a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Physics
Momentum: Collisions

Science and Mathematics
Education Research Group

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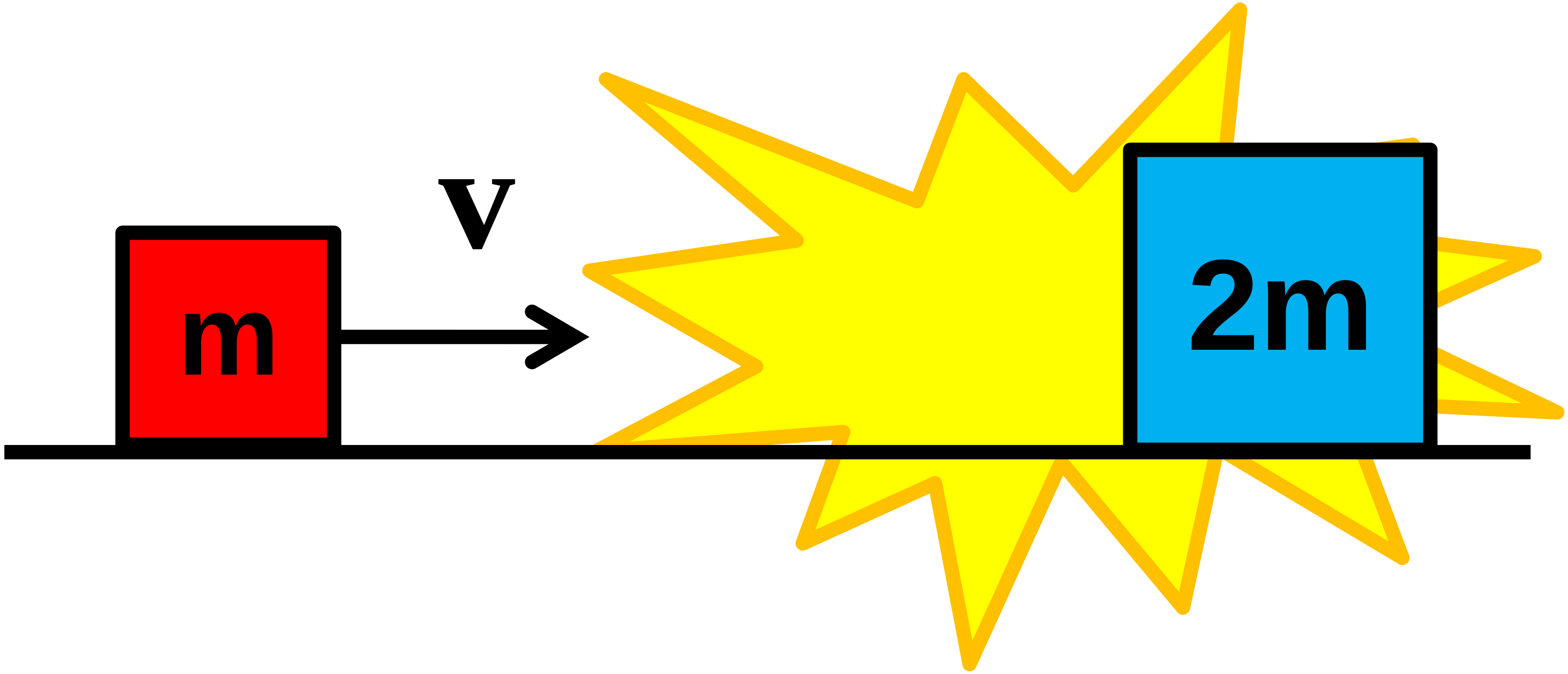
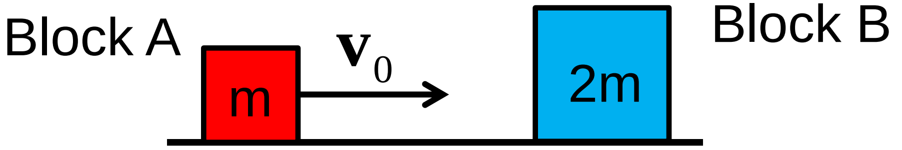
Two Block Collisions



Colliding Blocks I

Block A has an initial velocity $\mathbf{v}_0$. Block B is stationary and has twice the mass of block A.



