a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Physics
Momentum: Elastic Collisions

Science and Mathematics
Education Research Group

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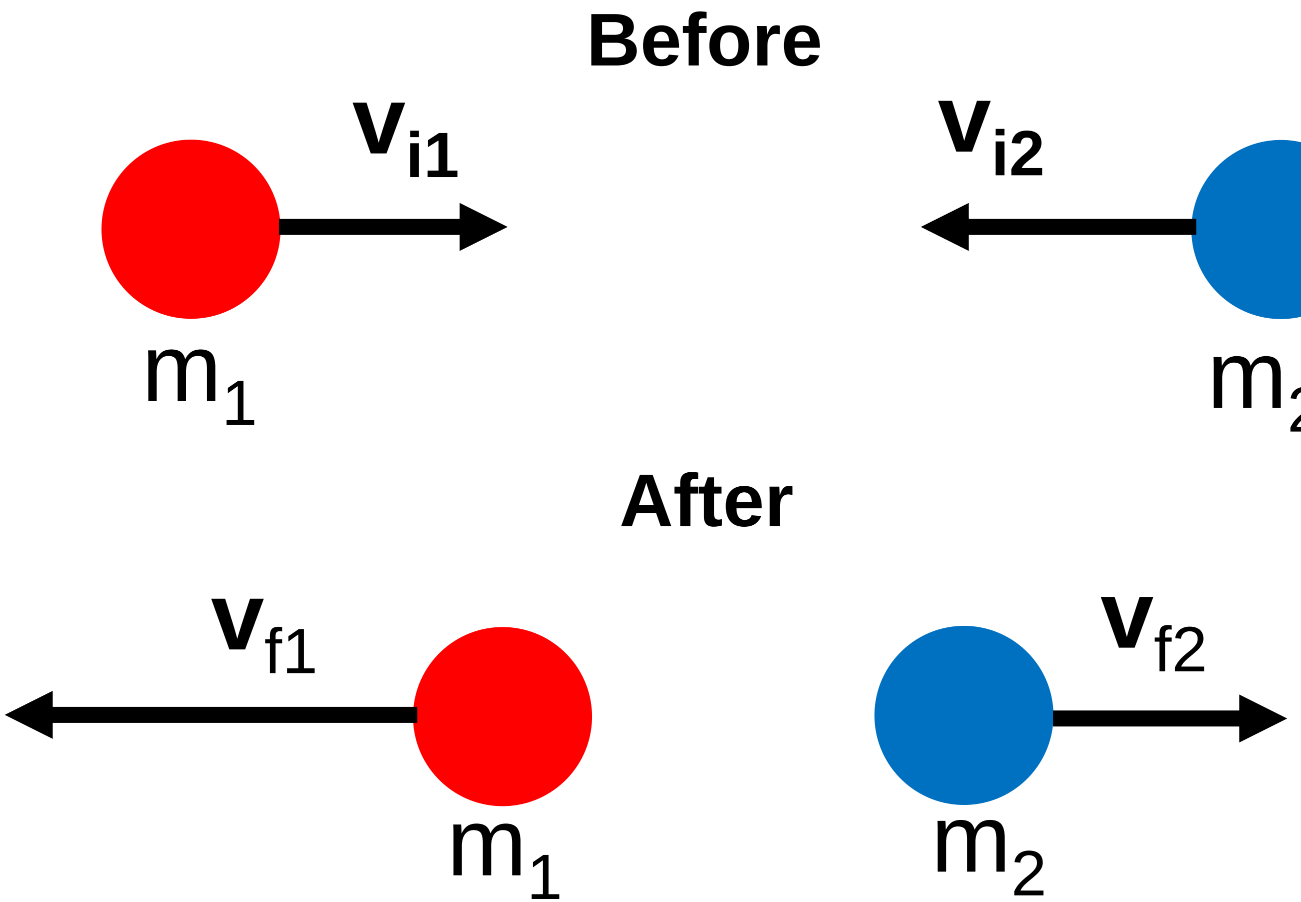
Center of Mass



Center of Mass I

A ball with mass m_1 has an initial speed v_{i1} travelling to the right. Another ball with mass m_2 has an initial speed v_{i2} travelling to the left. The balls m_1 and m_2 collide elastically. After the collision, the balls travel in the opposite directions with final speeds of v_{f1} and v_{f2} respectively. What is the total momentum of this system after the collision?

