a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Physics
1D-Kinetmatics:
Average Velocity

Science and Mathematics
Education Research Group

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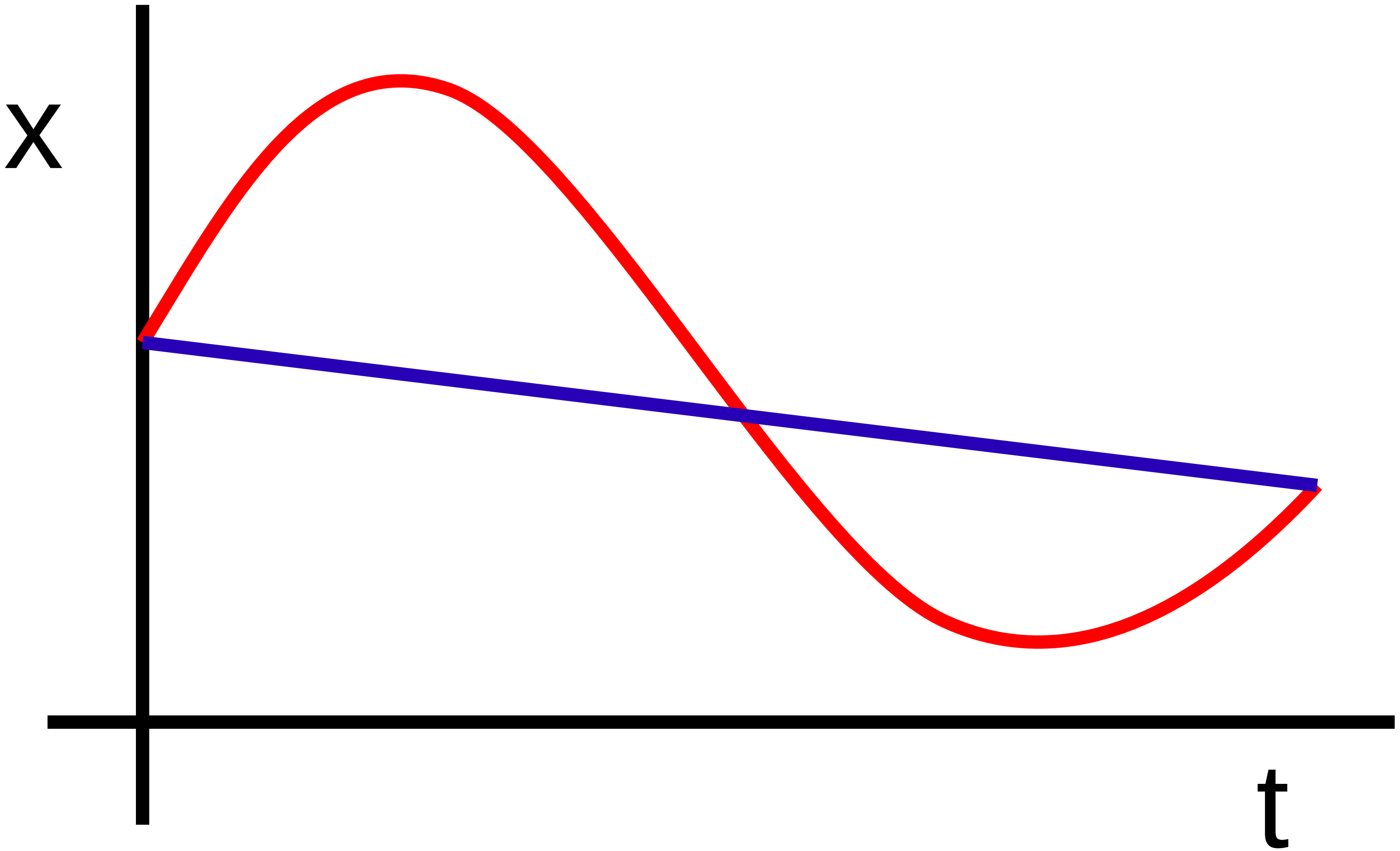
Average Velocity



Average Velocity I

A car spends 1 hour travelling at 30 km/h along a straight track. The car spends the next hour travelling at 50 km/h.

The average velocity of the car is the change in its displacement divided by the total time taken. What is the average velocity of the car over the two hours it travels?

A. 30 km/h

B. 40 km/h

C. 50 km/h

D. 80 km/h

E. Between 30 km/h and 50 km/h

Solution

Answer: B

Justification: Average velocity is defined as:

$$\bar{v} = \frac{\text{change in displacement}}{\text{change in time}} = \frac{\Delta x}{\Delta t}$$

In the first hour, the car travels 30 km since it moves at a <u>constant</u> velocity of 30 km/h. $\Delta x = vt = 30\ \text{km/h} \times 1\ \text{h} = 30\ \text{km}$

In the second hour, the car travels 50 km since it moves at 50 km/h. The total displacement over the two hours is 30 km + 50 km = 80 km. The average velocity of the car is:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{30\ \text{km} + 50\ \text{km}}{2\ \text{h}} = 40\ \text{km/h}$$

Average Velocity II

A car travels at 30 km/h along a straight 1 km track. After some time, its velocity instantly changes to 50 km/h.

