a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Physics
Special Relativity

Science and Mathematics
Education Research Group

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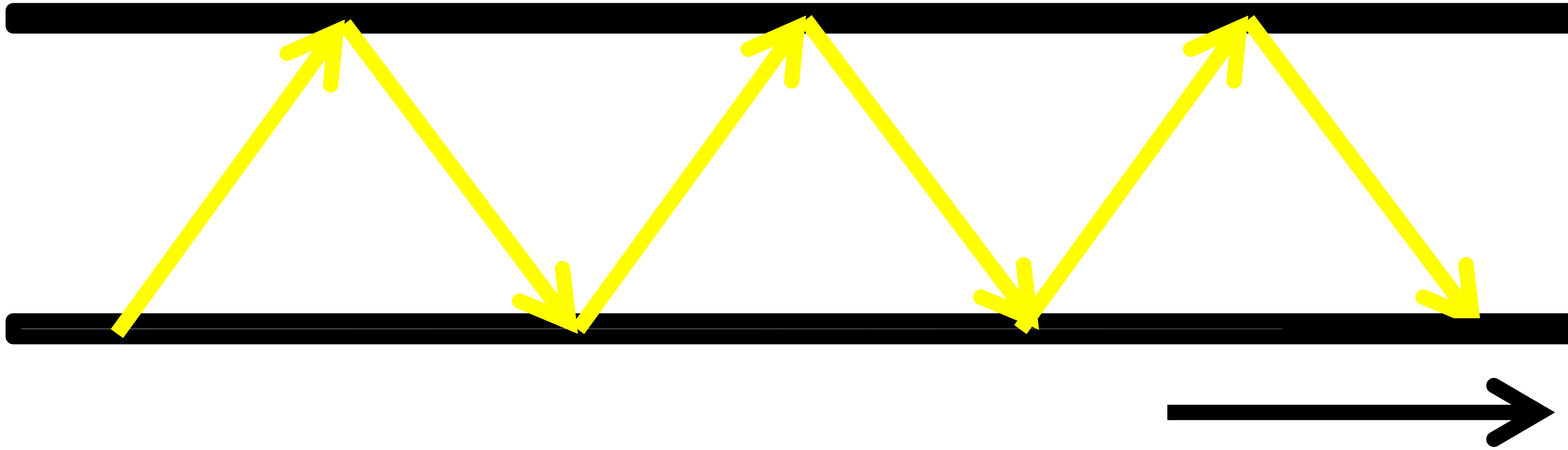
Trapped Light

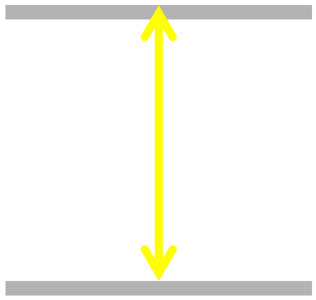


Trapped Light I

A spaceship has a peculiar clock which consists of a light beam trapped between two parallel mirrors. According to a crew member on the spaceship, every 300 million cycles the clock emits a flash signifying 1 second has passed. How far are the two mirrors?

A. 0.5 m
B. 1 m
C. 1.5 m
D. 2 m
E. None of the above

Assume that the speed of light is equal to 300 million m/s



Solution

Answer: A

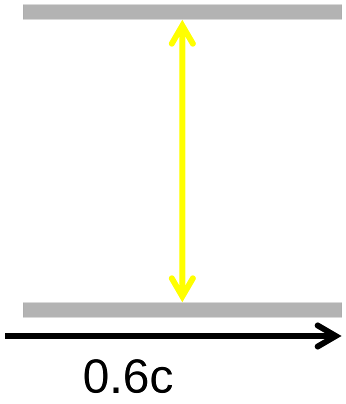
Justification: Consider the situation in the reference frame of the crew member on the spaceship. In one second, light travels 3×10^8 m ≈ 300 000 000 m. The clock only emits a pulse every 300 million cycles. The light goes from one side to the other and back during one cycle.

The distance between the mirrors is 0.5 m as the light has to travel 1 m per cycle if the clock is to emit a pulse once every 300 million cycles.

Trapped Light II

The spaceship now moves to the right at a constant speed of 0.6c, where c is the speed of light. From the point of view of a crew member on the spaceship, what is the period of flashes from the clock?

A. 0.8 s
B. 1 s
C. 1.25 s
D. 2 s
E. None of the above



Solution

Answer: B

Justification: The observer is moving in the same reference frame as the clock, so for him, the clock is still at rest and nothing changed compared to the situation when the spaceship was at rest relatively to Earth. According to the special theory of relativity the speed of light is constant in all inertial reference frames. Therefore for him, the clock still flashes every second like it would if the space ship was not moving.

