a place of mind

FACULTY OF EDUCATION

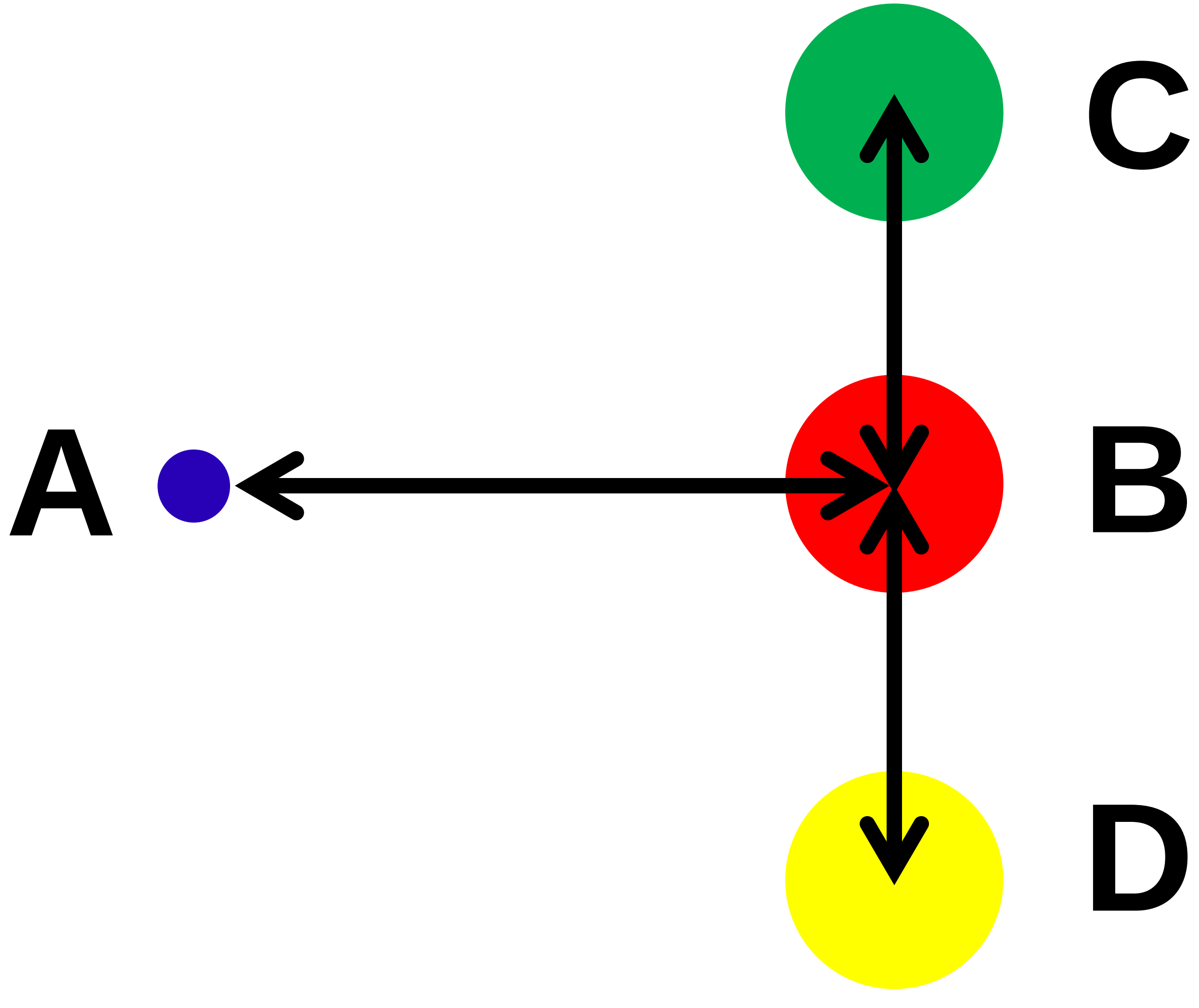
Department of
Curriculum and Pedagogy

Physics
Gravitation: Force

Science and Mathematics
Education Research Group

Supported by UBC Teaching and Learning Enhancement Fund 2012-2013

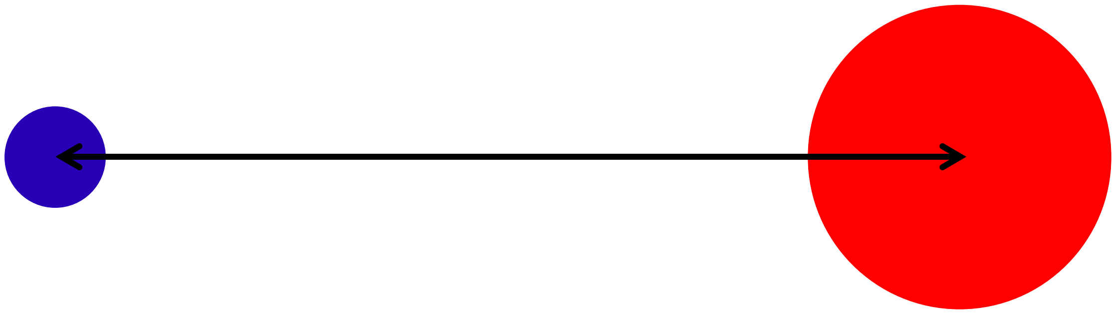
Planetoids



Planetoids I

Two planetoids of unequal mass are separated by an unknown distance. What can be said of the gravitational force on each mass?

A. The force on the smaller mass is greater
B. The force on the larger mass is greater
C. They are equal
D. No idea



Solution

Answer: C

Justification:

1. The equation for the gravitational force one planet exerts on another is given below. The equation does not change when considering the force 1 exerts on 2, or the force 2 exerts on 1. Therefore, the two forces are equal.

$$F_{1on2} = \frac{Gm_1m_2}{r^2} \quad \text{and} \quad F_{2on1} = \frac{Gm_2m_1}{r^2}$$

2. Another way of thinking about this problem: By Newton's third law the force the smaller mass exerts on the larger mass is equal to the force the larger mass exerts on the smaller mass.

Planetoids II

If $m_C = m_B$, and the distances are as shown (with each of the lines equal to r and perpendicular), what is the magnitude of the force A exerts on C as compared to the force A exerts on B?