What is the average velocity of the car during the time it is on the track?

A. Less than 30 km/h

B. Exactly 40 km/h

C. Between 30 km/h and 50 km/h

D. Greater than 50 km/h

Solution

Answer: C

Justification: We do not know how long the car is travelling at 30 km/h or how long the car is travelling at 50 km/h. Since we do not know the total time the car spends travelling on the track, we cannot find the exact average velocity of the car.

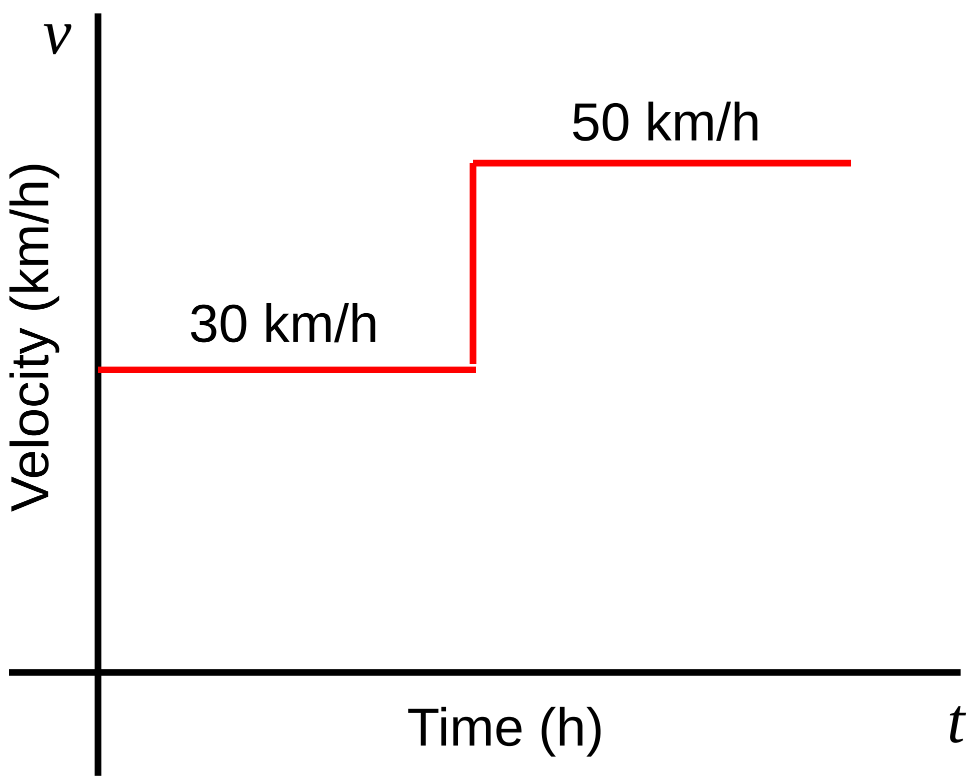
However, the car never travels slower than 30 km/h. This means the average velocity should not be slower than 30 km/h. The car also never travels faster than 50 km/h, so the average velocity should not be faster than 50 km/h.

The average velocity of the car be anywhere between 30 km/h and 50 km/h, depending on how long it spends travelling at each velocity. The longer the car spends travelling at 50 km/h, the closer the average velocity gets to 50 km/h.

Average Velocity III

The velocity of a car travelling along a straight track is shown below.

What is the average velocity of the car?



A. 30 km/h

B. 40 km/h

C. 50 km/h

D. Between 30 km/h and 50 km/h

E. Not enough information

Solution

Answer: B

Justification: We do not know exactly how long the car spends travelling at 30 km/h and 50 km/h.

However, the graph tells us that the car spends an equal amount of time travelling at 30 km/h as it does travelling at 50 km/h.

Therefore, we should expect the average velocity of the car to be the average between 30 km/h and 50 km/h:

$$\bar{v} = \frac{30\ \text{km/h} + 50\ \text{km/h}}{2} = 40\ \text{km/h}$$

Alternative Solution

Answer: B

Justification: We do not know exactly how long the car spends travelling at 30 km/h and 50 km/h. However, the graph tells us that the car spends the same amount of time travelling at 30 km/h as it does travelling at 50 km/h.

Let the total time the car travels be *t* hours. The car spends *t*/2 hours travelling at 30 km/h, and *t*/2 hours travelling at 50 km/h. From the formula (displacement) = (velocity) × (time):

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{30\left(\frac{t}{2}\right)\text{km} + 50\left(\frac{t}{2}\right)\text{km}}{t\ \text{h}} = 15\ \text{km/h} + 25\ \text{km/h} = 40\ \text{km/h}$$

Notice how the unknown *t* gets cancelled in the equation.

