a place of mind

FACULTY OF EDUCATION

Department of
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Mathematics
Energy: Thermal Energy

Science and Mathematics
Education Research Group

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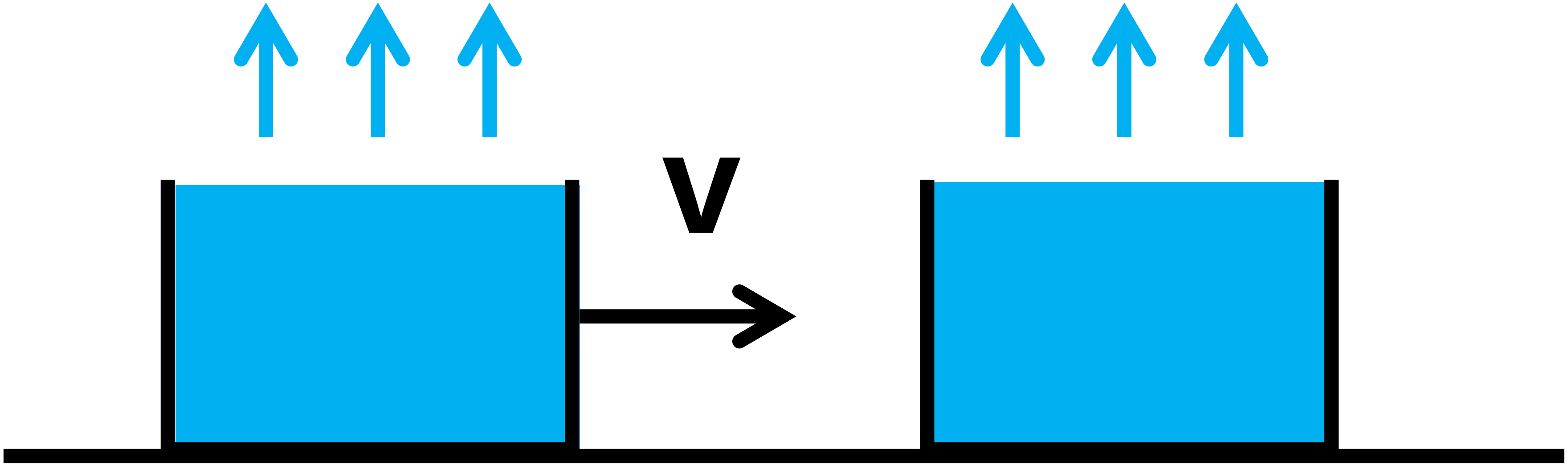
Colliding Water Tanks



Colliding Water Tanks I

A tank of water, of mass m, sliding across the table at speed v collides with an identical tank at rest. What is the momentum of the first tank?

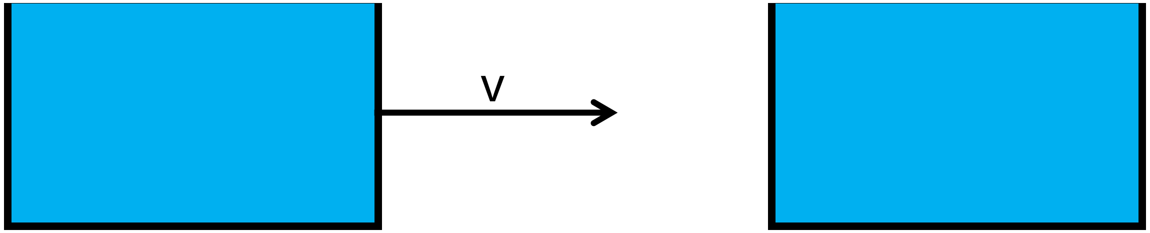
$A.\ mv$

$B.\ \dfrac{mv^2}{2}$

$C.\ \dfrac{mv}{2}$

$D.\ \dfrac{mv}{3}$

$E.$ not enough information



Solution

Answer: A

Justification: In this instance, we are only interested in momentum. Since the tanks are moving slowly, we use a classical definition of momentum. The definition of non-relativistic momentum is: $\mathbf{p} = m\mathbf{v}$.

