a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Physics
Vector Components

Science and Mathematics Education Research Group

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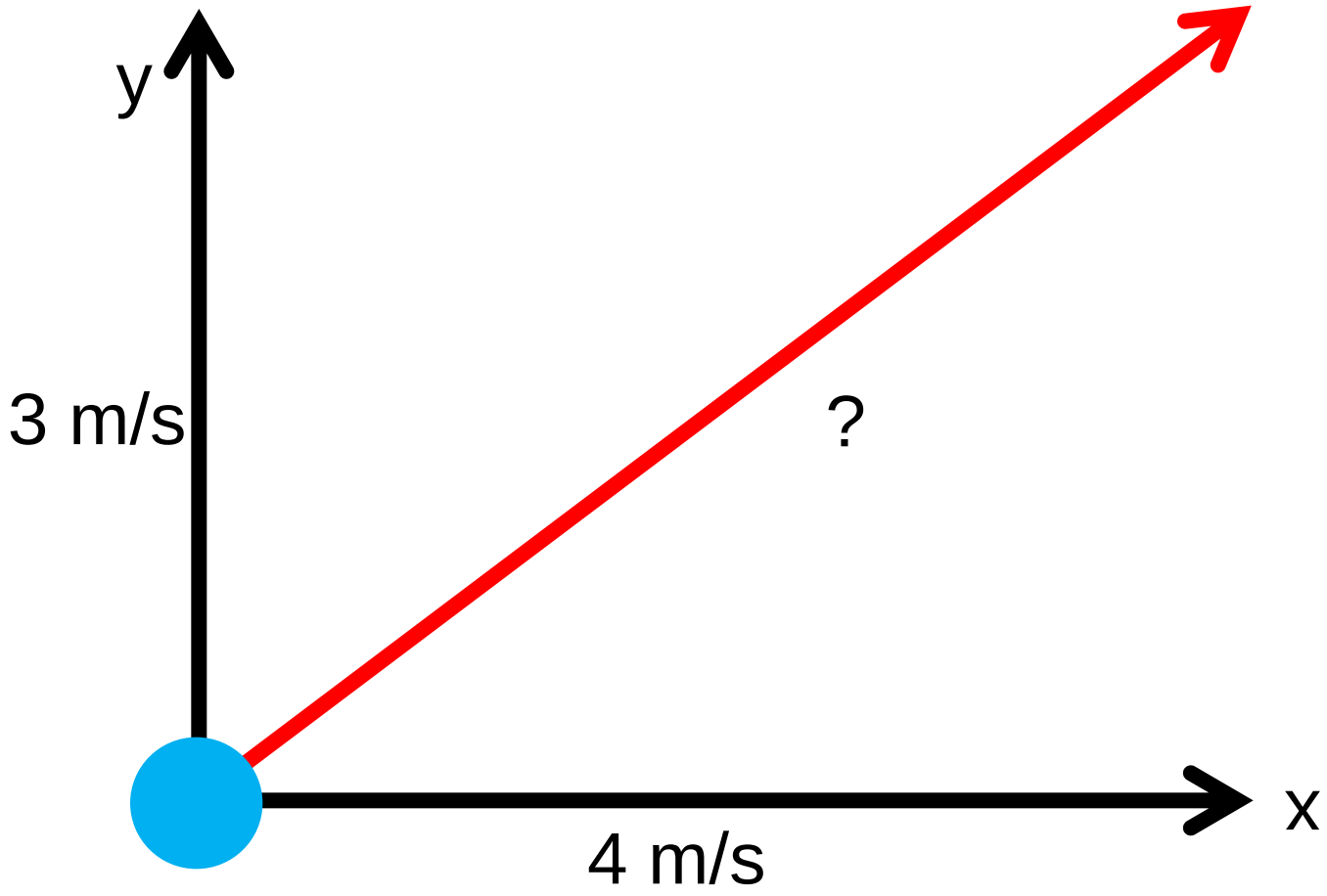
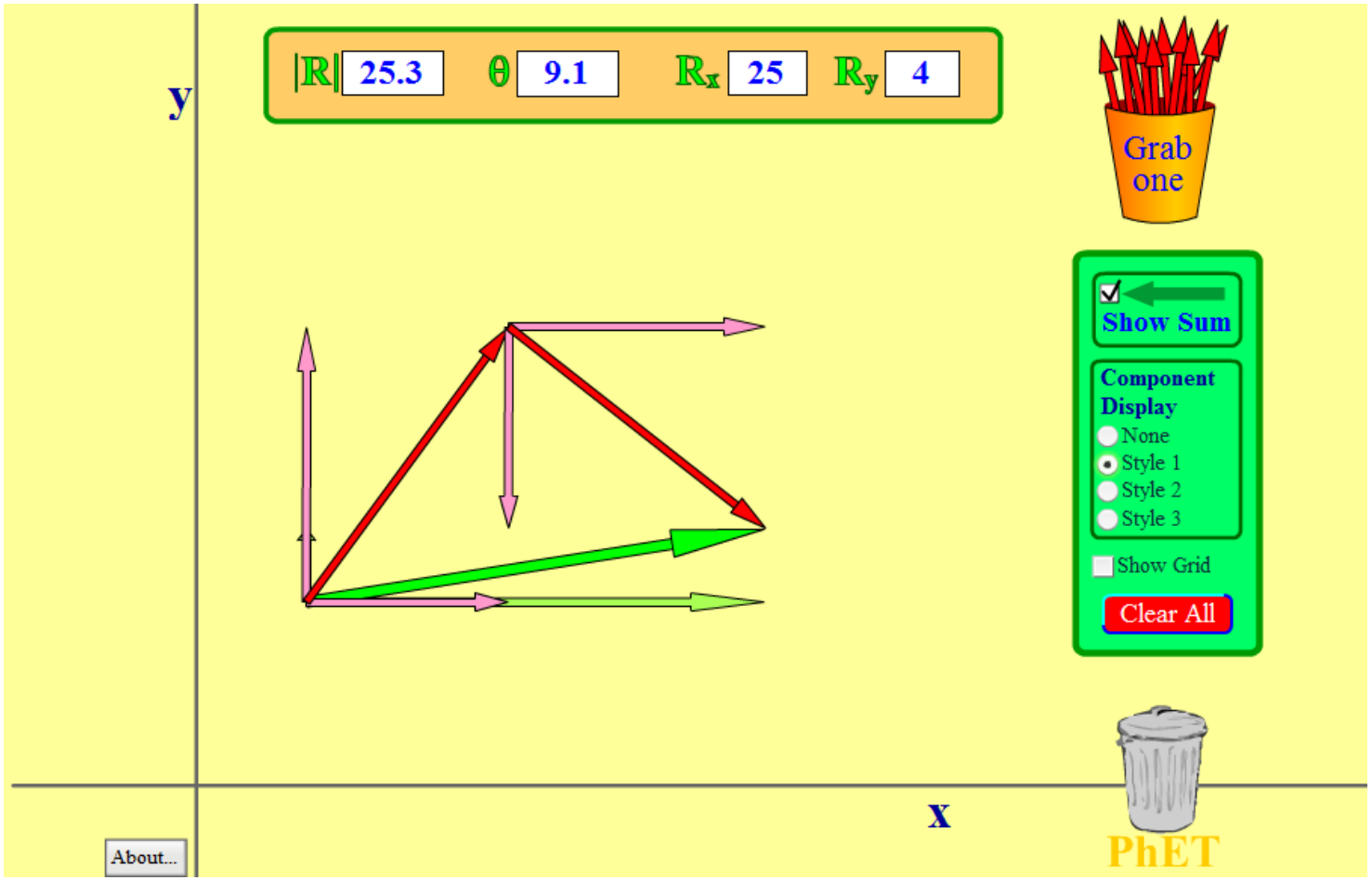
Vector Components