Block A collides into Block B. Consider the system consisting of only blocks A and B. Which of the following forces are internal?

A. The force Block A exerts onto Block B and vise-versa, due to the collision: $\mathbf{F}_{\mathbf{AonB}}$ and $\mathbf{F}_{\mathbf{BonA}}$

B. The gravitational force the earth exerts onto the blocks, $\mathbf{F}_{\mathbf{gonA}}$ and $\mathbf{F}_{\mathbf{gonB}}$

C. The normal force the table exerts onto the blocks, $\mathbf{F}_{\mathbf{NonA}}$ and $\mathbf{F}_{\mathbf{NonB}}$

D. The friction force exerted by the table onto the blocks, $\mathbf{F}_{\mathbf{friconA}}$ and $\mathbf{F}_{\mathbf{friconB}}$

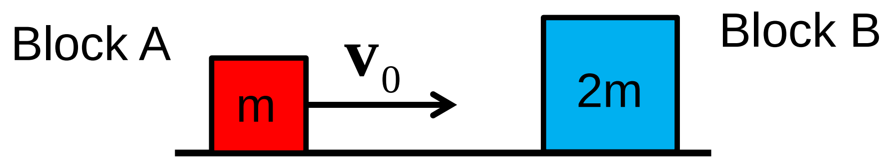
E. All of the above

Solution

Answer: A

Justification: The forces internal to the system are the forces exerted by the objected included in the system. In our case these are the forces blocks exert onto each other since the system consists only of Block A and Block B. The forces exerted by objects in the surroundings (such as the earth and table) onto the system are the external forces.

Colliding Blocks II



During the collision, Block A exerts a force $\mathbf{F}_{\mathbf{AonB}}$ onto Block B, while Block B exerts a force $\mathbf{F}_{\mathbf{BonA}}$ onto Block A. Which of the following correctly describes the relationship between $|\mathbf{F}_{\mathbf{AonB}}|$ and $|\mathbf{F}_{\mathbf{BonA}}|$?

A. $|\mathbf{F}_{\mathbf{BonA}}| = 0.5|\mathbf{F}_{\mathbf{AonB}}|$

B. $|\mathbf{F}_{\mathbf{BonA}}| = |\mathbf{F}_{\mathbf{AonB}}|$

C. $|\mathbf{F}_{\mathbf{BonA}}| = 2|\mathbf{F}_{\mathbf{AonB}}|$

D. $|\mathbf{F}_{\mathbf{BonA}}| = 4|\mathbf{F}_{\mathbf{AonB}}|$

Solution

Answer: B

Justification: According to Newton's Third Law:

$$\mathbf{F}_{B\,on\,A} = -\mathbf{F}_{A\,on\,B}$$

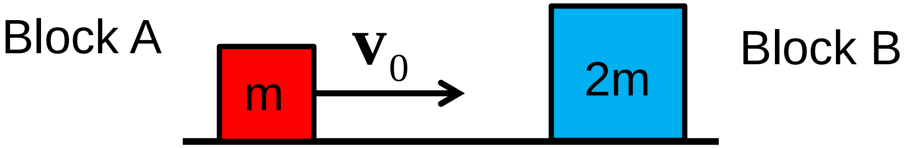
$$\left|\mathbf{F}_{B\,on\,A}\right| = \left|\mathbf{F}_{A\,on\,B}\right|$$

Therefore, internal forces cannot generate a net force on a system and thus cannot change the momentum of the system (the two blocks). Assuming no friction and $\mathbf{F_g} = -\mathbf{F_N}$, there will be no external net force on the system.

