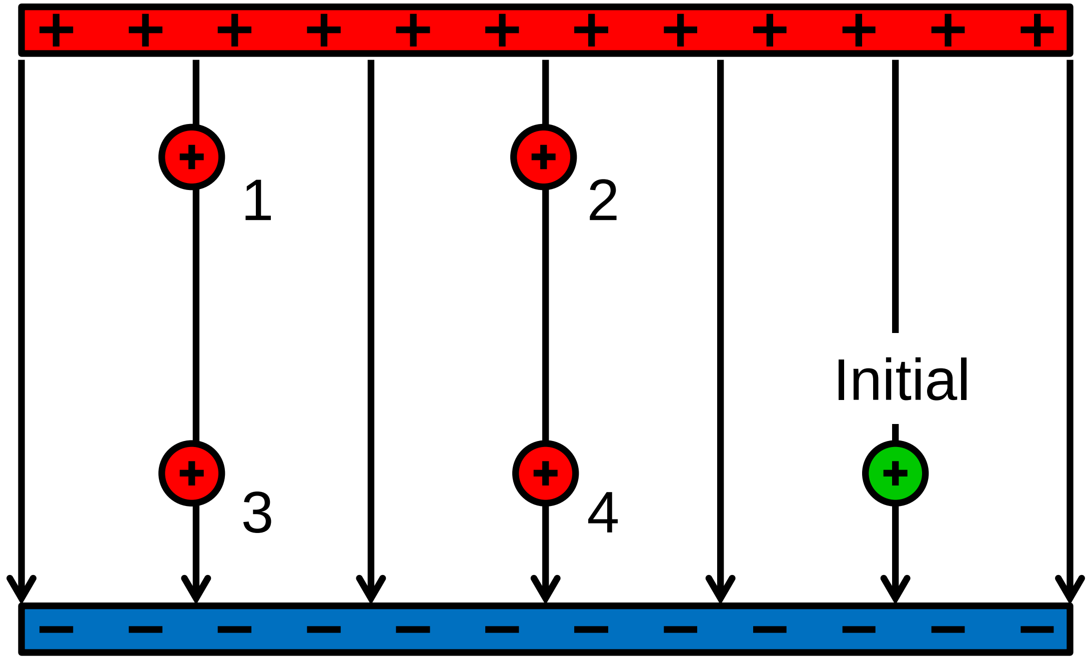
$$\Delta U = -W = 0 \text{ J}$$

Electric Potential IV

The green positive charge marks the initial position of a proton. It is then moved to one of the four final positions.

At which final point will the change in electric potential energy be the greatest?

A. Point 1
B. Point 1 or 2
C. Point 3
D. Point 3 or 4
E. Point 1 or 3



Solution

Answer: B

Justification: In the previous question, we saw that there is no change in electric potential energy when a charge is moved perpendicular to the electric field. Therefore, charges at point 1 and point 2 will have the same electric potential energy. Similarly, charges at point 3, point 4, and the initial point will have the same electric potential energy.

The greatest change in electric potential energy occurs when the charge is moved to either point 1 or point 2. The same amount of work is required to move a charge from point 1 and point 2, even though point 1 is a farther distance away.

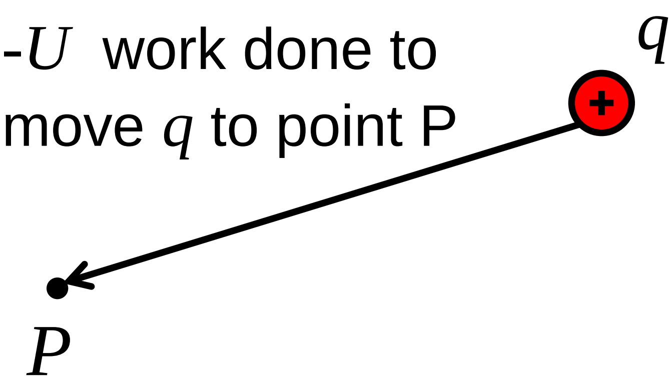
Electric Potential

The <u>electric potential difference</u> between two points is defined as the change in electric potential energy per unit of charge.