Colliding Water Tanks II

A tank of water of mass m, sliding across the table at speed v with mass m collides with an identical tank at rest. Before the collision, what is the momentum of the second tank?

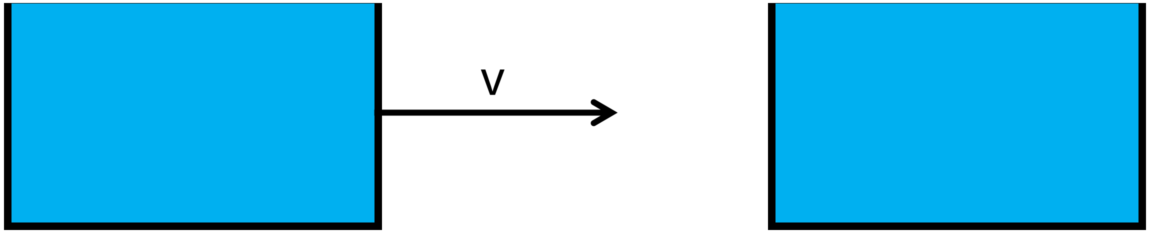
A. mv

B. $\dfrac{mv^2}{2}$

C. $\dfrac{mv}{2}$

D. $\dfrac{mv}{3}$

E. 0



Solution

Answer: E

Justification: The momentum of an object is defined as $\mathbf{p} = m\mathbf{v}$. Since the tank is not moving, it has no momentum.

Colliding Water Tanks III

A tank of water with mass m, sliding across the table at speed v with mass m collides with an identical tank at rest. What is the momentum of the tanks after they collide and stick together? (inelastic collision)

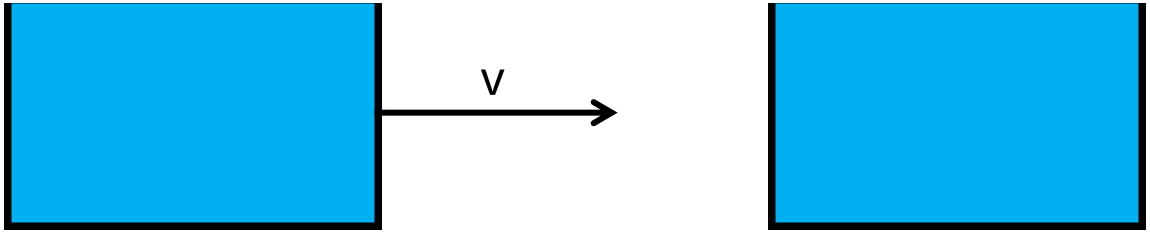
A. mv

B. $\dfrac{mv^2}{2}$

C. $\dfrac{mv}{2}$

D. $\dfrac{mv}{3}$

E. 0



Solution

Answer: A

Justification: The total momentum of a closed system does not change, due to conservation of momentum. A closed system means that during the collision the tanks only interact with each other (there are no other forces acting on them).

Colliding Water Tanks IV

A tank of water sliding across the table at speed v with mass m collides with an identical tank at rest. What is the final speed of the tanks if they collide and stick together?