Average Velocity IV

A car travels along a straight path to a city 40 km away. The car then travels back to its starting location. The trip to the city and back takes a total time of 2 hours.

What was the average velocity of the car during the trip?

A. 0 km/h
B. 20 km/h
C. 40 km/h
D. 80 km/h
E. Not enough information

Solution

Answer: A

Justification: Let the starting position of the car be $x_i = 0\text{ km}$. Upon reaching the city, the car's displacement is 40 km. When the car returns to its starting location, the car's total displacement is $x_f = 0\text{ km}$ again (the car is 0 km away from its starting location).

The average velocity of the car over the 2 hours is:

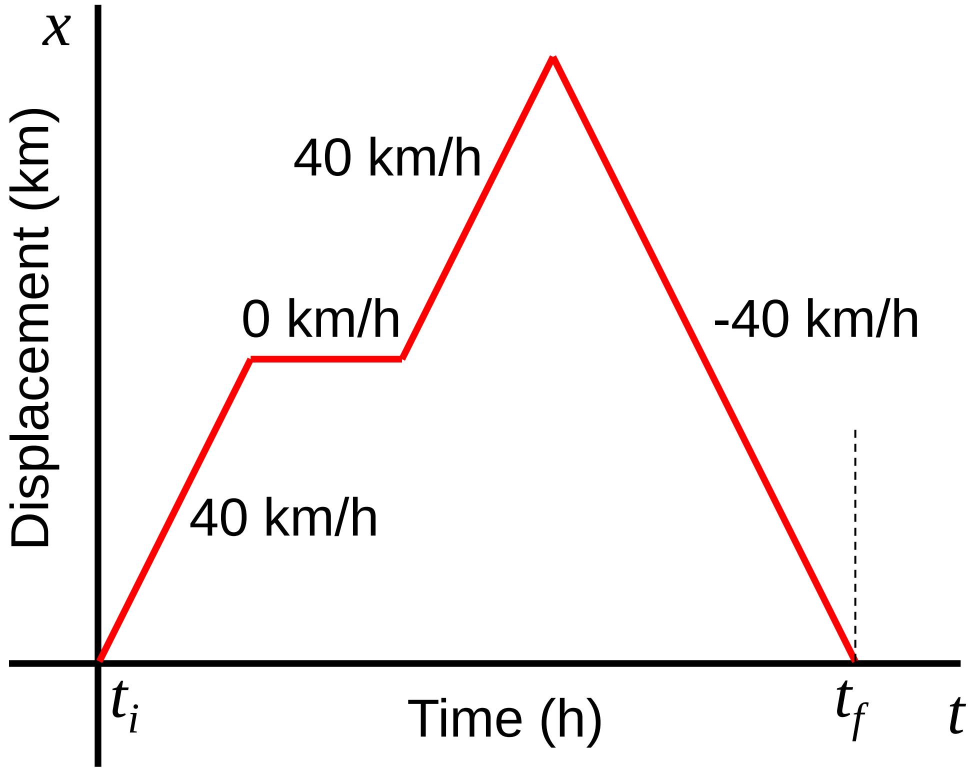
$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{2\text{ h}} = \frac{0\text{ km} - 0\text{ km}}{2\text{ h}} = 0\text{ km/h}$$

Note: The car travels a total <u>distance</u> of 80 km in 2 hours, so its average <u>speed</u> is 40 km/h.

Average Velocity V

The displacement of a car over time is shown in the graph below

What was the average velocity of the car between t_0 and t_f?



A. 0 km/h
B. 20 km/h
C. 40 km/h
D. Between 0 and 40 km/h
E. Not enough information

Solution

Answer: A

Justification: The displacement of the car at t_i is $x_i = 0\text{ km}$. The displacement of the car at t_f is $x_f = 0\text{ km}$. The average velocity of the car is therefore 0 km/h.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{2\text{ h}} = \frac{0\text{ km} - 0\text{ km}}{2\text{ h}} = 0\text{ km/h}$$

Just like in the previous question, the car returns to its starting location. Average velocity does not depend on the path the car takes to get from x_i and x_f. Average velocity only depends on the initial and final positions of the car – the total displacement.

Average Velocity VI

Car A travels 50 km/h east for 2 hours. The car then remains stationary for the next 2 hours.

Car B travels at a velocity equal to Car A's average velocity for four hours.

Suppose Car A and Car B both start at a $x_i = 0 \text{ km}$. Which car has the largest displacement after 4 hours?

A. Car A

B. Car B

C. Both cars have the same displacement

D. Not enough information

Solution

Answer: C

Justification: Car A travels 100 km during its first two hours of travel, and then remains stationary for the next two hours. The final displacement of Car A is $x_f = 100\ \text{km}$.