$$\Delta V = \frac{\Delta U}{q}$$

Units: 1 V = 1 J/C

-U work done to move q to point P



initially at a point far away where $U = 0$

If we let V = 0 at a reference point far away, the <u>electric potential</u> at a point is defined as:

$$V = \frac{U}{q}$$

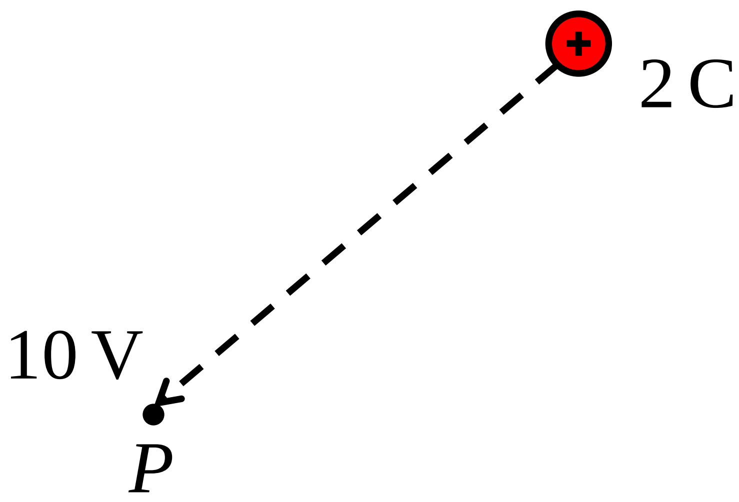
where U will be the electric potential energy of a charge q that is moved to the point with electric potential V.

Electric Potential V

The electric potential at a point P is 10 V. Suppose we moved a 2 C charge to this point. The charge is initially far away so that its electric potential is 0.

What is the electric potential energy of the 2 C charge at point P?

A. 5 J
B. 10 J
C. 20 J
D. 5 J/C
E. 10 J/C



Solution

Answer: C

Justification: From the definition of electric potential, we can solve for the electric potential energy:

$$V = \frac{U}{q} \quad \Rightarrow \quad U = Vq$$

The energy of the 2 C charge after it has been moved to the point with electric potential 10 V is:

$$U = Vq = 10\,\frac{\text{J}}{\text{C}} \times 2\,\text{C} = 20\,\text{J}$$

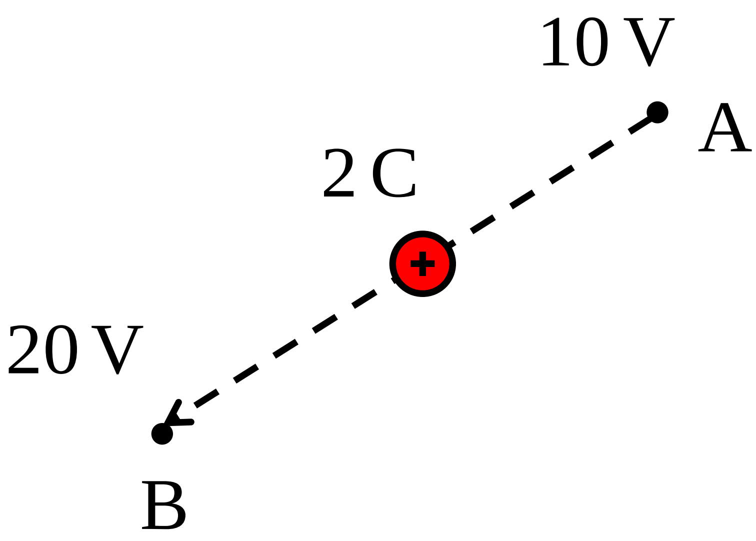
Make sure that electric potential energy has units of energy (J in this case).

Electric Potential VI

The electric potential at the point A is 10 V while the electric potential at point B is 20 V.

What is the difference in electric potential energy of the 2 C charge moving from point A to point B?

A. 10 J
B. 15 J
C. 20 J
D. 30 J
E. 60 J



Solution

Answer: C

Justification: The electric potential energy of the 2 C charge at point A is:

$$U_A = V_A q = 10\ \mathrm{V} \times 2\ \mathrm{C} = 20\ \mathrm{J}$$

At point B, which is at a higher electric potential of 20 V, the charge has electric potential energy of:

$$U_B = V_B q = 20\ \mathrm{V} \times 2\ \mathrm{C} = 40\ \mathrm{J}$$

The work required to move the 2 C charge from a electric potential of 10 V to 20 V is:

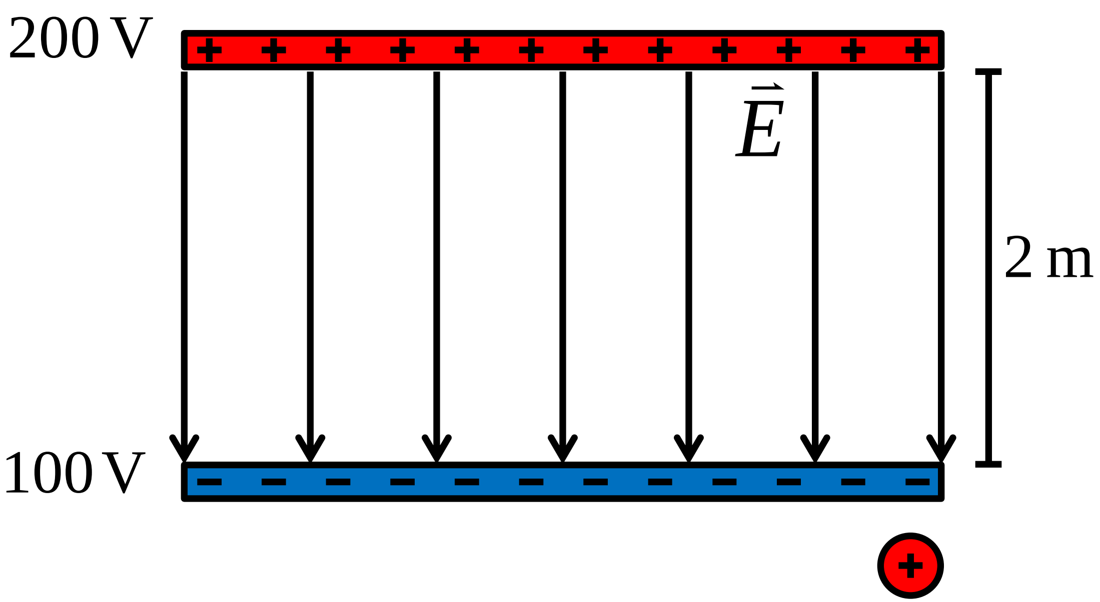
$$\Delta U = U_B - U_A = q(V_B - V_A) = 20\ \mathrm{J}$$

Electric Potential VII

The electric potential at the positive plate is 200 V while the electric potential of the negative plate is 100 V. The plates are separated by distance 2 m.

What is the magnitude of the electric field between the plates?

A. 50 N/C
B. 100 N/C
C. 200 N/C
D. 300 N/C
E. 400 N/C



Solution

Answer: A

Justification: The difference in electric potential energy between two charged plates is given by:

$$\Delta U = qEd$$

From the definition of electric potential energy, the magnitude of the electric field between two parallel charged plates in terms of V and d is:

$$\Delta V = \frac{\Delta U}{q} = \frac{qEd}{q}, \quad \Rightarrow \quad E = \frac{\Delta V}{d}$$

$$E = \frac{200\ \mathrm{V} - 100\ \mathrm{V}}{2\ \mathrm{m}} = 50\ \frac{\mathrm{N}}{\mathrm{C}}$$

Double check that the units are correct:

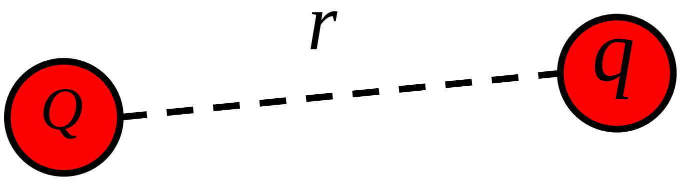
$$\frac{\mathrm{V}}{\mathrm{m}} = \frac{1}{\mathrm{m}}\frac{\mathrm{J}}{\mathrm{C}} = \frac{\mathrm{N}}{\mathrm{C}}$$

The direction of the electric field points from high potential to low potential.

Electric Potential of a Single Charge

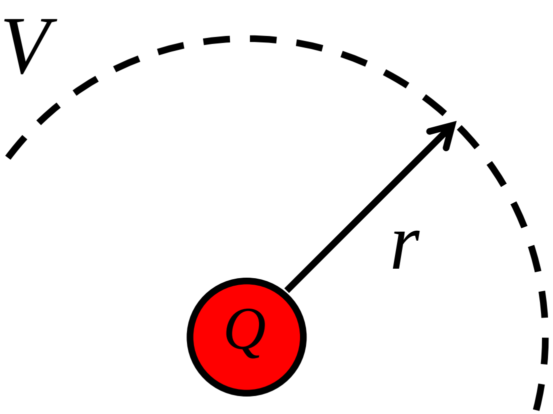
The electric potential energy of two charges separated by a distance r is:

$$U = \frac{kQq}{r}$$



The electric potential of a distance r away from a single charge Q is:

$$V = \frac{kQ}{r}$$



Note: Calculus is required to derive these results.