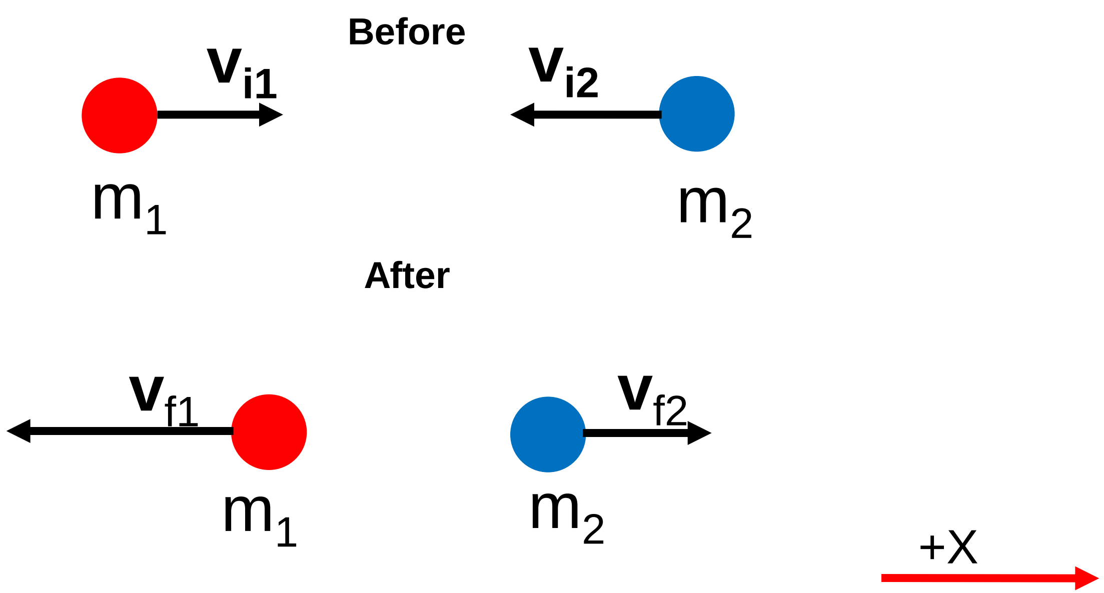
A. $m_1\mathbf{v_{i1}} + m_2\mathbf{v_{i2}}$

B. $m_1\mathbf{v_{i1}} - m_2\mathbf{v_{i2}}$

C. $m_1\mathbf{v_{i1}} + m_2\mathbf{v_{f2}}$

D. $m_1\mathbf{v_{f1}} - m_2\mathbf{v_{f2}}$

E. No idea



Left is negative and right is positive.

Solution part 1

Answer: A

Justification: Momentum is conserved. The total momentum before the collision and after the collision is the same and it equals the sum of the initial momenta (plural of momentum) or the sum of the final momenta: $\mathbf{p}_{\mathbf{tot}} = \mathbf{p}_{\mathbf{i}} = m_1\mathbf{v}_{\mathbf{i1}} + m_2\mathbf{v}_{\mathbf{i2}} = \mathbf{p}_{\mathbf{f}} = m_1\mathbf{v}_{\mathbf{f1}} + m_2\mathbf{v}_{\mathbf{f2}}$

Momentum is a vector, as is velocity. The direction of the velocity is already included within the vector (which is shown in bold). If the velocities were not in bold then the answer would be B because v_{i2} would need the negative to be shown (provided positive is to the right). However, since $\mathbf{v}_{\mathbf{i2}}$ already includes the direction and having another negative sign is repetitive.

C is mixing before and after. D has the same problem as B, it is including a negative sign when it is already included in the velocity vector.

Solution part 2 (additional example)

Justification: To help students understand, an example might be helpful…

A 3-kg object moves to the right at 4 m/s. It collides head-on with a 6-kg object moving to the left at 3 m/s. What is the total momentum?

Answer: The total momentum before the collision is equal to the total momentum after the collision. Keep in mind that momentum is a vector quantity so direction matters.

$P_i = m_1\mathbf{v_{i1}} + m_2\mathbf{v_{i2}}$ = (3kg)(4m/s)+(6kg)(-3m/s) = 12kg*(m/s)-18kg*(m/s) = -6kg*(m/s).

Total momentum before collision is 6 kg*(m/s) to the left. Which will also be the total momentum after the collision.

Units: Unlike many other physical quantities, momentum doesn't have a special name for its unit, so it is kg*m/s.