In this question set, any bolded variable is considered a vector.

Vector Addition with PhET



http://phet.colorado.edu/en/simulation/vector-addition

Vector Components I

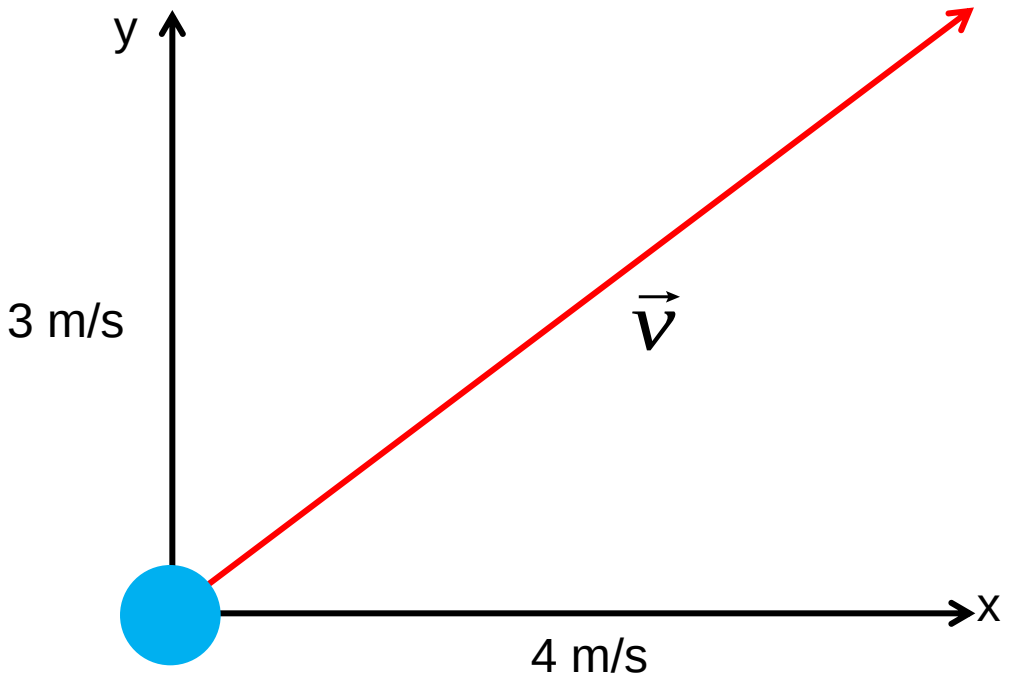
A ball is travelling with velocity **v**. The x-component of the velocity is 4 m/s, and the y-component is 3 m/s. What is the magnitude of the velocity of the ball?

