a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

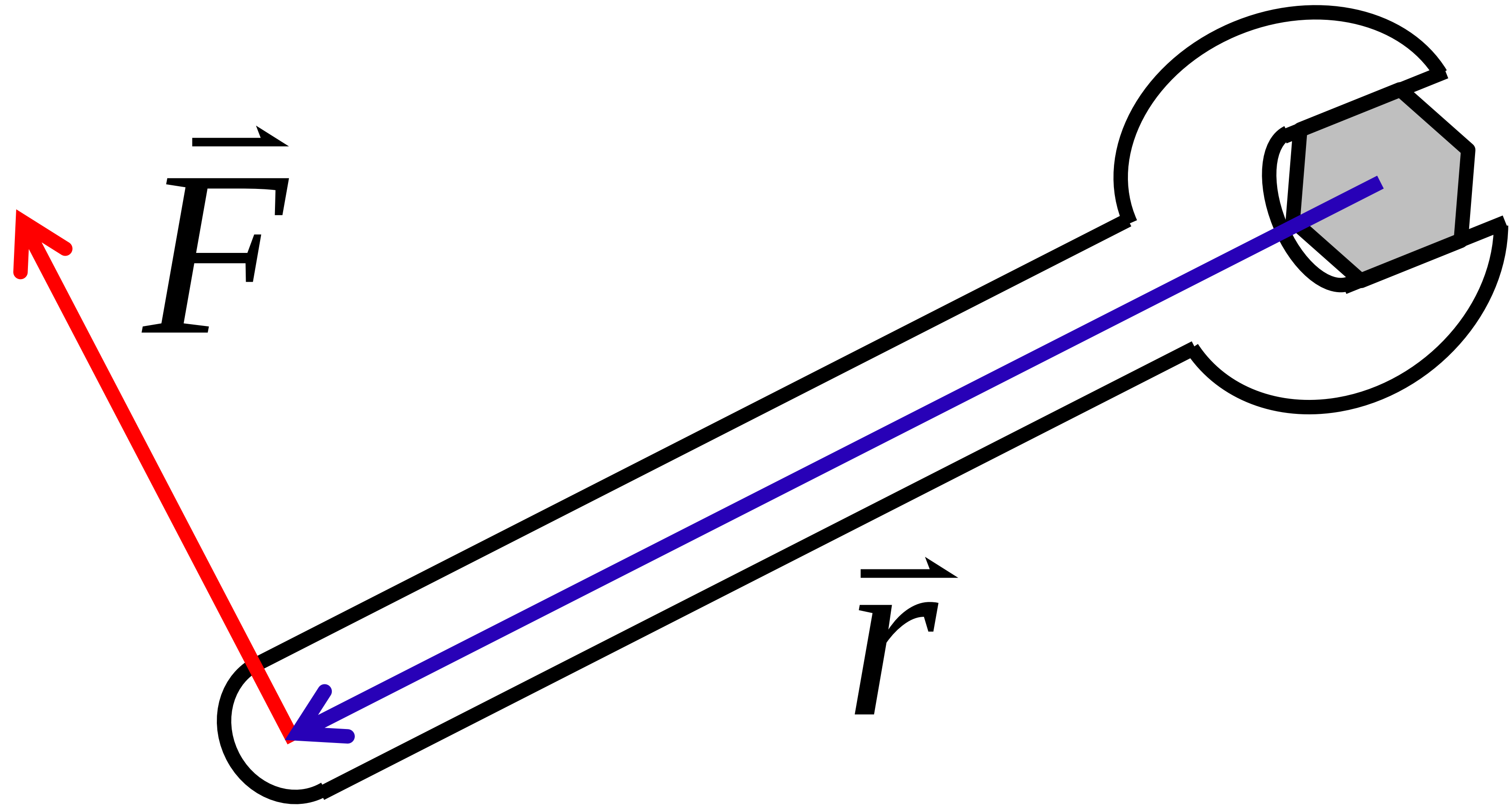
Physics

Equilibrium: Torque

Science and Mathematics
Education Research Group

Supported by UBC Teaching and Learning Enhancement Fund 2012-2014

Introduction to Torque



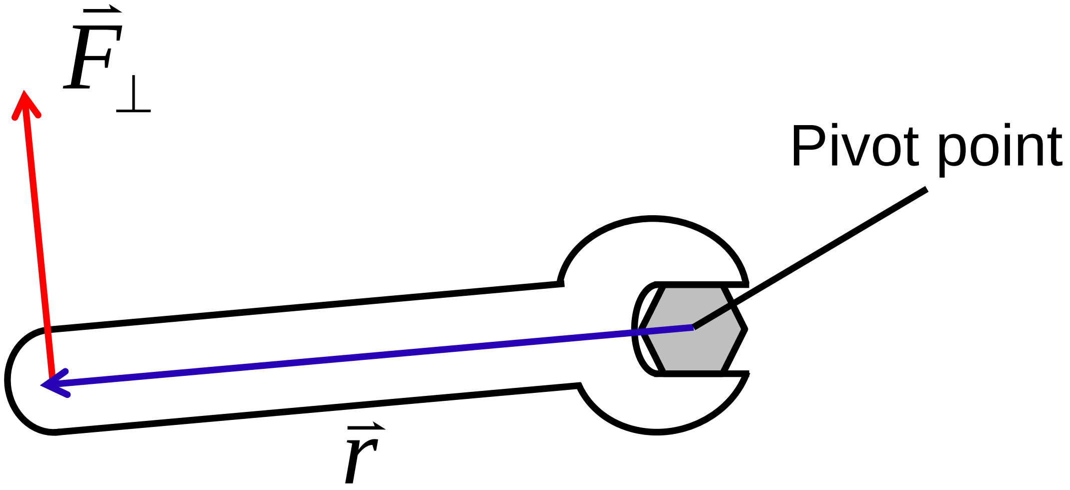
The Magnitude of Torque

The magnitude of torque on an object is defined as

$$\tau = rF_{\perp}$$

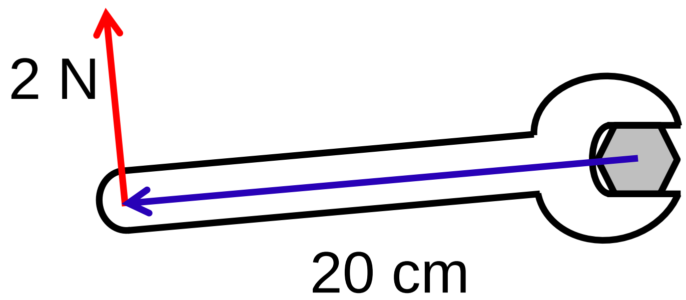
r → The distance between the pivot point and the location the force is applied

$F_{\perp}$ → The component of force that is perpendicular to the lever arm



Torque I

A force of 2 N is applied to a 20 cm long wrench at the location shown.



What is the magnitude of torque relative to the bolt?

A. $\tau = 40\ \mathrm{N}$

B. $\tau = 0.40\ \mathrm{N}$

C. $\tau = 40\ \mathrm{Nm}$

D. $\tau = 4.0\ \mathrm{Nm}$

E. $\tau = 0.40\ \mathrm{Nm}$

$$\boxed{\tau = rF_{\perp}}$$

Solution

Answer: E

Justification: Magnitude of torque is defined by:

$$\tau = rF_{\perp}$$

The quantity r is measured in meters (m), while the quantity F is measured in newtons (N). Therefore, torque has units of Nm. This eliminates answers A and B.

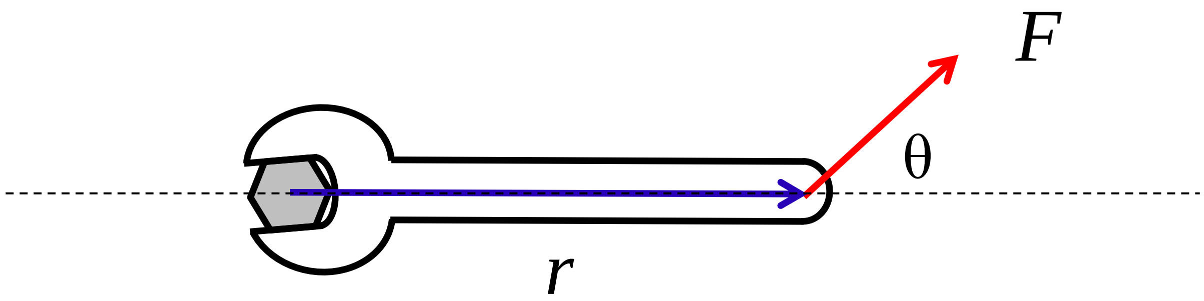
Since the force applied is perpendicular to the wrench, we can calculate the magnitude of the torque from the formula:

$$\tau = rF_{\perp} = (20\,\mathrm{cm})(2\ \mathrm{N}) = (0.20\,\mathrm{m})(2\ \mathrm{N}) = 0.40\ \mathrm{Nm}$$

Notice we have to convert 20 cm into 0.20 m so that torque is measured in Nm.

Torque II

A force F is now applied at an angle θ as shown:



What is the magnitude of torque relative to the bolt?

A. $\tau = rF$

B. $\tau = rF\theta$

C. $\tau = rF\cos\theta$

D. $\tau = rF\sin\theta$

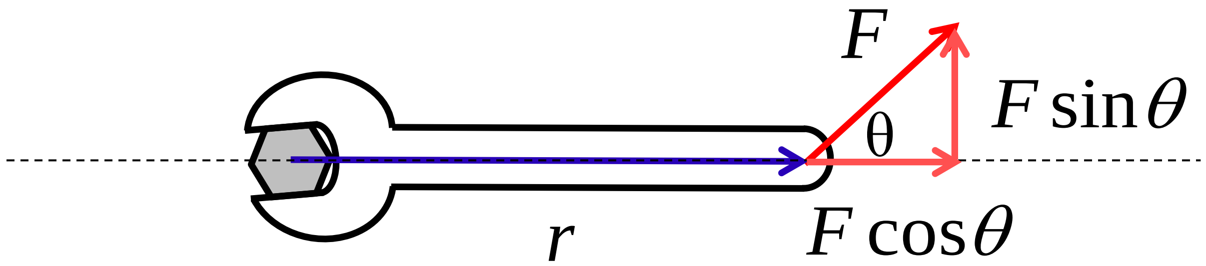
E. $\tau = rF\tan\theta$

$$\boxed{\tau = rF_{\perp}}$$

Solution

Answer: D

Justification: We must find the component of force that is perpendicular to the lever. This can be found by breaking F into vector components:



$$F_{\perp} = F\sin\theta$$

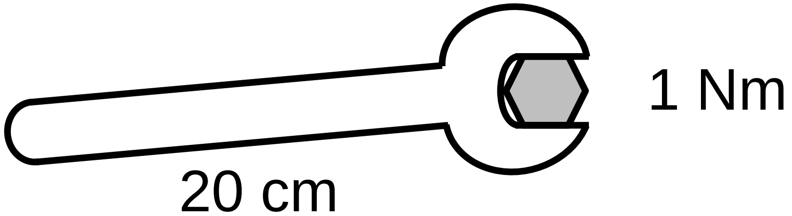
This gives us the equation for the magnitude of torque:

$$\tau = rF\sin\theta$$

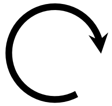
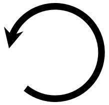
Notice that when $\theta = 90°$, we have that $F_{\perp} = F$, which simplifies to our original formula.

Torque III

A force is applied to the end of a 20 cm wrench so that the bolt begins to rotate. The net magnitude of torque on a bolt is 1 Nm.



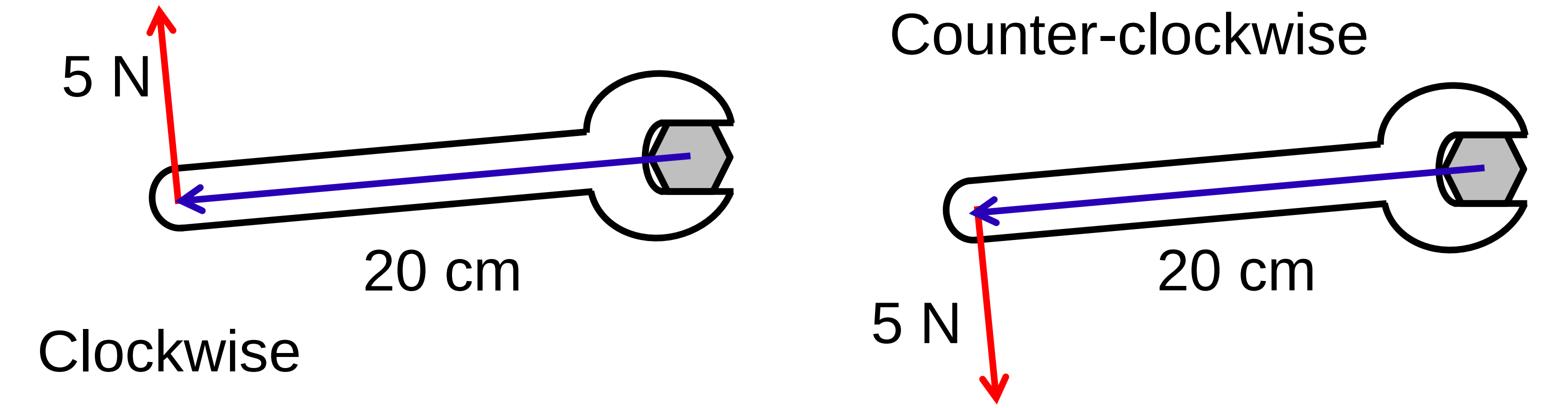
What direction does the bolt rotate?

A. Clockwise 
B. Counter-clockwise 
C. Not enough information

Solution

Answer: C

Justification: The magnitude of the torque does not tell us which way an object will turn. For example, in both scenarios shown below the magnitude of the torque is 1 Nm.



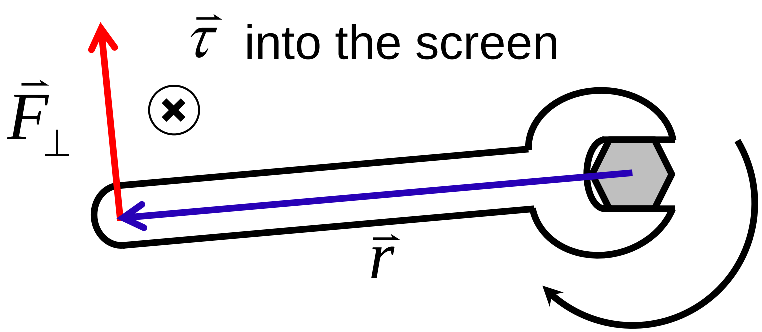
In order to tell us the direction of rotation, torque must be defined as a vector. The direction of the torque vector will tell us which way the pivot will rotate.

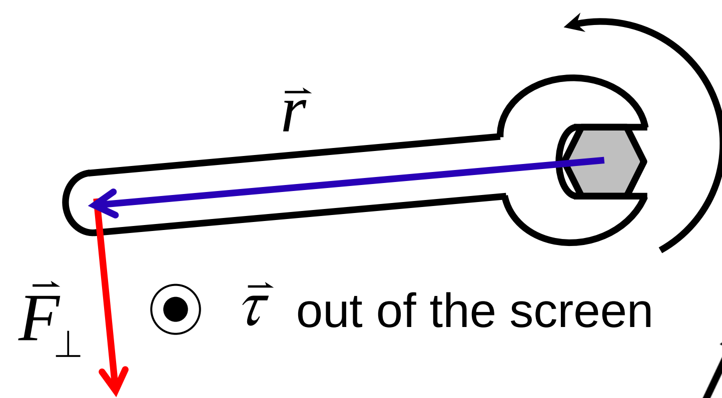
Extend Your Learning: Points of Interest

