a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Physics

Equilibrium: Torque

Science and Mathematics
Education Research Group

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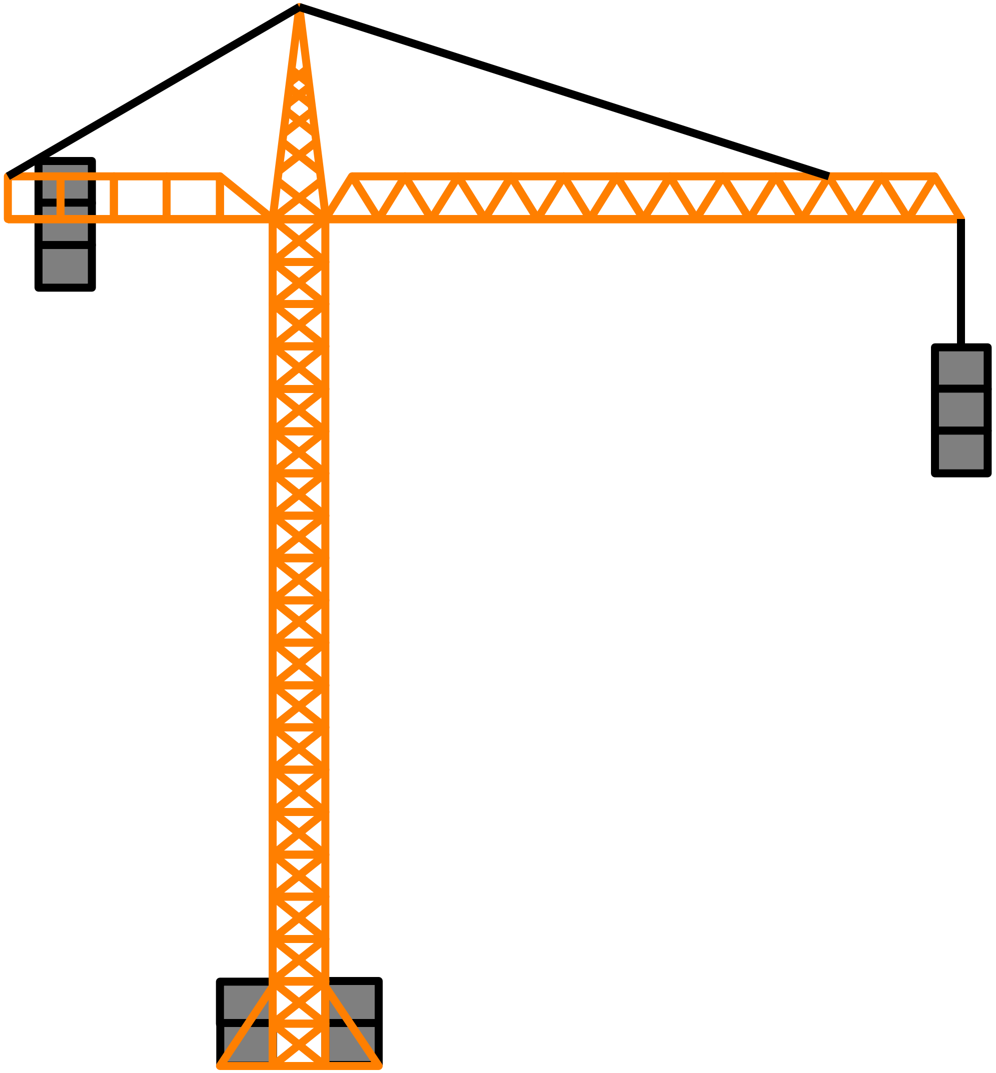
Crane



Crane I

A. $m_1 g r_1$ ↻

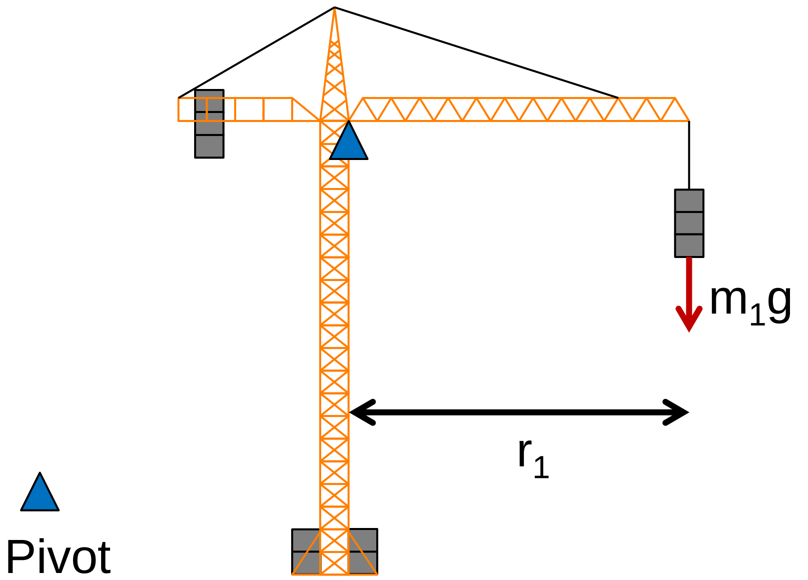
B. $m_1 g r_1$ ↺

C. $m_1 r_1$ ↻

D. $m_1 r_1$ ↺

E. $m_1 g r_1$ ↙

The length of the right crane arm is r_1 and the mass of the load is m_1. Using the vertical structure of the crane as a pivot, what is the magnitude and direction of the torque exerted by the mass?