The average velocity of Car A is 25 km/h:

$$\bar{v}_A = \frac{\Delta x}{\Delta t} = \frac{100\ \text{km}}{4\ \text{h}} = 25\ \text{km/h}$$

After 4 hours of travelling at 25 km/h, the final displacement of Car B is $x_f = 100\ \text{km}$.

$$\Delta x = \bar{v}t = 25\ \text{km/h} \times 1\ \text{h} = 100\ \text{km}$$

Both cars have the same final displacement.

Alternative Solution

Answer: C

Justification: Let the final displacement of Car A be x_f after Δt hours. From the definition of average velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

$$\bar{v}\Delta t = x_f - x_i$$

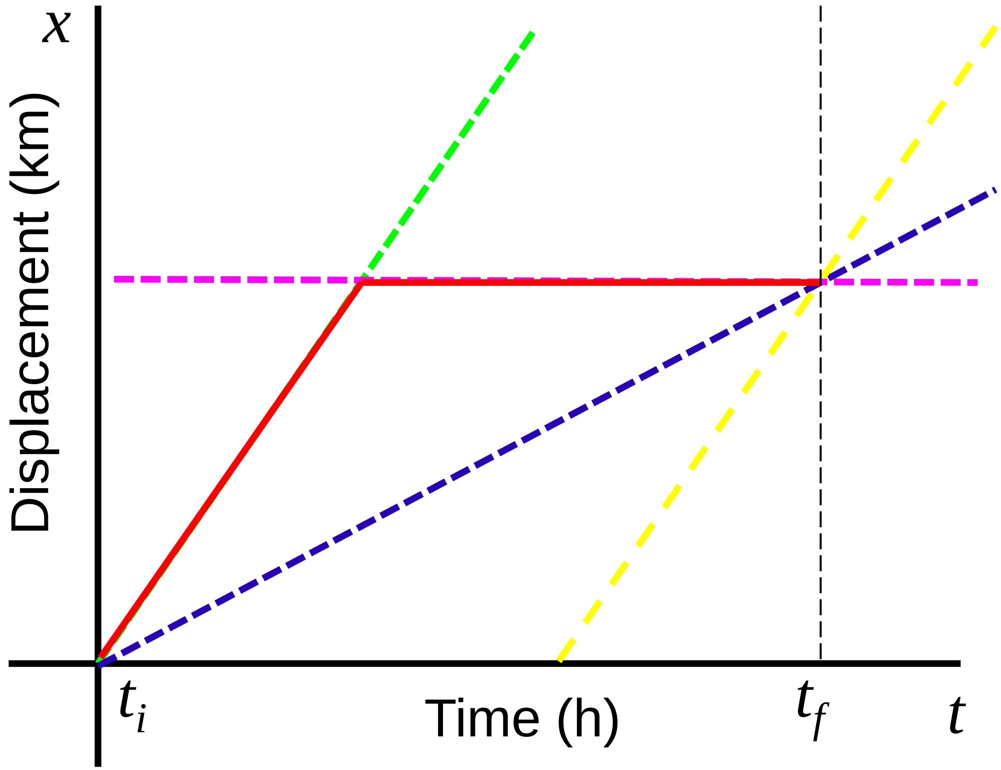
$$\bar{v}\Delta t + x_i = x_f$$ (x_i is often chosen to be 0 km)

It Car A and B start at the same position, Car B (moving at Car A's average velocity) will reach x_f at the same time as Car A. Average velocity only depends on displacement, not the path a car takes to get from x_i to x_f.

Average Velocity VII

The displacement of car A over time is shown in red below. Car B starts at 0 km at $t_0 = 0\ \mathrm{h}$ and travels at Car A's average velocity.

Which line best represents Car B's displacement over time?



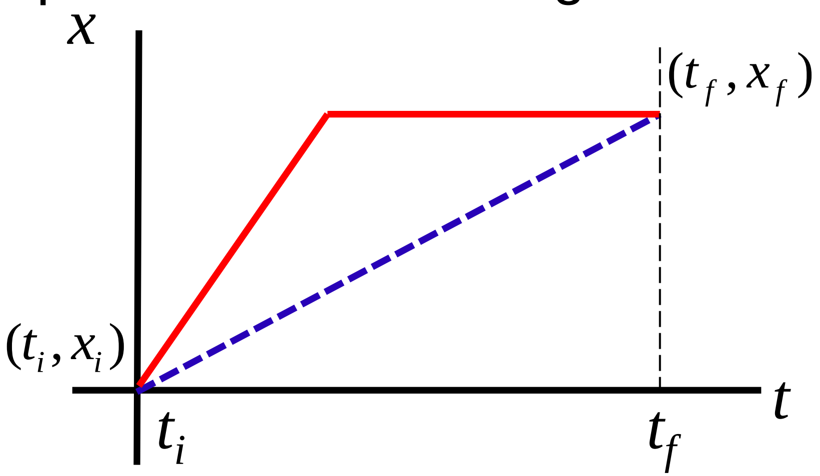
A. Blue
B. Yellow
C. Green
D. Purple
E. None of the above

Solution

Answer: A

Justification: Both Car A and Car B start at the same position ($x_i = 0$ km). Car A and Car B must therefore have the same final displacement at t_f since they have the same average velocity (see previous question).