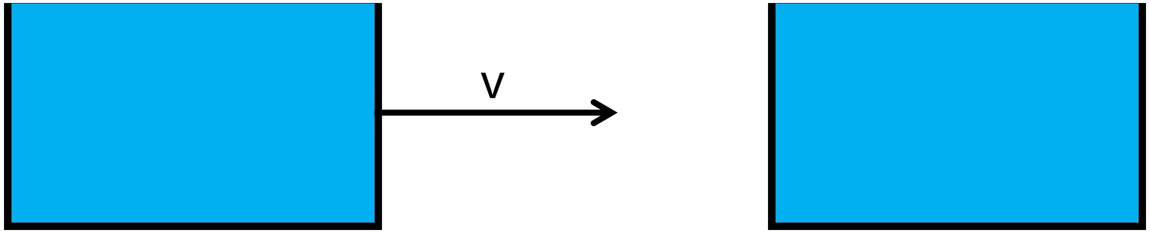
$A.\ v$

$B.\ \dfrac{v}{2}$

$C.\ \dfrac{v}{3}$

$D.\ \dfrac{v}{4}$

$E.$ Not enough information



Solution

Answer: B

Justification: Momentum is conserved in an inelastic collision, so the final momentum is equal to the initial momentum. The mass doubles, so the speed must halve for the momenta to be equal. We denote the speed of the tanks after the collision as u:

$$mv + 0 = 2mu$$

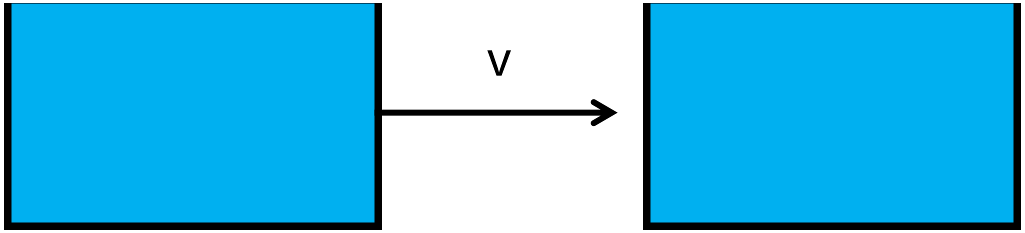
$$mv = 2mu$$

$$u = \frac{v}{2}$$

Colliding Water Tanks V

A tank of water with mass m, sliding across the table with speed v collides with an identical tank at rest. What can you say about the mechanical energy in this perfectly inelastic collision?

A. Mechanical energy is conserved
B. Mechanical energy is lost
C. Mechanical energy is gained
D. All mechanical energy turned into heat
E. None of the above



Solution

Answer: B

Justification: Even without doing the calculation, we know that some of the mechanical energy will be converted into heat (the water will start shaking and splashing), sound and other forms of energy. Thus the mechanical energy will be converted into other forms of energy. We can say that the mechanical energy will be lost.

Colliding Water Tanks VI

A tank of water with mass m, sliding across the table with speed v collides with an identical tank at rest. How much energy is lost in this perfectly inelastic collision?

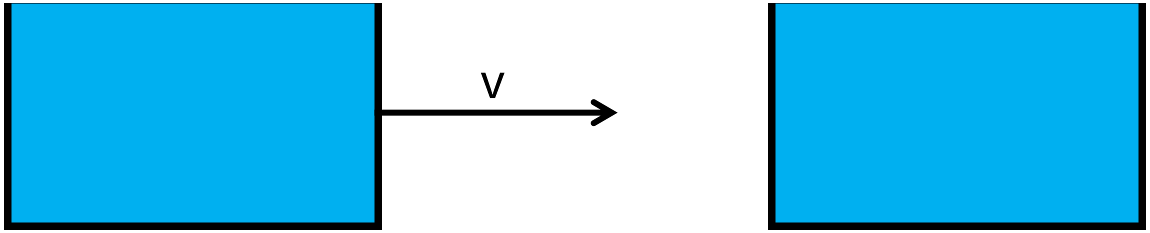
$A.\ mv^2$

$B.\ \dfrac{mv^2}{2}$

$C.\ \dfrac{mv^2}{4}$

$D.\ 0$

$E.\ \text{Not enough information}$



Solution

Answer: C

Justification: We begin with a kinetic energy of $\frac{1}{2}mv^2$ from the moving tank. After it collides, the tanks, now with a collective mass of 2m, travelling at speed $v/2$.