Electric Potential VIII

Consider the arrangement of two positive and two negative charges shown below. Each is positioned 1 m away from the origin.

What is the electric potential at the center of the four charges?

A. $V = 0\ \text{V}$

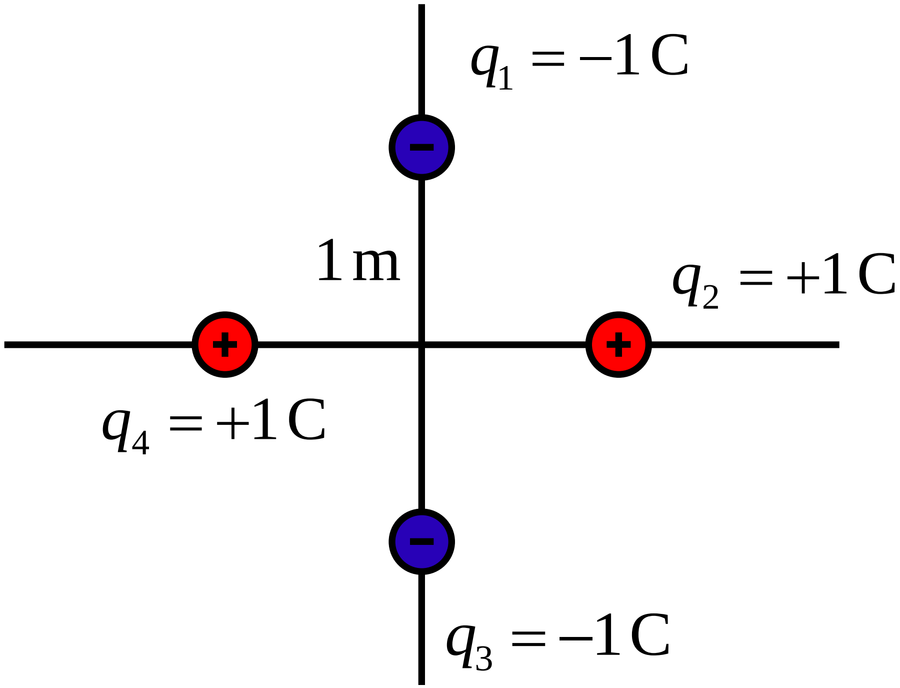
B. $V = 2k\ \text{V}$

C. $V = -2k\ \text{V}$

D. $V = 4k\ \text{V}$

E. $V = -4k\ \text{V}$

$$k = \frac{1}{4\pi\varepsilon_0} = 9\cdot 10^9\ \frac{\text{Nm}^2}{\text{C}^2}$$



Solution

Answer: A

Justification: The electric potential at the origin is the sum of the electric potentials due to each charge.

$$V_{origin} = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3} + \frac{kq_4}{r_4}$$
$$= \frac{k}{1\,\mathrm{m}}(-1\,\mathrm{C} + 1\,\mathrm{C} - 1\,\mathrm{C} + 1\,\mathrm{C})$$
$$= 0\,\mathrm{V}$$

The distance to the origin is 1 m for all the charges.

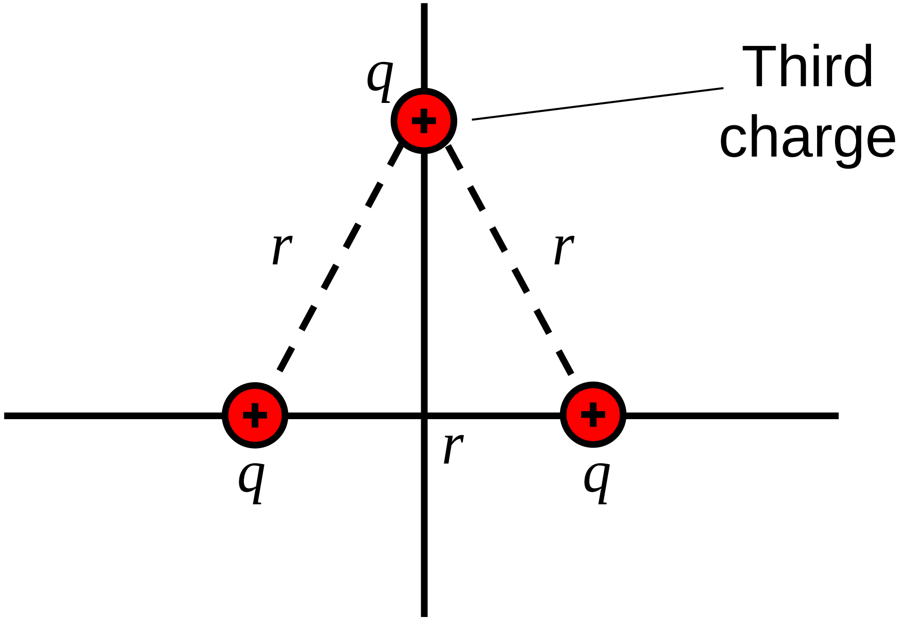
Notice that since electric potential is a scalar. Unlike electric fields and forces, we do not have to add vectors, which often makes calculations much easier.