Solution

Answer: A

Justification: The expression for the magnitude of torque is Fr, where F is the force at a distance r from the pivot (notice, the distance is measured as the shortest distance from the line of force to the pivot). In the picture, the mass is pulling downwards, which is **clockwise** in respect to the pivot. We sat that the torque vector has a sense – its direction can be either clockwise or counter-clockwise.

You can also find the direction by using the right hand rule:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Crane II

What is the magnitude of the net torque exerted in the clockwise direction by the weight of the right arm, m_2, and the weight of the load, m_1?

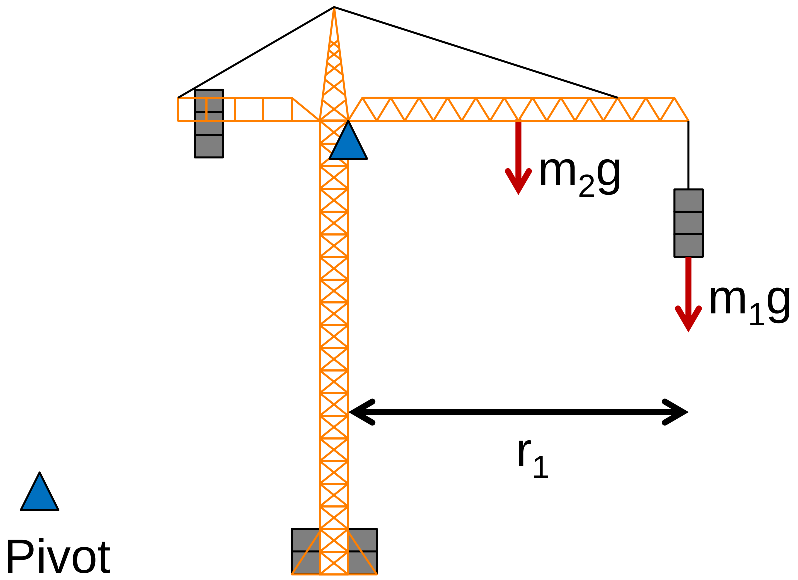
$$A.\ (m_1 + m_2)gr_1$$

$$B.\ \left(\frac{m_1}{2} + m_2\right)gr_1$$

$$C.\ \left(m_1 + \frac{m_2}{2}\right)gr_1$$

$$D.\ \left(m_1 + \frac{m_2}{4}\right)gr_1$$

$$E.\ \left(m_1 + \frac{m_2}{2}\right)r_1$$



Solution

Answer: C

Justification: From the previous question we know the torque exerted by the load is m_1gr_1. The weight of the right arm is m_2g, and it is centered on the arm. Thus, the torque exerted by the weight of the arm is:

$$m_2\left(\frac{r_1}{2}\right)$$

Since two torques are acting in the same direction, we then find the answer by adding the torque exerted by the weight of the arm to the torque exerted by the load. This is the only correct answer.

$$\tau = \left(m_1 + \frac{m_2}{2}\right)gr_1$$

Crane III

What is the vertical component of tension (T_y) in the cable required to maintain equilibrium in the right arm of the crane? ($\Sigma\tau=0$)

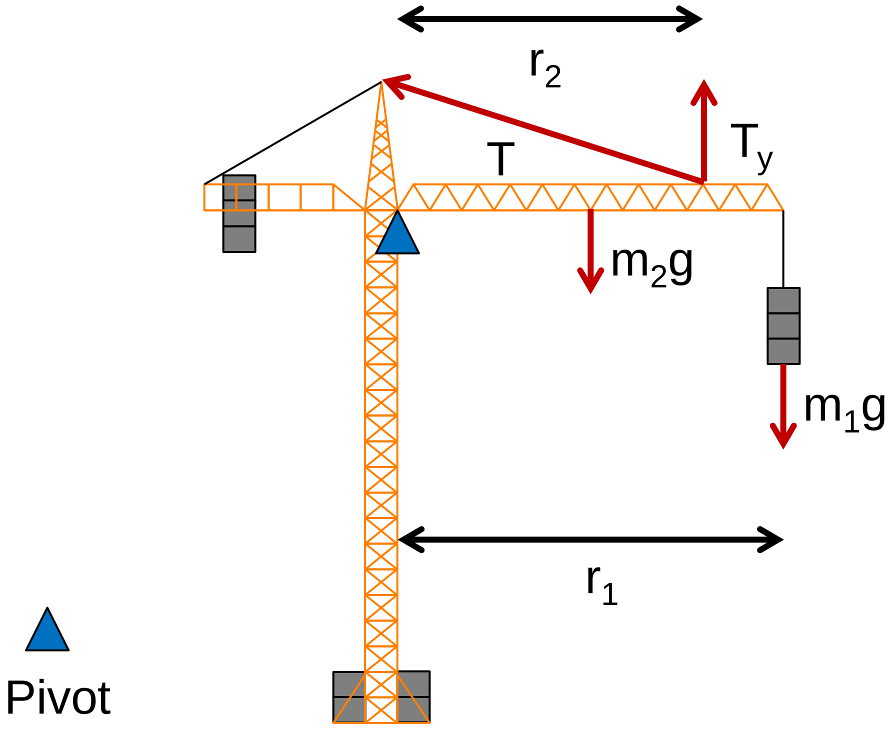
A. $\left(m_1 + \frac{m_2}{2}\right) g \frac{r_1}{r_2}$