Plugging this into the expression for kinetic energy:

$$E_k = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$$

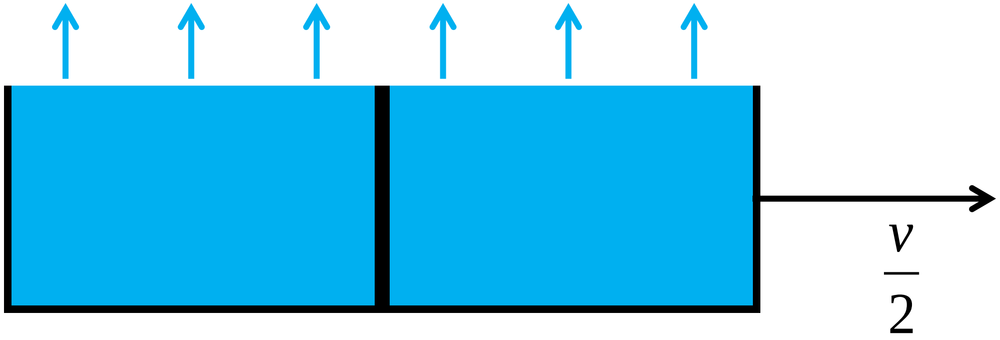
Subtracting this from our original kinetic energy, we get:

$$\frac{1}{2}mv^2 - \frac{1}{4}mv^2 = \frac{1}{4}mv^2$$

Colliding Water Tanks VII

Suppose the tanks themselves are weightless and the system is at 0 °C. At what initial speed should the tank travel so that enough energy is released in the collision for the water to start boiling? Assume that all of the energy goes into the water and use c for the specific heat capacity of water.

A. $\sqrt{(800c)}$
B. $\sqrt{(400c)}$
C. $\sqrt{(200c)}$
D. 0
E. Not enough information



Solution

Answer: A

Justification: We know from question 2 that $\frac{1}{4}mv^2$ is the amount of energy released. The relation between temperature and specific heat is Q=mcΔT, where Q is energy. Plugging in our variables we have

$$\frac{1}{4}mv^2 = 2mc\Delta T$$

$$mv^2 = 8mc\Delta T \Rightarrow v^2 = 8c\Delta T \Rightarrow v = \sqrt{8c\Delta T}$$

We need the temperature to be raised from 0 °C to 100 °C, so our answer is $v=\sqrt{(800c)}$. This may not seem like much but considering that c is about 4×10^3 J/(kgK), the speed would have to be about 1.8×10^3 m/s to boil water, not to mention that not all of the energy goes into the water in a practical situation. Notice, the mass of water doesn't affect the speed needed to boil it.

Colliding Water Tanks VIII

Assuming the same situation as question 3, if all of the water boils, how much momentum is carried away by escaping water molecules?

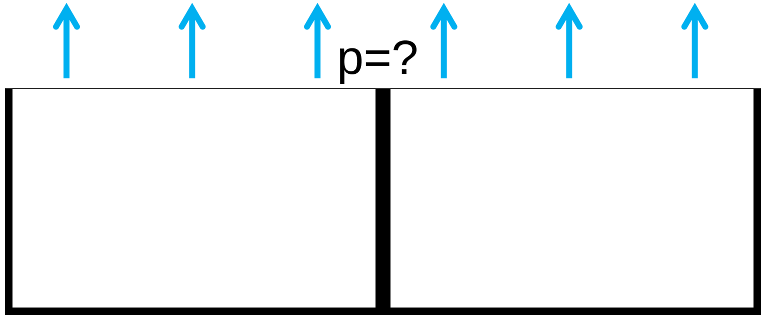
$A.\ 2mv$

$B.\ mv$

$C.\ \dfrac{mv}{2}$

$D.\ 0$

$E.$ Not enough information



Solution

Answer: B

Justification: All of the momentum will be carried away by water molecules because the tanks themselves were assumed to have no mass. The velocity of the tanks afterwards, however, is indeterminate.