Torque as a vector is defined as a cross product:

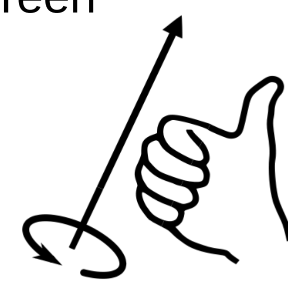
$$\vec{\tau} = \vec{r} \times \vec{F}_{\perp} \qquad \left|\vec{\tau}\right| = \left|\vec{r} \times \vec{F}_{\perp}\right| = \left|\vec{r}\right| \cdot \left|\vec{F}_{\perp}\right| \sin\theta$$

The torque vector is perpendicular to r and F. When torque points into the screen, the rotation is clockwise. When torque points out of the screen, the rotation is counter-clockwise.





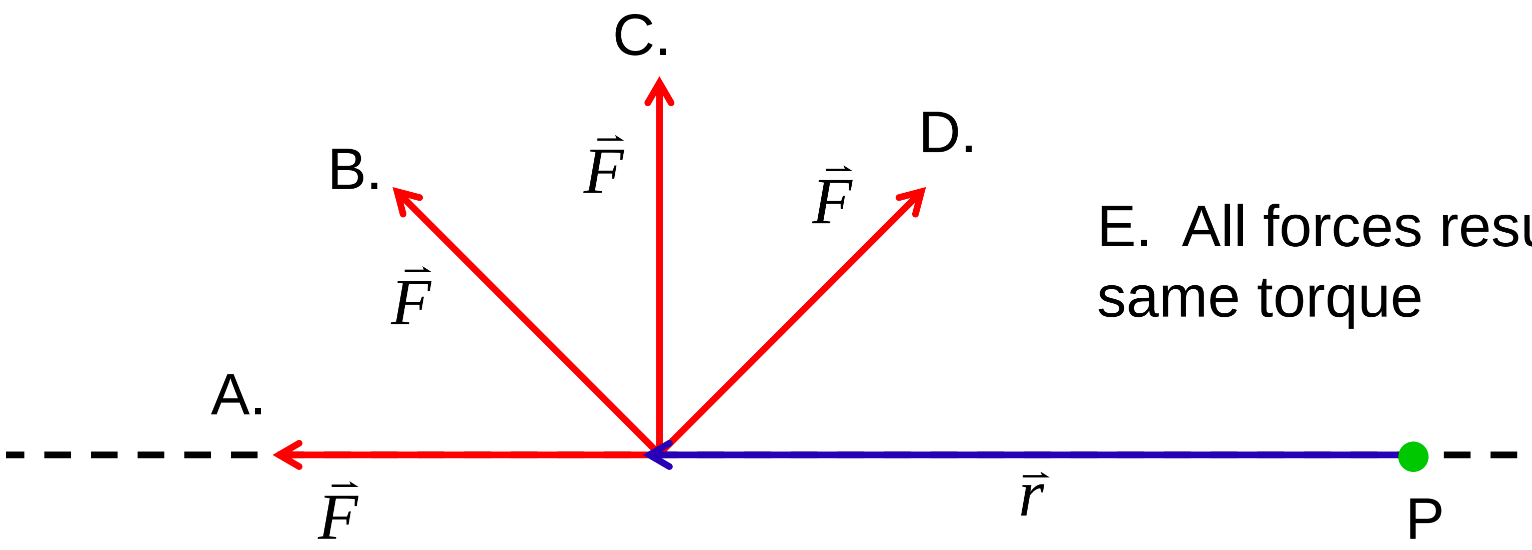
The direction of torque and the direction of rotation can be remembered using the right-hand rule:



Torque IV

Consider applying a force $\vec{F}$ relative to the point P.

Which of the following forces will result in the largest torque?



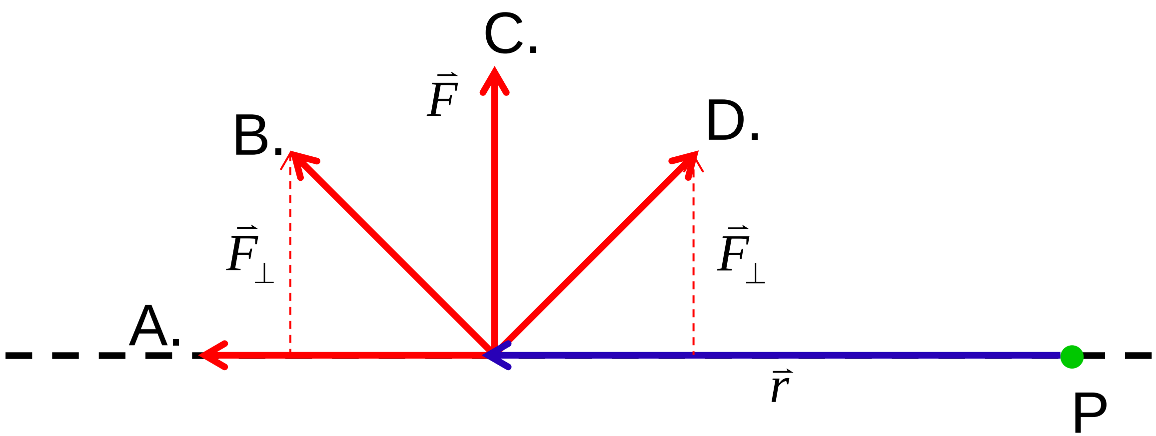
E. All forces result in the same torque

Note: All the forces $\vec{F}$ shown have the same magnitude.