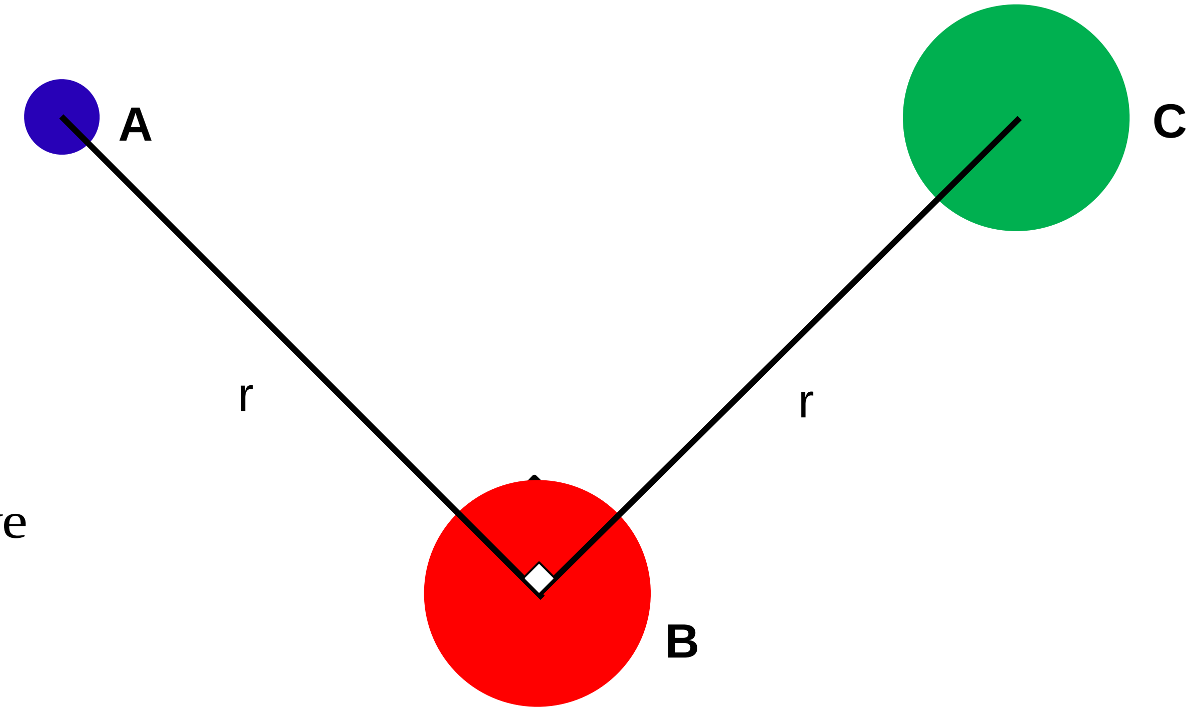
$A.\ \sqrt{2}F_{AonB}$

$B.\ F_{AonB}$

$C.\ \dfrac{F_{AonB}}{\sqrt{2}}$

$D.\ \dfrac{F_{AonB}}{2}$

$E.$ None of the above



Solution

Answer: D

Justification: The gravitational force is proportional to the inverse of the distance squared ($F \alpha \frac{1}{r^2}$). The distances between A and B, and B and C are both r. By the Pythagorean theorem, the distance between A and C is $r\sqrt{2}$. Therefore:

$$F_{AonC} = G\frac{m_A m_C}{\left(r\sqrt{2}\right)^2} = G\frac{m_A m_C}{2r^2} = G\frac{m_A m_B}{2r^2} = \frac{F_{AonB}}{2} \text{ (Remember } m_B = m_C\text{)}$$

Thus, the force between A and C is 2 times weaker than the force between A and B.

Planetoids III

If m_C is equal to m_B, and is twice m_A, and the distances are as shown (with each of the lines equal to r and perpendicular), what is the magnitude of the net force on B, in terms of the force A exerts on B?