Center of Mass II

A ball with mass m_1 has an initial speed v_{i1} travelling to the right. Another ball with mass m_2 has an initial speed v_{i2} travelling to the left. m_1 and m_2 collide elastically. After the collision, the balls travel in the opposite directions with final speeds of v_{f1} and v_{f2} respectively. What is the velocity of the center of mass, v_{cm}, of this system?

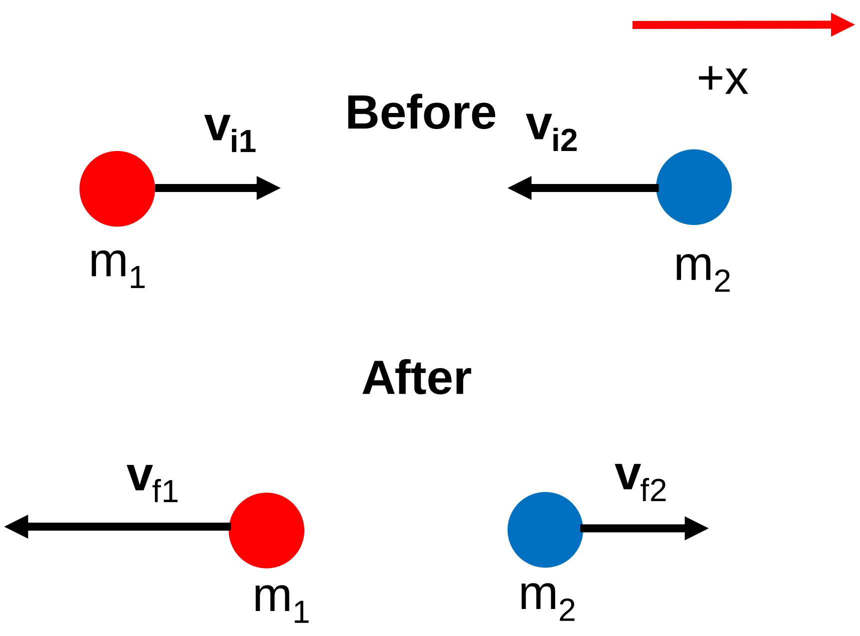
$$A.\ \frac{m_1\overrightarrow{v_{i1}}+m_2\overrightarrow{v_{i2}}}{m_1-m_2}$$

$$B.\ \frac{m_1\overrightarrow{v_{i1}}-m_2\overrightarrow{v_{i2}}}{m_1+m_2}$$

$$C.\ \frac{m_1\overrightarrow{v_{i1}}+m_2\overrightarrow{v_{i2}}}{m_1+m_2}$$

$$D.\ \frac{m_1\overrightarrow{v_{i1}}-m_2\overrightarrow{v_{i2}}}{m_1-m_2}$$

$E.$ No Idea



Solution

Answer: C

Justification: $\mathbf{p}_{tot} = M_{tot}\mathbf{v}_{cm}$, where from last slide

$\mathbf{p}_{tot} = \mathbf{p}_i = m_1\mathbf{v}_{i1} + m_2\mathbf{v}_{i2}$, therefore

$$\overrightarrow{v_{cm}} = \frac{\overrightarrow{p_{tot}}}{M_{tot}} = \frac{m_1\overrightarrow{v_{i1}} + m_2\overrightarrow{v_{i2}}}{M_{tot}}$$

M_{tot} is the total mass of the system = $m_1 + m_2$

Therefore,

$$\overrightarrow{v_{cm}} = \frac{m_1\overrightarrow{v_{i1}} + m_2\overrightarrow{v_{i2}}}{m_1 + m_2}$$

Notice if we wanted to find the magnitude of the velocity of centre of mass (the speed of the centre of mass, then we would have written:

$$v_{cm} = \left|\frac{m_1 v_{i1} - m_2 v_{i2}}{m_1 + m_2}\right|$$

Center of Mass III

In the center of mass reference, $\mathbf{v}_{\mathbf{cm}}' = 0$ (' means in center of mass reference frame). To switch to this frame of reference every velocity is decreased by $\mathbf{v}_{\mathbf{cm}}$ (for example $\mathbf{v}_{\mathbf{i1}}' = \mathbf{v}_{\mathbf{i1}} - \mathbf{v}_{\mathbf{cm}}$). What is the new expression for the total momentum of the system relative to the center of mass reference frame?