B. $\left(m_1 + \frac{m_2}{2}\right) g \frac{r_2}{r_1}$

C. $\left(m_1 + \frac{m_2}{2}\right) g r_1$

D. $\left(m_1 + \frac{m_2}{2}\right) g r_2$

E. $\left(m_1 + \frac{m_2}{2}\right) r_1 r_2$



Solution

Answer: A

Justification: The total clockwise torque from the masses is

$$\left(m_1 + \frac{m_2}{2}\right) g r_1$$

To retain equilibrium, there must be an equal and opposite torque exerted by the tension in the string, so the sum of torques equals zero (no rotation). From this, we know that ,

$$T_y r_2 = \left(m_1 + \frac{m_2}{2}\right) g r_1$$

Rearranging this expression, we find the answer.

$$T_y = \left(m_1 + \frac{m_2}{2}\right) g \frac{r_1}{r_2}$$

Crane IV

$$A.\ T_y$$

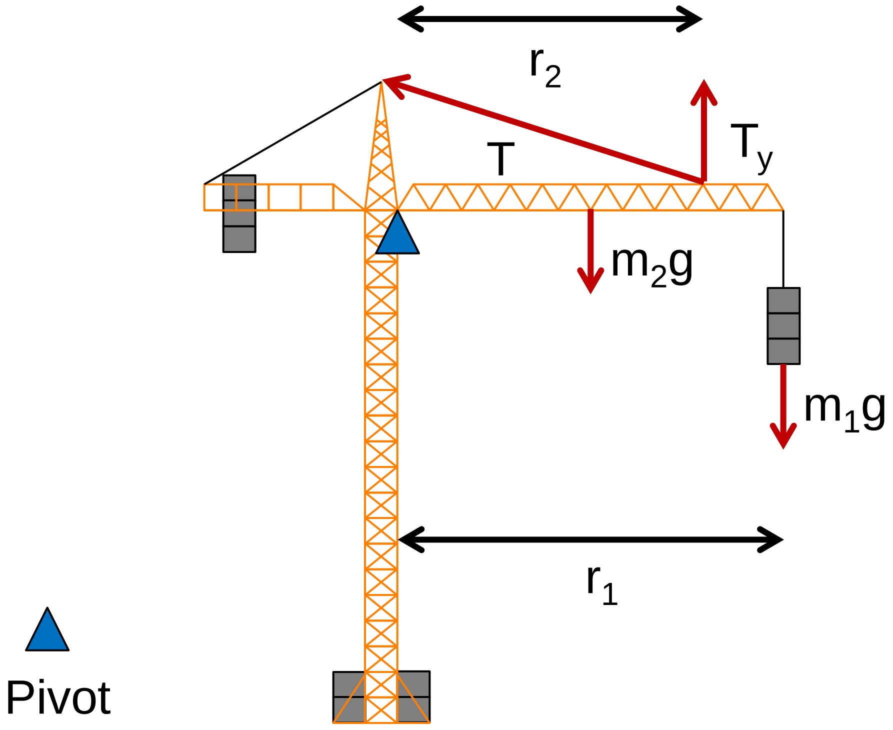
$$B.\ \frac{T_y}{\sin\theta}$$

$$C.\ \frac{T_y}{\cos\theta}$$

$$D.\ \frac{T_y}{\tan\theta}$$

$$E.\ \frac{T_y}{\tan^2\theta}$$

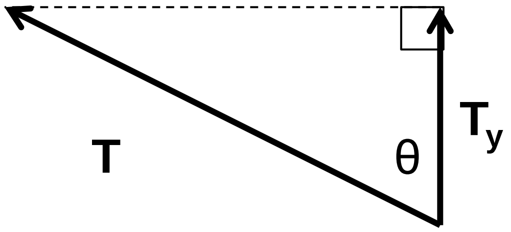
Suppose the cable makes an angle of θ from the horizontal. What must the tension in the cable be to maintain equilibrium? (T_y is the vertical component of the tension)



Solution

Answer: C

Justification: From trigonometry, $T_y = T\cos\theta$. T can be found by simply dividing both parts by the $\cos\theta$.



$$T_y = T\cos\theta$$

$$T = \frac{T_y}{\cos\theta}$$