$$m_B = m_C = 2m_A$$

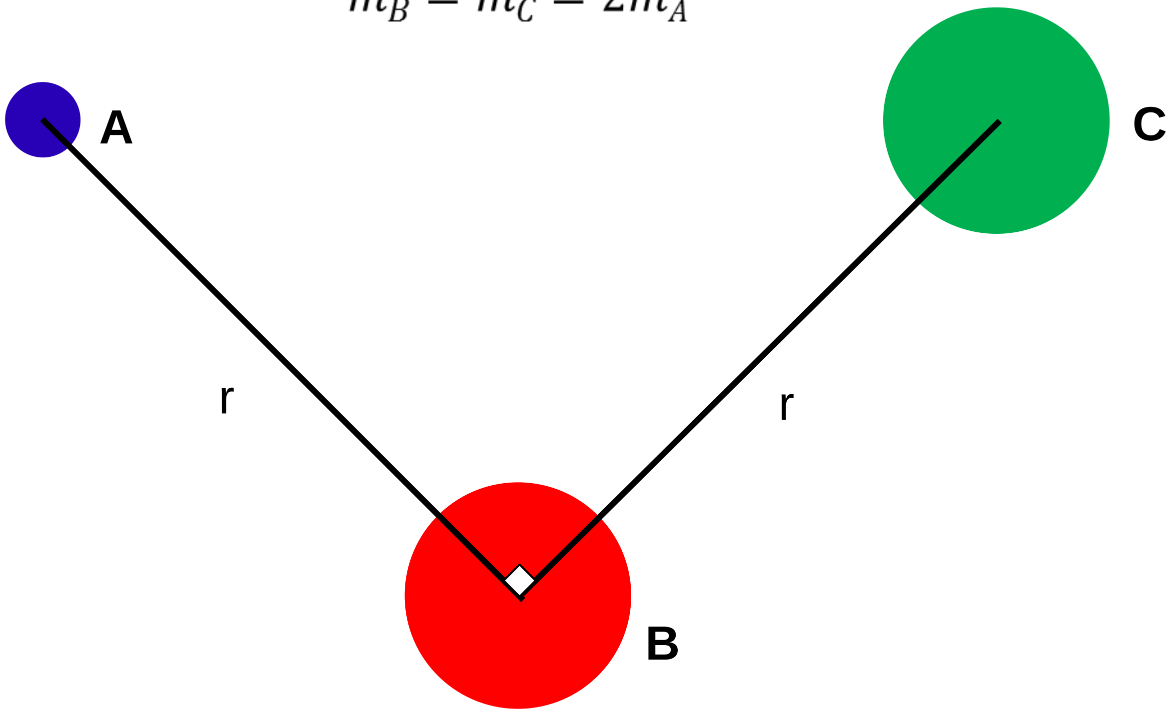
$A.\ \sqrt{2}F_{AonB}$

$B.\ F_{AonB}$

$C.\ \sqrt{3}F_{AonB}$

$D.\ \dfrac{F_{AonB}}{2}$

$E.\ \sqrt{5}F_{AonB}$

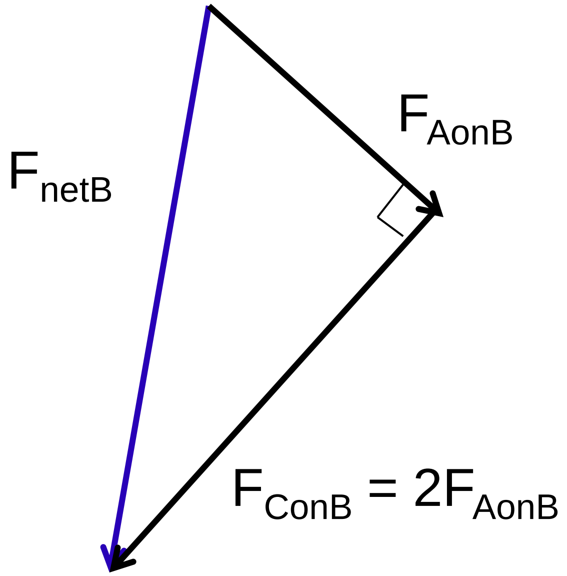


Solution

Answer: E

Justification: Since the gravitational force scales linearly with mass, the force C exerts on B is twice as large as the force A exerts on B:

$$F_{ConB} = G\frac{m_C m_B}{r^2} = G\frac{2m_A m_B}{r^2} = 2G\frac{m_A m_B}{r^2} = 2F_{AonB} \quad \text{(Remember } m_C = 2m_A\text{)}$$