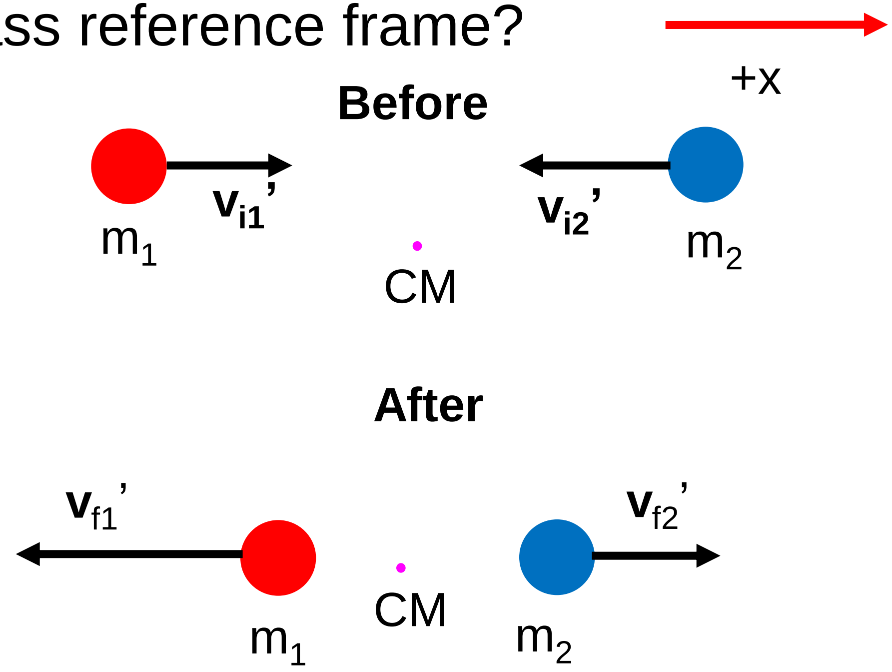
A. $m_1\mathbf{v}_{\mathbf{i1}}' + m_2\mathbf{v}_{\mathbf{i2}}' = m_1\mathbf{v}_{\mathbf{f1}}' - m_2\mathbf{v}_{\mathbf{f2}}'$

B. $m_1\mathbf{v}_{\mathbf{i1}}' + m_2\mathbf{v}_{\mathbf{i2}}' = m_1\mathbf{v}_{\mathbf{f1}}' + m_2\mathbf{v}_{\mathbf{f2}}' = 0$

C. $m_1\mathbf{v}_{\mathbf{i1}}' - m_2\mathbf{v}_{\mathbf{i2}}' = m_1\mathbf{v}_{\mathbf{i1}}' + m_2\mathbf{v}_{\mathbf{i2}}'$

D. $m_1\mathbf{v}_{\mathbf{i1}}' - m_2\mathbf{v}_{\mathbf{i2}}' = m_1\mathbf{v}_{\mathbf{i1}}' - m_2\mathbf{v}_{\mathbf{i2}}' = 0$

E. No idea



Solution

Answer: B

Justification: As in last question, $\mathbf{p}_{TOT}' = M_{tot}\mathbf{v}_{cm}'$ where $\mathbf{v}_{cm}' = 0$. Therefore, $\mathbf{p}_{tot}' = M_{tot}\mathbf{v}_{cm}' = 0$.

$\mathbf{p}_{tot}' = \mathbf{p}_i' = \mathbf{p}_f'$

$\mathbf{p}_{tot} = m1\mathbf{v}_{\mathbf{i1}}\mathbf{'} + m_2\mathbf{v}_{\mathbf{i2}}' = m_1\mathbf{v}_{\mathbf{f1}}\mathbf{'} + m_2\mathbf{v}_{\mathbf{f2}}\mathbf{'} = 0$

There should be no negative signs in the equation, since the velocity is a vector and already includes the direction.

The total equation should also equal zero.

Center of Mass IV

We know $\mathbf{p}_i' = m_1\mathbf{v}_{i1}' + m_2\mathbf{v}_{i2}'$, which is also equal to $m_1\mathbf{v}_{i1}' = -m_2\mathbf{v}_{i2}'$. The same is true for $\mathbf{p}_f' = m_1\mathbf{v}_{f1}' + m_2\mathbf{v}_{f2}'$, which is also equal to $m_1\mathbf{v}_{f1}' = -m_2\mathbf{v}_{f2}'$.