$$\mathbf{F}_{net} = \frac{\Delta \mathbf{p}_{sys}}{\Delta t} = \mathbf{0}$$

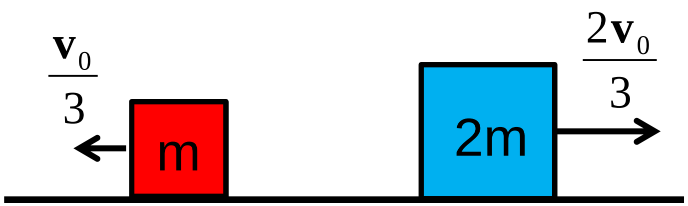
Colliding Blocks III

Initial State:

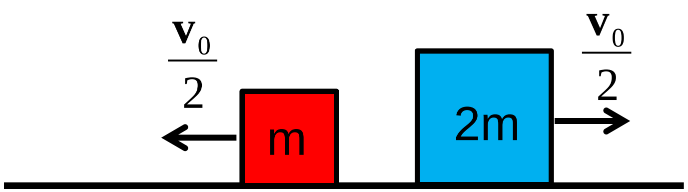


According to the law of momentum conservation, which of the following is NOT a possible final state after the collision? Assume no friction.

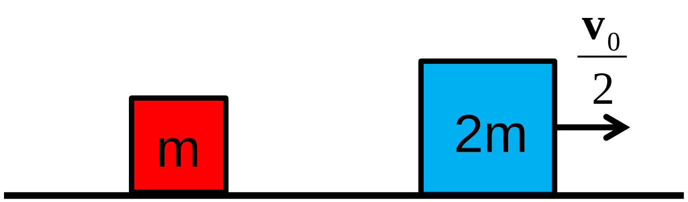
A.



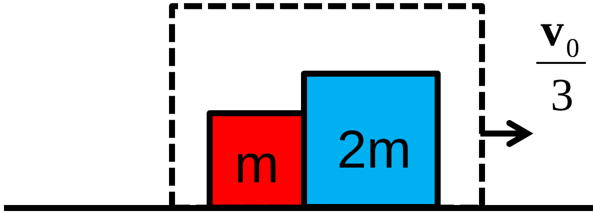
B.



C.



D.



Solution

Answer: B

Justification: Momentum is conserved in all the collisions except for B.

$$\mathbf{p}_i = m\mathbf{v}_0$$

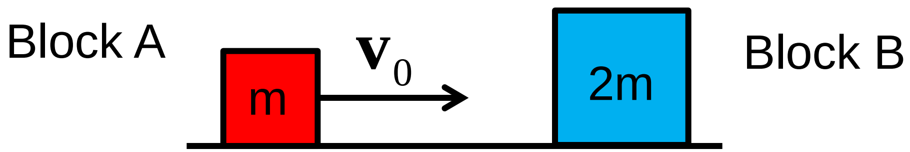
$$\mathbf{p}_f = m\frac{(-\mathbf{v}_0)}{2} + 2m\frac{\mathbf{v}_0}{2} = \frac{m\mathbf{v}_0}{2}$$

Since the final momentum of the system does not equal the initial momentum, this collision outcome is not possible.

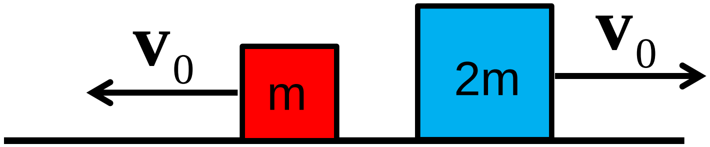
Additionally, the final kinetic energy in B is greater than the initial kinetic energy of the system. The mechanical energy of the system cannot increase as a result of collision, it can only decrease.

Colliding Blocks IV

Initial State:



Jeremy must predict the final state of the collision. After taking conservation of momentum into consideration, Jeremy comes up with the following final state:



Is this collision outcome possible?

A. Yes

B. No

C. Cannot be determined

Solution

Answer: B

Justification: Momentum is conserved in the proposed collision. However, the final kinetic energy is 3 times larger than the initial kinetic energy.