Trapped Light III

A. 0.8 s

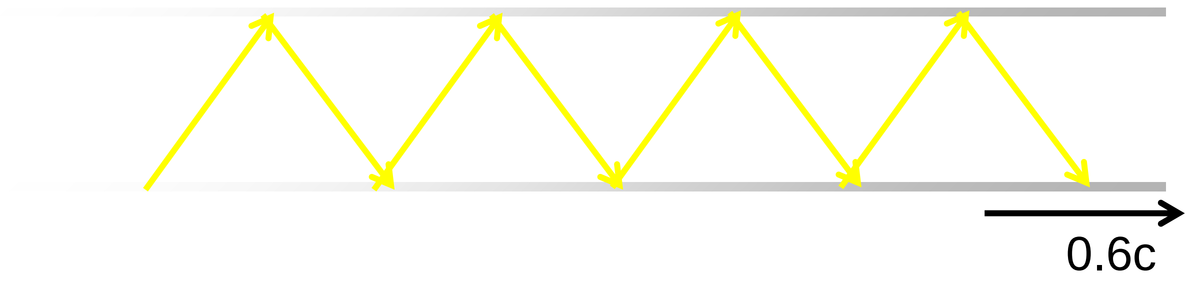
B. 1 s

C. 1.25 s

D. 2 s

E. None of the above

The spaceship now moves at a constant speed of 0.6c to the right. An outside observer may claim that the clock is now inaccurate, as the light appears to travel in a diagonal instead of a straight line. From the point of view of the observer, what is the period of flashes of this clock?



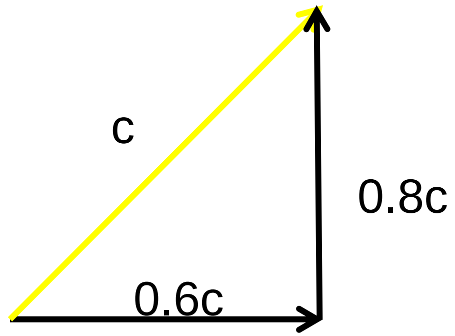
Solution

Answer: C

Justification: The clock emits a pulse every time the light has travelled 300 million cycles, or 300000 km. Since the speed of light is constant in every inertial reference frame, the vertical component of velocity of the light beam according to the outside observer is equal to $\sqrt{c^2-(0.6c)^2}=0.8c$ (using Pythagorean Theorem).

The period of 300 million cycles to an outside observer is therefore

$$\frac{300000000m}{0.8c}=1.25s$$



Trapped Light IV

A. $\gamma = \sqrt{c^2 - v^2}$

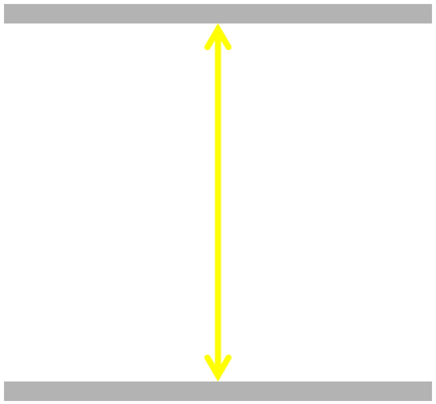
B. $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

C. $\gamma = \dfrac{c}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

D. $\gamma = \dfrac{1}{\sqrt{1 + \dfrac{v^2}{c^2}}}$

E. none of the above

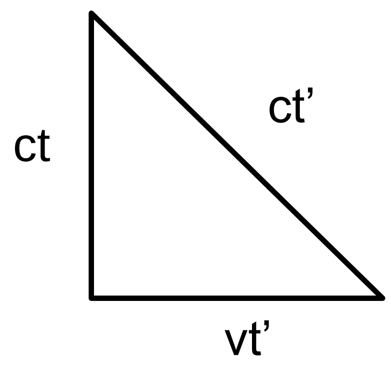
Inside the spaceship, according to a space ship crew member, light travels in a straight line and at the same constant speed. However, the person inside the spaceship sees the clock pulse every second while the person on the outside sees the clock pulse every 1.25 seconds. A possible conclusion would be that the time has slowed inside the spaceship by a factor of γ. What is the equation for γ in terms of velocity and the speed of light?



Solution

Answer: B

Justification: From the previous question we know that the vertical velocity of the light from the observer's perspective is $\sqrt{(c^2-v^2)}$. The vertical distance that the light travels in time t is ct.



$$(ct')^2 = (ct)^2 + (vt')^2$$

$$c^2t'^2 = c^2t^2 + v^2t'^2$$

$$c^2t'^2 - v^2t'^2 = c^2t^2$$

$$t'^2\left(c^2 - v^2\right) = c^2t^2$$

$$t'^2 = \frac{c^2t^2}{c^2 - v^2} \rightarrow t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Note: t is the time in the reference frame of the space ship, while t' is the time in the reference frame of the outside observer.