Car B therefore travels at constant speed from x_i to x_f. This is represented as a straight line from (t_i, x_i) to (t_f, x_f).

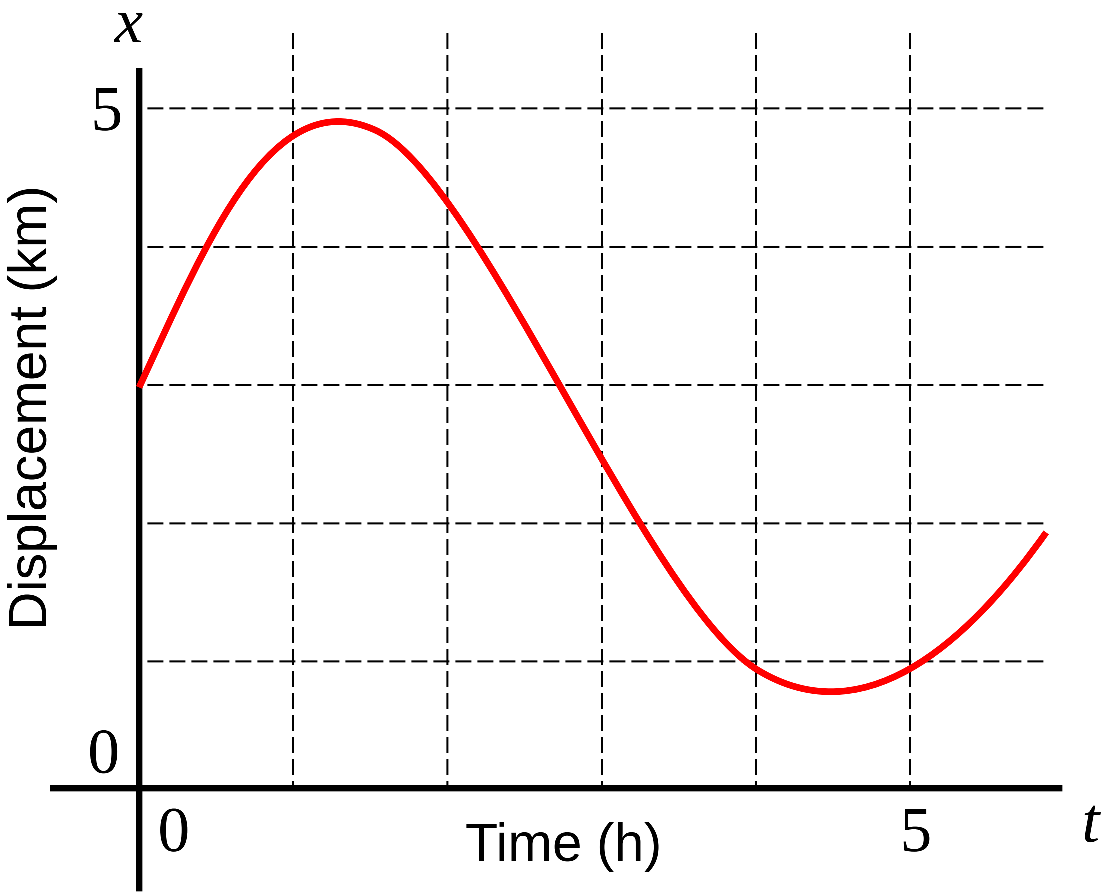


Slope of a line from (t_i, x_i) to (t_f, x_f):

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i}$$

Average Velocity VIII

What is the average velocity of the object from $t = 0$ h to $t = 5$ h?