The two forces F_{AonB} and F_{ConB} are perpendicular to each other, and can be added using Pythagorean Theorem:

$$F_{net} = \sqrt{\left(F_{AonB}\right)^2 + \left(F_{ConB}\right)^2} = \sqrt{\left(F_{AonB}\right)^2 + \left(2F_{AonB}\right)^2}$$

$$F_{net} = \sqrt{5\left(F_{AonB}\right)^2} = \sqrt{5}F_{AonB}$$

Planetoids IV

If m_C is equal to m_B and m_D, and is twice m_A, and the distances are as shown (with each of the lines equal to r and perpendicular), what is the magnitude of the net force on A?

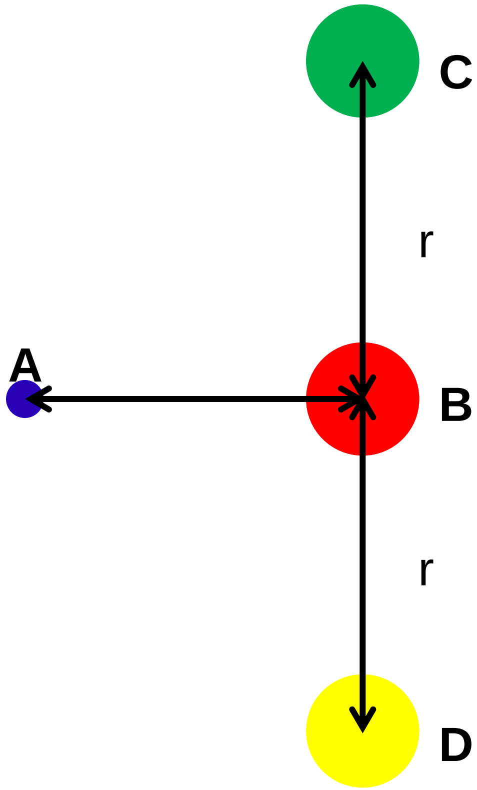
$A. \left(1+\frac{1}{\sqrt{2}}\right)F_{AonB}$