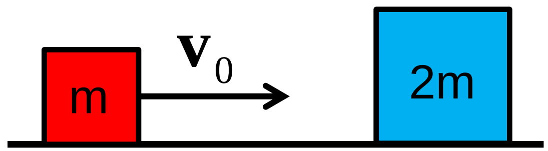
$$E_{k,i} = \frac{1}{2} m \left|\mathbf{v}_0\right|^2$$

$$E_{k,f} = \frac{1}{2} m \left|\mathbf{v}_0\right|^2 + \frac{1}{2}(2m)\left|\mathbf{v}_0\right|^2 = \frac{3}{2} m \left|\mathbf{v}_0\right|^2$$

The final kinetic energy may be less than or equal to the initial kinetic energy, but not greater.

Colliding Blocks V

Initial state: Block A

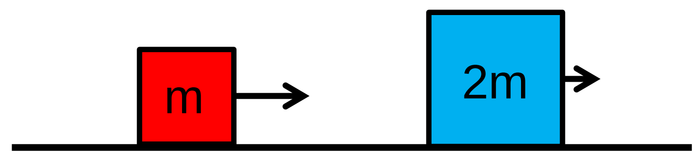


Block B

Final states:

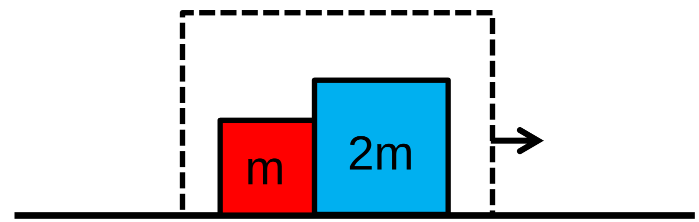
1.

2.



3.

4.



Rank the final states from greatest to least kinetic energy.
(Momentum is conserved in all these collisions)

A. $E_{k,\,1} > E_{k,\,3} > E_{k,\,2} > E_{k,\,4}$

B. $E_{k,\,3} > E_{k,\,1} > E_{k,\,2} > E_{k,\,4}$

C. $E_{k,\,1} > E_{k,\,2} > E_{k,\,3} > E_{k,\,4}$

D. $E_{k,\,1} > E_{k,\,3} > E_{k,\,4} > E_{k,\,2}$

Solution

Answer: A $E_{k,1} > E_{k,3} > E_{k,2} > E_{k,4}$

Justification: Kinetic energy is a scalar quantity that depends on speed, unlike momentum which depends on velocity.

Collision 1 has the most kinetic energy since Block A is moving left. In order to conserve momentum, Block B must have a large velocity towards the right.

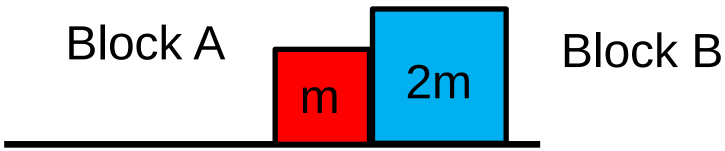
In collision 3, Block A is no longer moving left but is now stationary. Block B must have a smaller velocity to the right than in collision 1 in order to conserve momentum because of its larger mass.

In collision 2, both blocks are moving to the right. Thus both blocks must be moving slowly in order to have a final momentum equal to mv_0.

Collision 4 has the least kinetic energy since it is the maximally inelastic case where the two blocks stick together after the collision.

Colliding Blocks VI

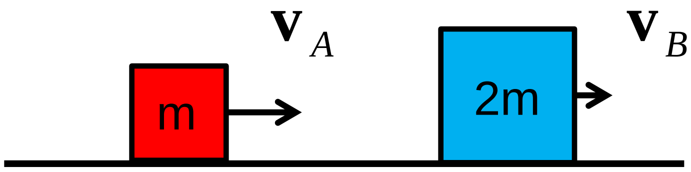
Final state:



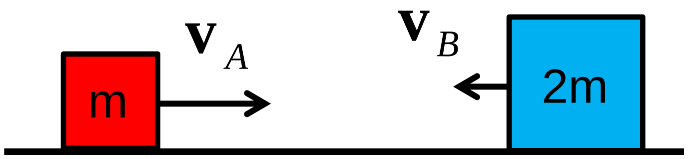
Suppose that after Block A and Block B collide, both blocks are at rest. All the kinetic energy has been transferred into other forms of energy such as heat. Which of the following must have been the initial state of the collision?