Electric Potential IX

Two positive charges q are originally placed a distance r apart. A third charge q is placed so that an equilateral triangle is formed between the three charges.

How many times larger is the electric potential energy of the triangle configuration, compared to when there were only two charges?

A. They have the same energy
B. 2 times more energy
C. 3 times more energy
D. 4 times more energy
E. 6 times more energy



Solution

Answer: C

Justification: When there are only two charges, the electric potential energy of configuration is:

$$U_{2\text{ charges}} = k\frac{q_1 q_2}{r_{12}} = k\frac{q^2}{r}$$

(r_{12} is the distance between charge 1 and charge 2)

When the third charge is added, there are two new interactions between charges 1 and 3, and charges 2 and 3.

$$U_{3\text{ charges}} = k\frac{q_1 q_2}{r_{12}} + k\frac{q_1 q_3}{r_{13}} + k\frac{q_2 q_3}{r_{23}} = 3k\frac{q^2}{r}$$

$$q = q_1 = q_2 = q_3$$
$$r = r_{12} = r_{13} = r_{23}$$

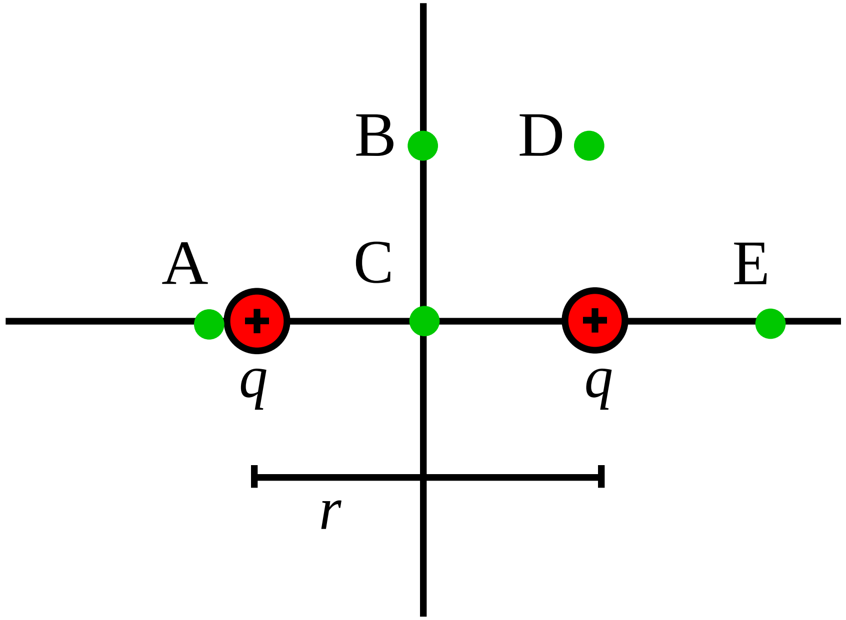
The electric potential energy of the three charge configuration is therefore three times larger than the two charge configuration.

Electric Potential X

Two identical positive charges q are originally placed a distance r apart (see diagram).

At which point should a third charge q be placed so that the electric potential energy of the three charge configuration is maximized?

A. Point A
B. Point B
C. Point C
D. Point D
E. Point E



Solution

Answer: A

Justification: When the third charge is placed, the configuration with the most electric potential energy will maximize:

$$U_{3\,\text{charges}} = k\frac{q_1 q_2}{r_{12}} + k\frac{q_1 q_3}{r_{13}} + k\frac{q_2 q_3}{r_{23}} = kq^2\left(\frac{1}{r_{12}} + \frac{1}{r_{13}} + \frac{1}{r_{23}}\right)$$

Since r_{12} is fixed, we must make r_{13} and r_{23} as small as possible so that U is as large as possible.

Notice that if $r_{13} << 1$, U will become very large, even if r_{23} is large. Therefore, if we put the third charge very close to one of the two existing charges, the electric potential energy will be very large. Releasing the charges will cause the charges to repel away from each other very quickly due to the strong Coulomb forces.