Since kinetic energy (E_k) is conserved, which statement is true for the squares of the momenta (eg, $(m_1\mathbf{v}_{i2}')^2$) if :

$$E_k = \frac{1}{2}m_1 v'^2_{i1} + \frac{1}{2}m_2 v'^2_{i2} = \frac{1}{2}m_1 v'^2_{f1} + \frac{1}{2}m_2 v'^2_{f2}$$

$$= \frac{(m_1 v'_{i1})^2}{m_1} + \frac{(m_2 v'_{i2})^2}{m_2} = \frac{(m_1 v'_{f1})^2}{m_1} + \frac{(m_2 v'_{f2})^2}{m_2}$$

A. $(m_1\mathbf{v}_{i1}')^2 = (m_1\mathbf{v}_{f1}')^2$

B. $(m_1\mathbf{v}_{i1}')^2 = -(m_2\mathbf{v}_{f2}')^2$

C. $(m_2\mathbf{v}_{i2}')^2 = -(m_1\mathbf{v}_{f1}')^2$

D. $(m_2\mathbf{v}_{i1}')^2 = (m_2\mathbf{v}_{f2}')^2$

E. No idea

Solution

Answer: A

Justification: Because momentum is conserved

$m_1\mathbf{v_{i1}}' = -m_2\mathbf{v_{i2}}'$ and $m_1\mathbf{v_{f1}}' = -m_2\mathbf{v_{f2}}'$

We plug it into $$E_k = \frac{(m_1 v'_{i1})^2}{m_1} + \frac{(m_2 v'_{i2})^2}{m_2} = \frac{(m_1 v'_{f1})^2}{m_1} + \frac{(m_2 v'_{f2})^2}{m_2}$$

$$= \frac{(m_1 v'_{i1})^2}{m_1} + \frac{(m_1 v'_{i1})^2}{m_2} = \frac{(m_1 v'_{f1})^2}{m_1} + \frac{(m_1 v'_{f1})^2}{m_2}$$

Because kinetic energy is conserved, the m_1 terms on both sides must match up, the terms m_2 must match as well:

$$\frac{(m_1 v'_{i1})^2}{m_1} = \frac{(m_1 v'_{f1})^2}{m_1}$$

$$(m_1 v'_{i1})^2 = (m_1 v'_{f1})^2$$

Center of Mass V

When you take the square root of both sides of $(m_1\mathbf{v}_{\mathbf{i1}}')^2=(m_1\mathbf{v}_{\mathbf{f1}}')^2$ the solutions are $m_1\mathbf{v}_{\mathbf{i1}}=m_1\mathbf{v}_{\mathbf{f1}}'$ (no collision) and $m_1\mathbf{v}_{\mathbf{i1}}'=-m_1\mathbf{v}_{\mathbf{f1}}'$. The second solution suggests that in the reference frame of the center of mass, the velocities simply reverse in direction after the collision. What would be the correct equation for $\mathbf{v}_\mathbf{f}$ in terms of $\mathbf{v}_\mathbf{i}$ and $\mathbf{v}_{\mathbf{cm}}$?

hint: $v_f' = v_f - v_{cm}$, $v_i' = v_i - v_{cm}$, $v_i' = -v_f'$

A. $\mathbf{v}_\mathbf{f} = 2\mathbf{v}_{\mathbf{cm}}-\mathbf{v}_\mathbf{i}$

B. $\mathbf{v}_\mathbf{f} = \mathbf{v}_\mathbf{i}-2\mathbf{v}_{\mathbf{cm}}$

C. $\mathbf{v}_\mathbf{f} = 2\mathbf{v}_{\mathbf{cm}}+\mathbf{v}_\mathbf{i}$

D. $\mathbf{v}_\mathbf{f} = -2\mathbf{v}_{\mathbf{cm}}-\mathbf{v}_\mathbf{i}$

E. No idea

Solution

Answer: A

Justification: We know $\mathbf{v}_i' = \mathbf{v}_\mathbf{i} - \mathbf{v}_\mathbf{cm}$ and $\mathbf{v}_\mathbf{f}' = \mathbf{v}_\mathbf{f} - \mathbf{v}_\mathbf{cm}$

Since $\mathbf{v}_\mathbf{i}' = -\mathbf{v}_\mathbf{f}'$, then $(\mathbf{v}_\mathbf{i} - \mathbf{v}_\mathbf{cm}) = -(\mathbf{v}_\mathbf{f} - \mathbf{v}_\mathbf{cm}) = -\mathbf{v}_\mathbf{f} + \mathbf{v}_\mathbf{cm}$

By rearranging $\mathbf{v}_\mathbf{f} = \mathbf{v}_\mathbf{cm} + \mathbf{v}_\mathbf{cm} - \mathbf{v}_\mathbf{i} = 2\mathbf{v}_\mathbf{cm} - \mathbf{v}_\mathbf{i}$