A.

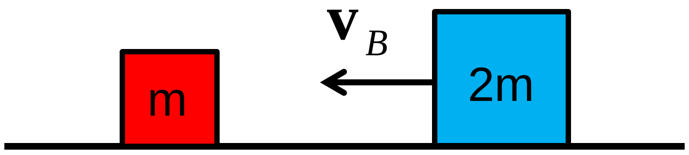
B.



C.



D.



E. All of the above initial states are possible

Solution

Answer: C

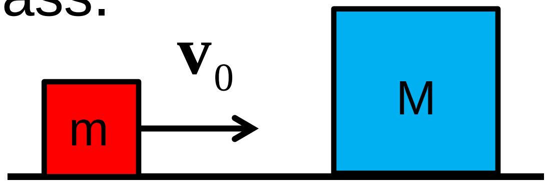
Justification: In the final state with both Block A and B at rest, there is no momentum and no kinetic energy. The initial momentum of the two blocks before the collision must have been zero.

Collisions A, B, and D are not possible because they have an initial momentum.

In collision B, Block A and Block B are moving in opposite directions. If Block A is moving twice as fast as Block B, then it is possible for the initial momentum to be zero.

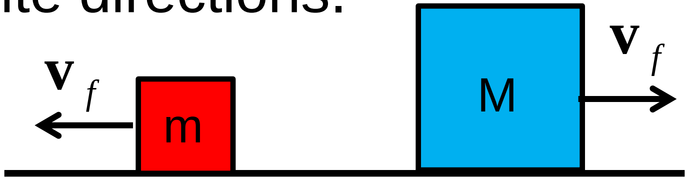
Colliding Block VII (Hard)

A block with initial velocity $\mathbf{v}_0$ is sliding towards a larger stationary block with unknown mass.



Initial state

After they collide *elastically*, the two blocks are moving at the same speed but in opposite directions.



Final state

Based on this information, what must be true about the relative masses of M and m?

A. M = m
B. M = 2m
C. M = 3m
D. M = 4m
E. M = ½ m

Solution (Part 1)

Answer: C

Justification:

Define x such that $M = xm$.
From the law of conservation of momentum:

$$mv_0 = -mv_f + xmv_f$$

$$v_0 = -v_f + xv_f$$

$$v_0 = (x-1)v_f \quad \text{Equation 1}$$

From conservation of energy (collision is elastic):

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}xmv_f^2$$

$$v_0^2 = v_f^2 + xv_f^2$$

$$v_0^2 = (x+1)v_f^2 \quad \text{Equation 2}$$

Solution (Part 2)

Substituting equation 1 into 2 gives:

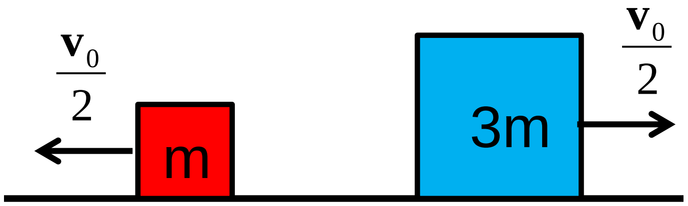
$$(x-1)^2 v_f^2 = (x+1) v_f^2$$

$$(x-1)^2 = (x+1) \qquad \text{since } v_f \neq 0$$

$$x^2 - 3x = 0$$

$$x = 3 \quad or \quad x = 0$$

Therefore, mass M must be 3 times as large as mass m.



Substituting *x* = *3* back into equation 1 shows that both the small mass and the larger mass will be moving at half the initial velocity.