Solution

Answer: C

Justification: We want to find the force with the largest perpendicular component relative to the lever arm.



Force C is applied perpendicular to $\vec{r}$, so it results in the maximum torque. Force A is parallel to $\vec{r}$, so it results in no torque. Forces B and D have both parallel and perpendicular components. These two forces apply an equal torque on P, although the magnitude is less than the torque created by force C.

Torque V

Consider the torque on P caused by the force $\vec{F}_1$ (shown in red). Another force $\vec{F}_i$, with equal magnitude, is applied so that the net torque on P is zero. Which force shown below is $\vec{F}_i$?

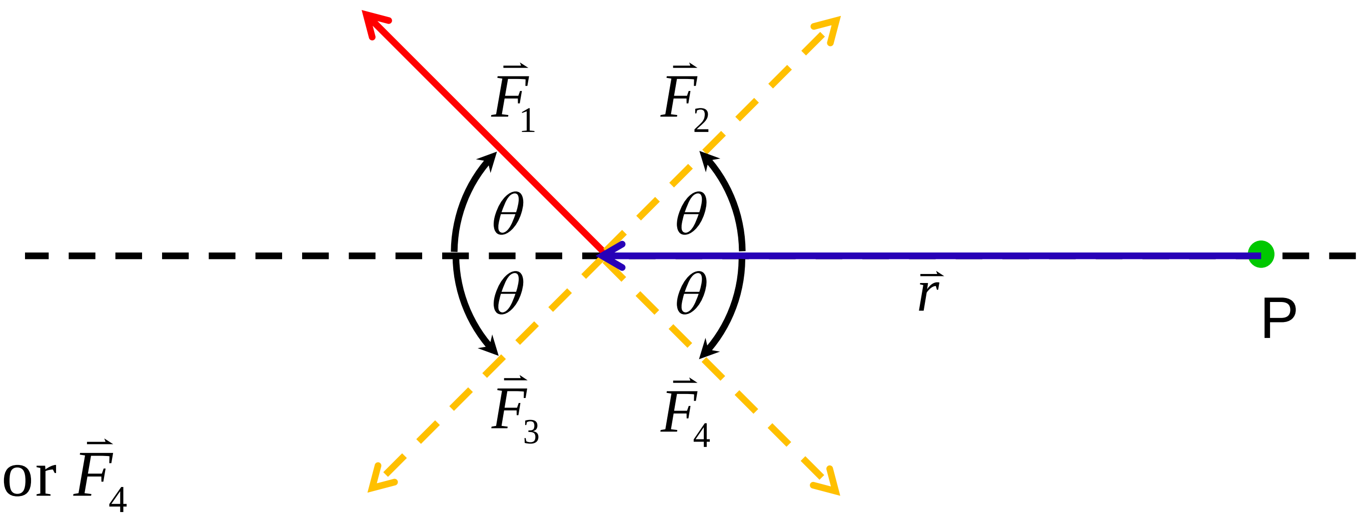
A. $\vec{F}_2$

B. $\vec{F}_3$

C. $\vec{F}_4$

D. Either $\vec{F}_3$ or $\vec{F}_4$

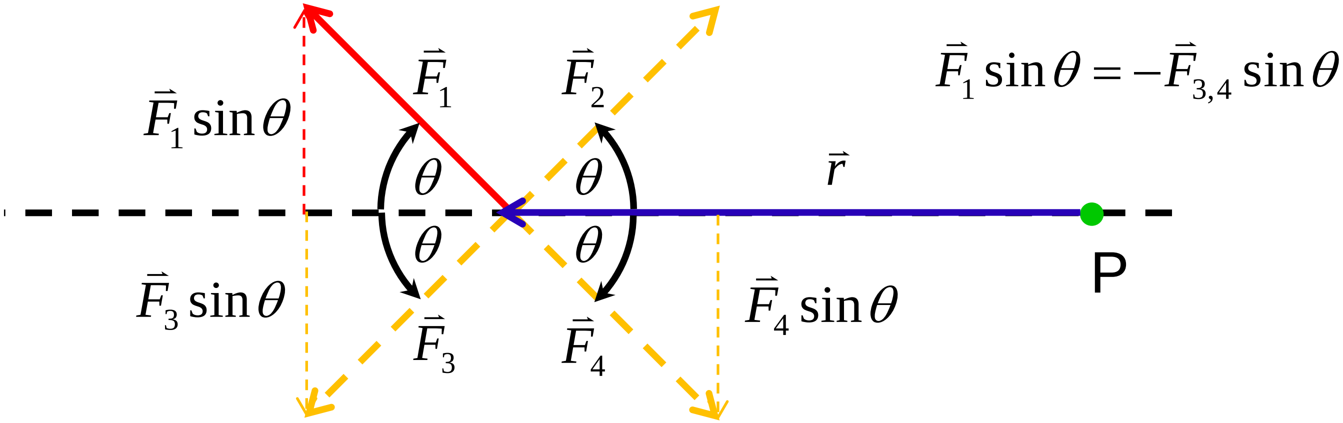
E. Either $\vec{F}_2$, $\vec{F}_3$ or $\vec{F}_4$



Solution

Answer: D

Justification: The torque caused by $\vec{F}_1$ on P is directed clockwise. Therefore, in order for there to be no net torque on P, $\vec{F}_i$ must generate a counter-clockwise torque. This eliminates answer A.