A. -0.4 km/h

B. -0.2 km/h

C. 0 km/h

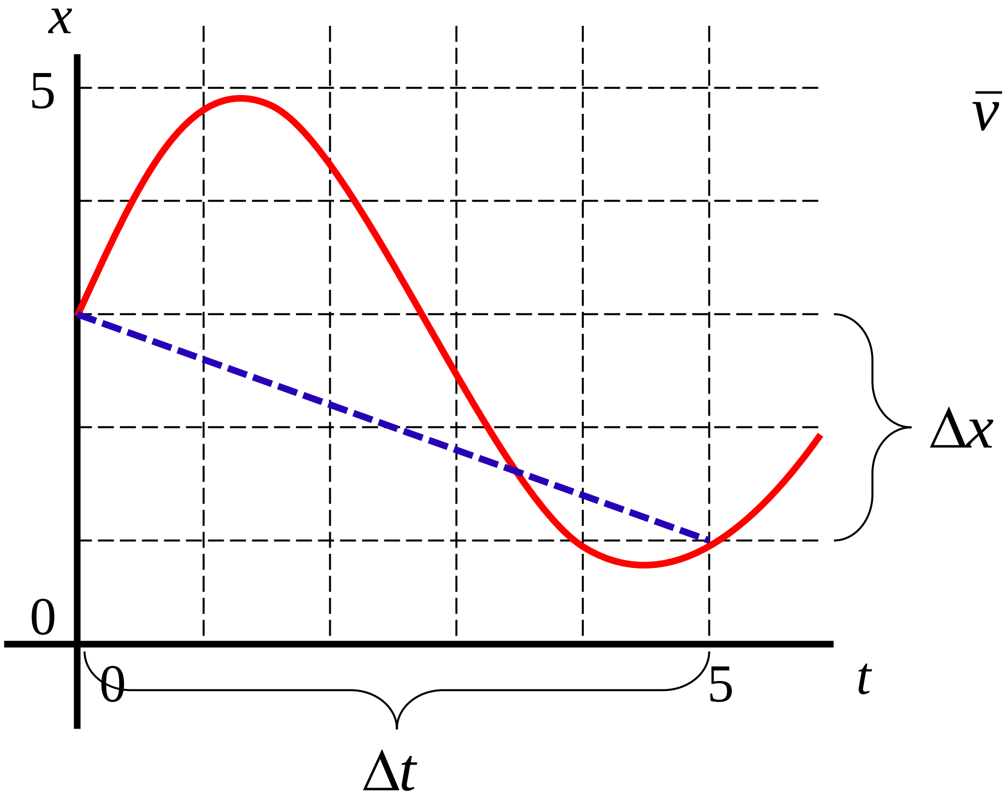
D. 0.2 km/h

E. 0.4 km/h

Solution

Answer: A

Justification: The average velocity from $t_i = 0$ h to $t_f = 5$ h is the slope of the line connecting the final and initial points.



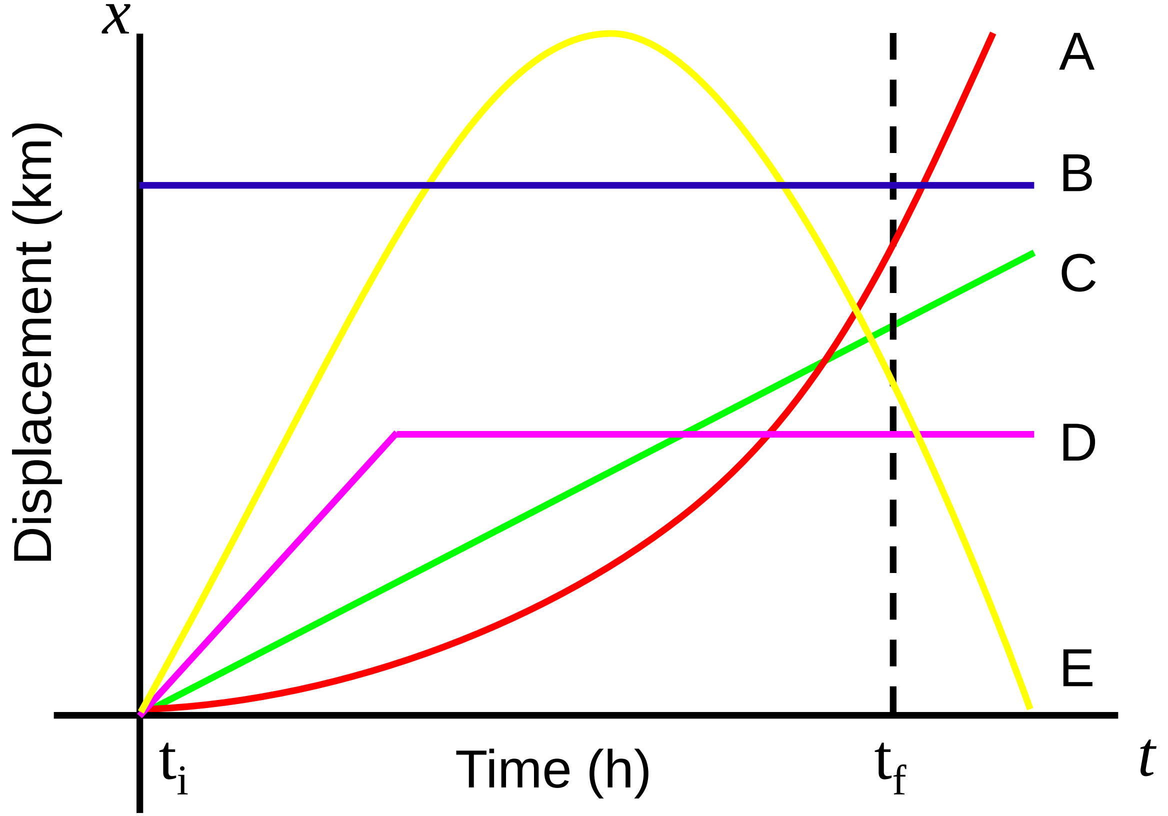
$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{1-3}{5-0} = \frac{-2}{5} \text{ km/h}$$

For an object moving left and right, a negative average velocity means that the final position of the object is to the left of its initial position.