$B. \frac{\sqrt{5}F_{AonB}}{2}$

$C. \left(1-\frac{1}{\sqrt{2}}\right)F_{AonB}$

$D. \left(1+\sqrt{2}\right)F_{AonB}$

$E.$ None of the above



$$m_B = m_C = m_D = 2m_A$$

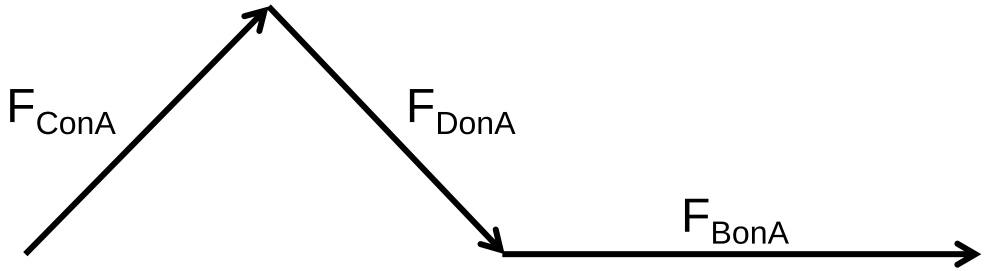
Solution

Answer: A

Justification: In question 2 we found that $F_{ConA} = \dfrac{F_{AonB}}{2}$.

Because D has the same mass as C and is located the same distance from A as C, the magnitudes of the forces exerted by each are the same, $F_{ConA} = F_{DonA}$. The forces are also perpendicular to each other.

Thus the net force exerted on A by C and D can be calculated using Pythagorean theorem. $\sqrt{\left(\dfrac{F_{BonA}}{2}\right)^2 + \left(\dfrac{F_{BonA}}{2}\right)^2} = \sqrt{2\dfrac{F_{BonA}{}^2}{4}} = \dfrac{F_{BonA}}{\sqrt{2}}$



Adding F_{BonA}

$$\frac{F_{BonA}}{\sqrt{2}} + F_{BonA} = F_{BonA}\left(\frac{1}{\sqrt{2}} + 1\right)$$

Note: According to Newton's third law: $F_{BonA} = F_{AonB}$ (equal magnitudes)

Planetoids V

If m_C is equal to m_B and m_D, and is twice m_A, and the distances are as shown (with each of the lines equal to r and perpendicular), what is the magnitude of the net force on B?

$$m_B = m_C = m_D = 2m_A$$

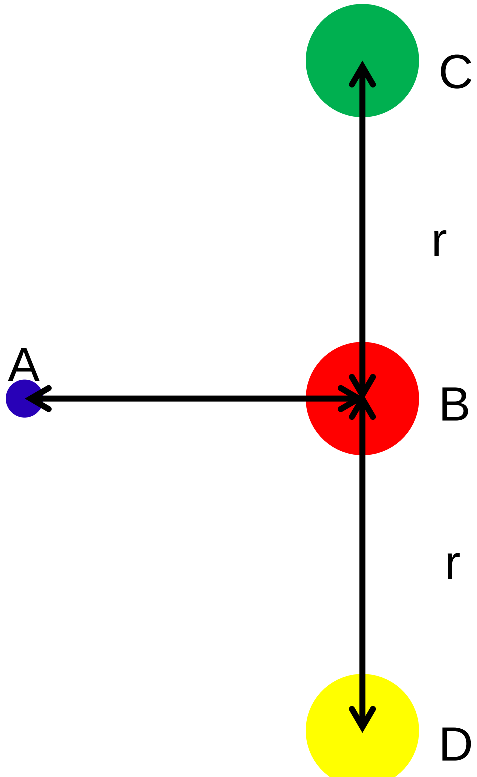
$$A. \left(1+\frac{1}{\sqrt{2}}\right)F_{AonB}$$

$$B. \frac{\sqrt{5}F_{AonB}}{2}$$

$$C. \frac{10F_{AonB}}{\sqrt{5}}$$

$$D. F_{AonB}$$

$$E. \left(1+\sqrt{2}\right)F_{AonB}$$



Solution

Answer: D

Justification: Because $m_C = m_D$, the forces exerted by C and D on B are equal in magnitude and opposite in direction. Therefore, they cancel each other out.

We are then left with the force that A exerts on B, F_{AonB}.