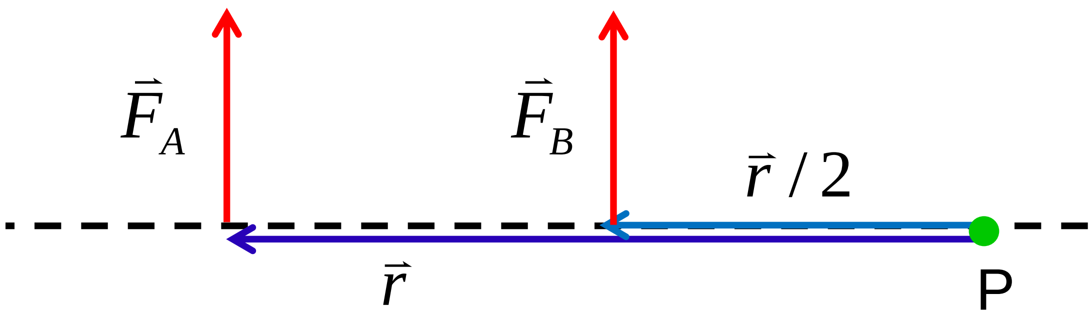


The perpendicular components of $\vec{F}_3$ and $\vec{F}_4$ are equal in magnitude and direction, which means they generate the same torque relative to P. The parallel components are equal and opposite, but do not affect the torque around P.

Torque VI

Two forces with magnitude F are applied at two different points, as seen below.

What is the net magnitude of torque relative to the point P?



A. $\tau = \dfrac{Fr}{2}$

B. $\tau = \dfrac{2Fr}{3}$

C. $\tau = Fr$

D. $\tau = \dfrac{3Fr}{2}$

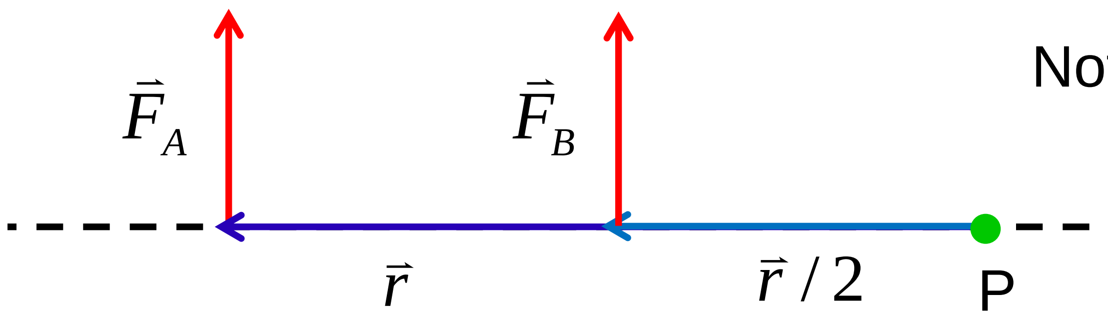
E. $\tau = 2Fr$

Solution

Answer: D

Justification: The torque from force A is $\tau_A = rF$ and directed clockwise.

The torque from force B is $\tau_B = \dfrac{r}{2}F$ and also directed clockwise.



Notice that: $\tau_A = 2\tau_B$

To find the net torque, we take the sum of the individual torques. Since both torques point clockwise, the net torque is also clockwise.

$$\tau_{net} = \frac{r}{2}F + rF = \frac{3r}{2}F \quad \text{(clockwise)}$$

Torque VII

A force F is applied at the point P as shown.

What is the net torque relative to the point P?

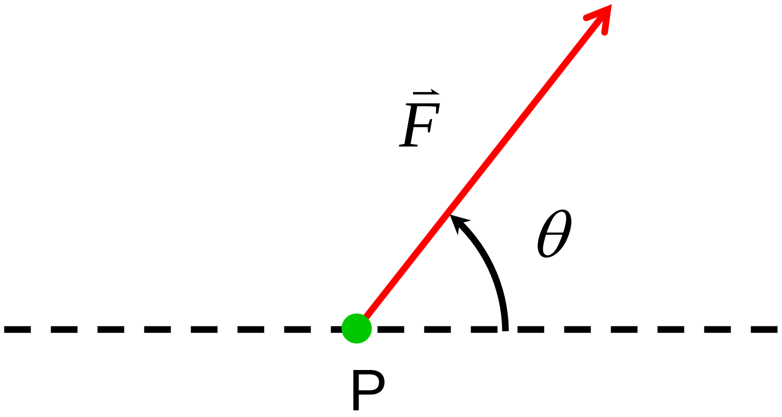
A. $\tau = 0$

B. $\tau = F\sin\theta$

C. $\tau = Fr$

D. $\tau = F$

E. $\tau = \text{infinite}$



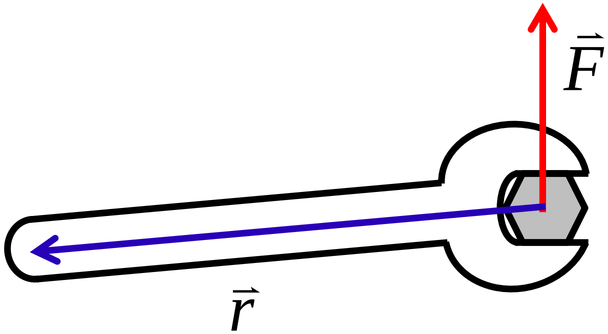
Solution

Answer: A

Justification: There is no lever from the pivot to where the force is applied. The distance *r* from the point P to the location that the force is applied is 0, so from our formula for the magnitude of torque:

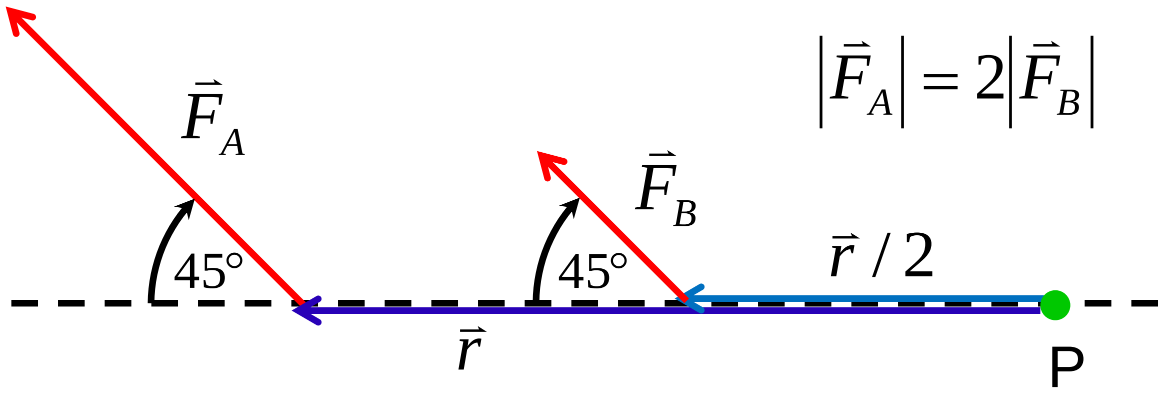
$$\tau = rF_{\perp} = 0 \cdot F_{\perp} = 0$$

Imagine applying a force on a bolt as shown. No matter how hard you push or which direction you push, the bolt will not turn because you are not applying a torque.



Torque VIII

Two forces are applied are applied at 45° to a lever. The magnitude of force A is twice the magnitude of force B.



What is the relationship between τ_A and τ_B?

A. $\tau_A = \tau_B$

B. $\tau_A = 2\tau_B$

C. $\tau_A = 4\tau_B$

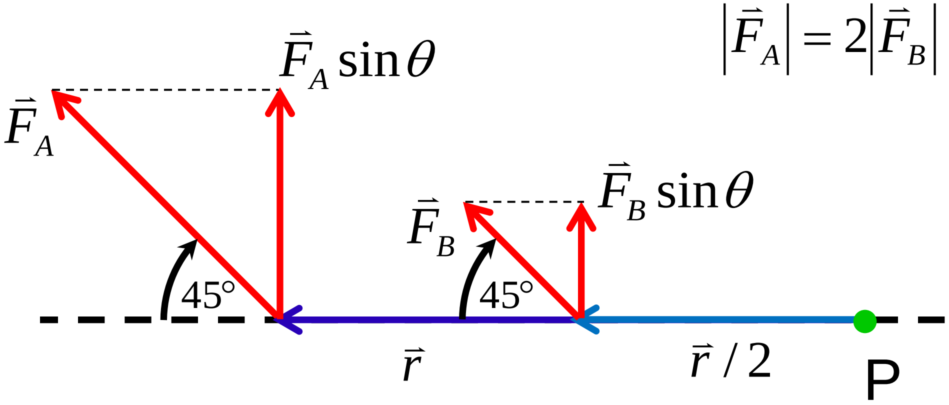
D. $\tau_A = 4\tau_B \sin(45°)$

E. $\tau_A = \dfrac{4\tau_B}{\sin(45°)}$

Solution

Answer: C

Justification:



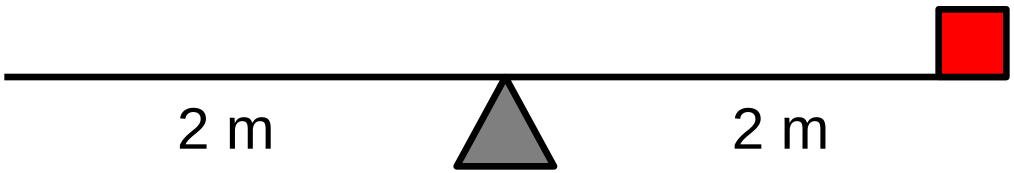
The torque from force B is: $\tau_B = \frac{r}{2} F_B \sin 45°$

The torque from force A is: $\tau_A = rF_A \sin 45° = 2rF_B \sin 45° = 4\tau_B$

Force B is half the magnitude of force A. Force B is applied at half the distance from the pivot point P. These two differences result in torque B being four times smaller than torque A. The 45° angle does not affect the ratio between torque A and B.

Torque IX

Consider a 4 m long seesaw with a 5 kg mass at the right end of the board.