Average Velocity IX

The displacement versus time graphs of 5 different cars are shown below.

Which car has the greatest average velocity between t_i and t_f?



A. Red

B. Blue

C. Green

D. Purple

E. Yellow

Solution

Answer: A

Justification: Car B has the same initial and final displacement, so its average velocity is 0 km/h.

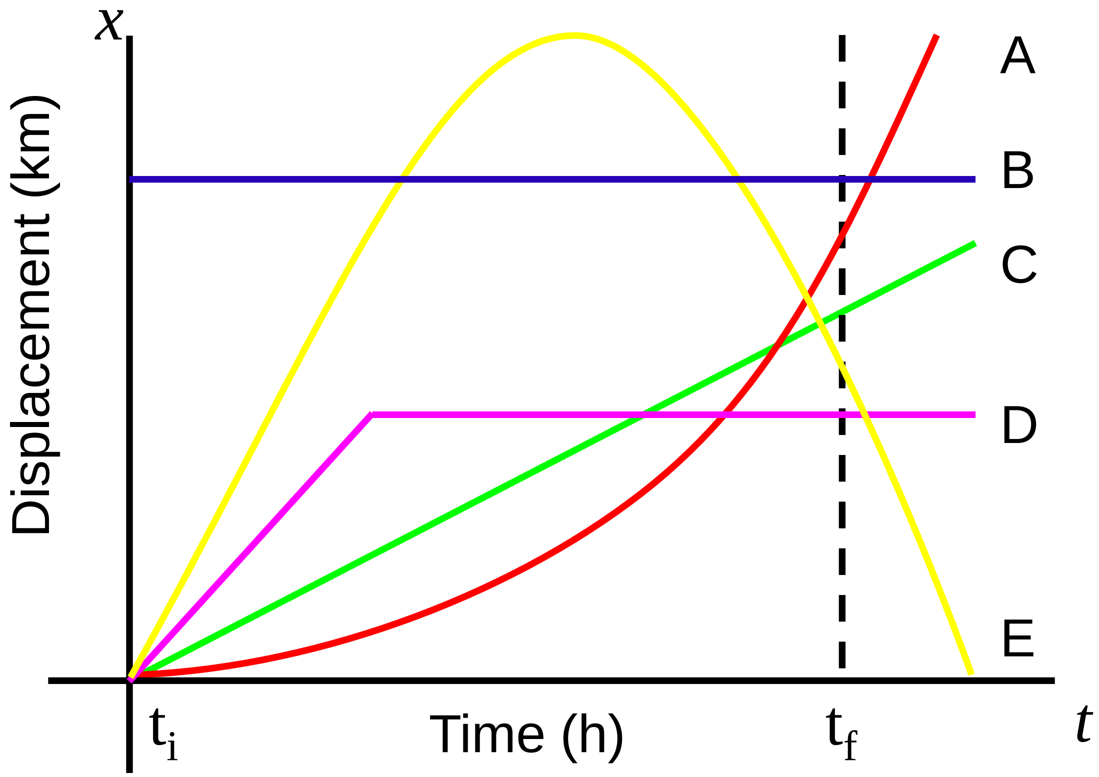
All the cars except for B start at $x_i = 0$. The car with the largest final displacement between A, B, C, and D will have the largest average velocity. The order from largest average velocity to smallest is:

$$A > C > E > D > B$$

Extend Your Learning: Displacement-Time Graphs

Describe the motion of each car between t_i to t_f.

Which car has the largest average speed? Which cars have the same average speed and average velocity?