What is the net torque relative to the pivot?

A. $\tau_{net} = 50\ \text{Nm} \quad \text{(clockwise)}$

B. $\tau_{net} = 100\ \text{Nm} \quad \text{(clockwise)}$

C. $\tau_{net} = 100\ \text{Nm} \quad \text{(counter-clockwise)}$

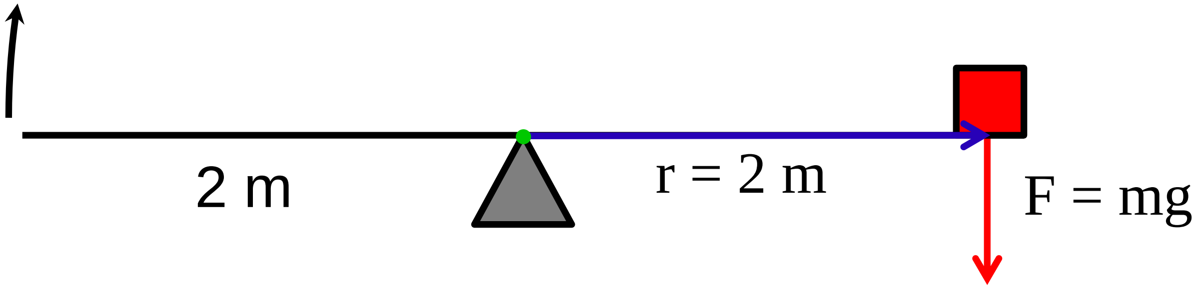
D. $\tau_{net} = 200\ \text{Nm} \quad \text{(clockwise)}$

E. $\tau_{net} = 200\ \text{Nm} \quad \text{(counter-clockwise)}$

Solution

Answer: B

Justification:



Even though the entire seesaw is 4 m long, the 5 kg mass is only 2 m away from the pivot. The mass applies a downward force F = mg. Therefore the net torque is:

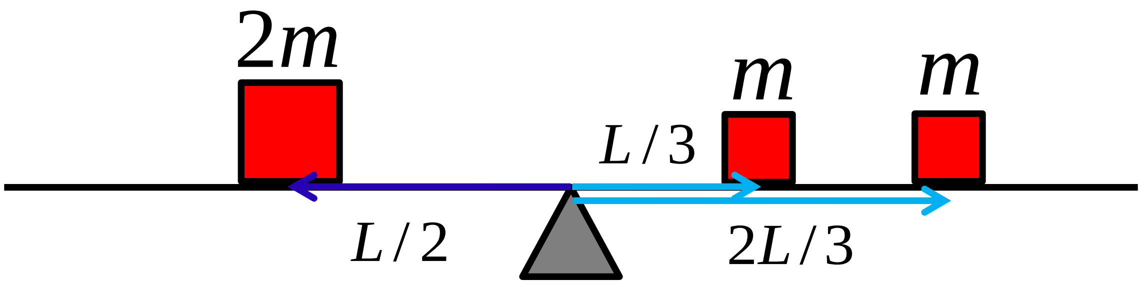
$$\tau_{net} = r(mg) = 2 \cdot 5(10) = 100\,\mathrm{Nm} \quad \text{(clockwise)}$$

The seesaw will tilt down to the right side, so the net torque is directed clockwise.

Will the net torque change if we also consider the mass of the seesaw board?

Torque X

Consider a seesaw of length 2L, with a pivot at the centre. Three masses are placed on the seesaw as shown.



Which way will the seesaw tilt if the masses are released?

A. Right (rotate counter-clockwise)

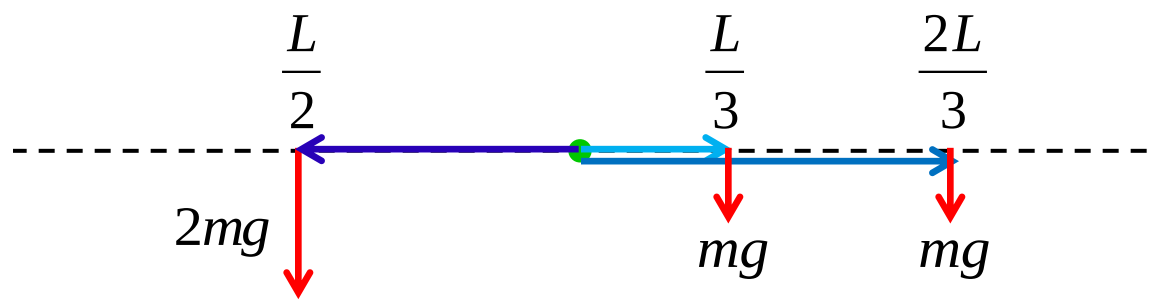
B. Left (rotate clockwise)

C. The seesaw will not tilt

Solution

Answer: C

Justification:



Masses to the left of the pivot generate counter-clockwise torques. The net counter-clockwise torque is:

$$\tau_{CW} = \frac{L}{2}(2mg) = Lmg$$

Masses to the right of the pivot generate clockwise torques. The net clockwise torque is:

$$\tau_{CCW} = \frac{L}{3}(mg) + \frac{2L}{3}(mg) = Lmg$$

The net torque around the pivot is 0 since the clockwise and counter-clockwise torques are equal, and the seesaw does not tilt.

Extend Your Learning: Simulation

Title: Balancing Act