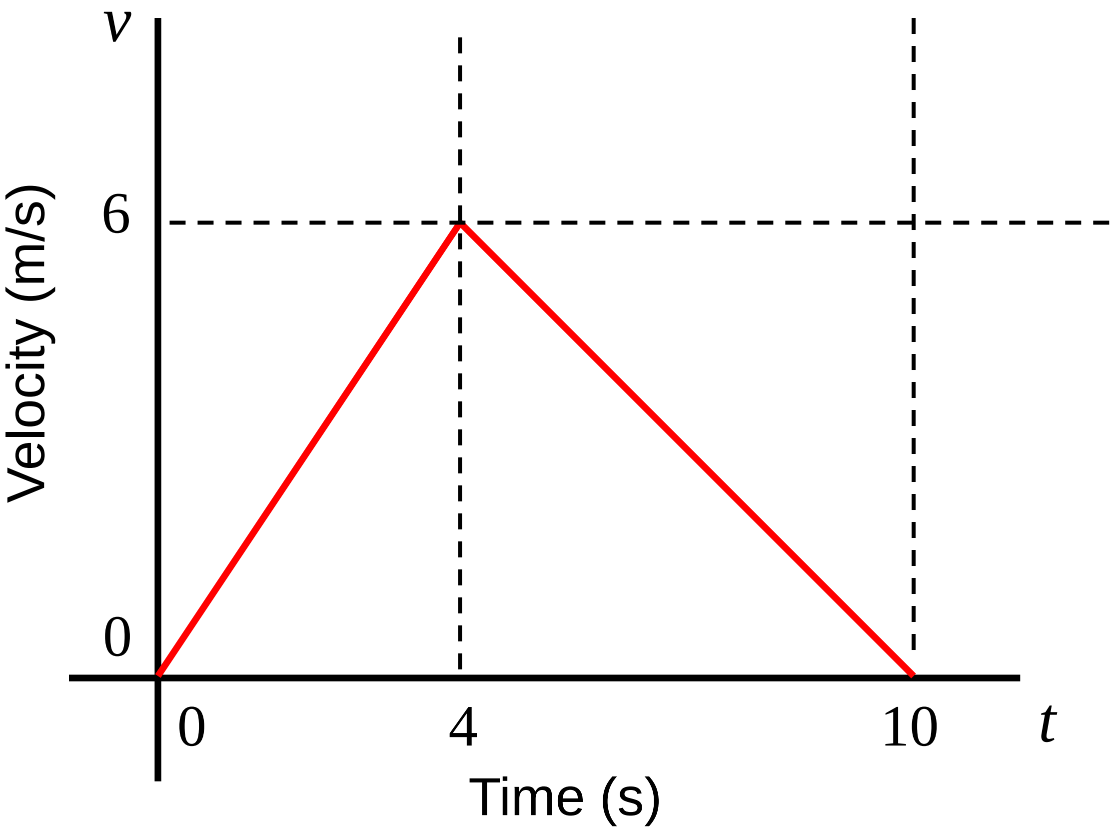
$$\overline{\text{speed}} = \frac{\Delta\text{distance}}{\Delta\text{time}}$$

$$\overline{\text{velocity}} = \frac{\Delta\text{displacement}}{\Delta\text{time}}$$

Average Velocity X

The graph below shows an object's velocity as it moves along a straight path.

What is the average velocity of the object between 0 s and 10 s?



A. -1 m/s

B. 1.5 m/s

C. 0 m/s

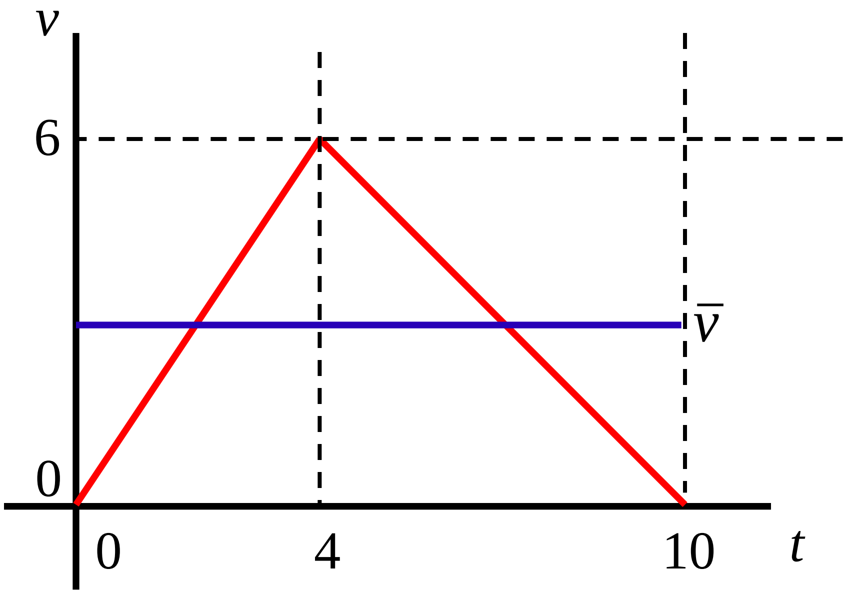
D. 3 m/s

E. 4 m/s

Solution

Answer: D

Justification: The displacement of the object is represented by the area under the graph. The area under a triangle with height 6 m/s and base 10 s is 30 m (notice the units!). Another object travelling with a velocity equal to this object's average velocity must be at the same final displacement after 10 seconds.



$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{30\ \text{m}}{10\ \text{s}} = 3\ \text{m/s}$$

Note: The area under the red line and blue line are equal

Alternative Solution

Answer: D

Justification: The displacement of an accelerating object is given by:

$$x = x_i + v_i \Delta t + \frac{1}{2} a (\Delta t)^2$$

For the first 4 seconds, x_i = 0 m, v_i = 0 m/s, and a = 1.5 m/s^2:

$$x = 0 + 0 + \frac{1}{2}(1.5)(4)^2 = 12 \text{ m}$$

For the next 6 seconds, x_i = 12 m, v_i = 6 m/s, and a = –1 m/s^2:

$$x = 12 + 6(6) + \frac{1}{2}(-1)(6)^2 = 30 \text{ m}$$

The average velocity from 0 s to 10 s: $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{30 \text{ m}}{10 \text{ s}} = 3 \text{ m/s}$