A. 3 m/s

B. 4 m/s

C. 5 m/s

D. 7 m/s



Solution

Answer: C, 5 m/s

Justification: Answers A and B are the magnitudes of the individual components of the resultant vector.

When adding vectors, to determine the magnitude of the resulting vector, you cannot just add the magnitudes of the two vectors.

The magnitude of a vector can be found by applying Pythagoras' theorem to its components.

$v^2 = (4\ m/s)^2 + (3\ m/s)^2$

$v^2 = 16\ m^2/s^2 + 9\ m^2/s^2$

$v = 5\ m/s$

Vector Components II

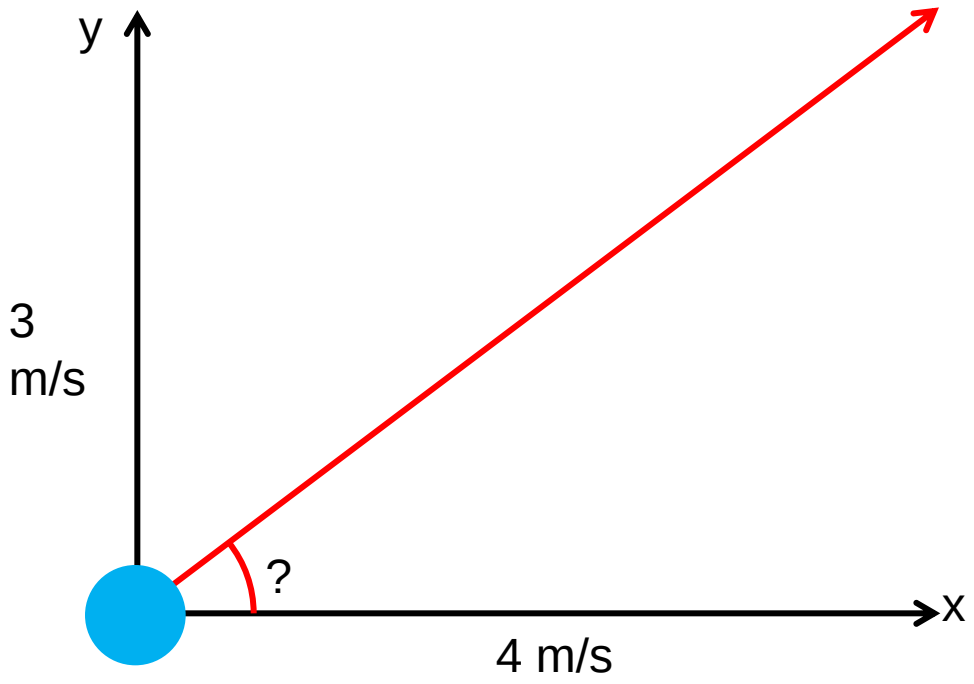
A. 12°

B. 28°

C. 37°

D. 53°

A ball is travelling 4 m/s in the x direction and 3 m/s in the y direction. At what angle is the ball moving above the x-axis?



Solution

Answer: C, 37°

Justification: The tangent of the angle between the vector and the x-axis is equal to the ratio of the y component to the x component.

From Pythagorean theorem, v_x= 4 m/s, v_y = 3 m/s, v= 5 m/s . We have three alternative solutions that give the same answer:

$$\tan\theta = \frac{v_y}{v_x} = \frac{3\ m/s}{4\ m/s} = 0.75 \Rightarrow \theta = \tan^{-1}(0.75) = \arctan(0.75) = 0.644 \text{ rad}=37^\circ$$

$$\sin\theta = \frac{v_y}{v} = \frac{3\ m/s}{5\ m/s} = 0.6 \Rightarrow \theta = \sin^{-1}(0.6) = \arcsin(0.6) = 0.644 \text{ rad}=37^\circ$$

$$\cos\theta = \frac{v_x}{v} = \frac{4\ m/s}{5\ m/s} = 0.8 \Rightarrow \theta = \cos^{-1}(0.8) = \arccos(0.8) = 0.644 \text{ rad}=37^\circ$$

Vector Components III

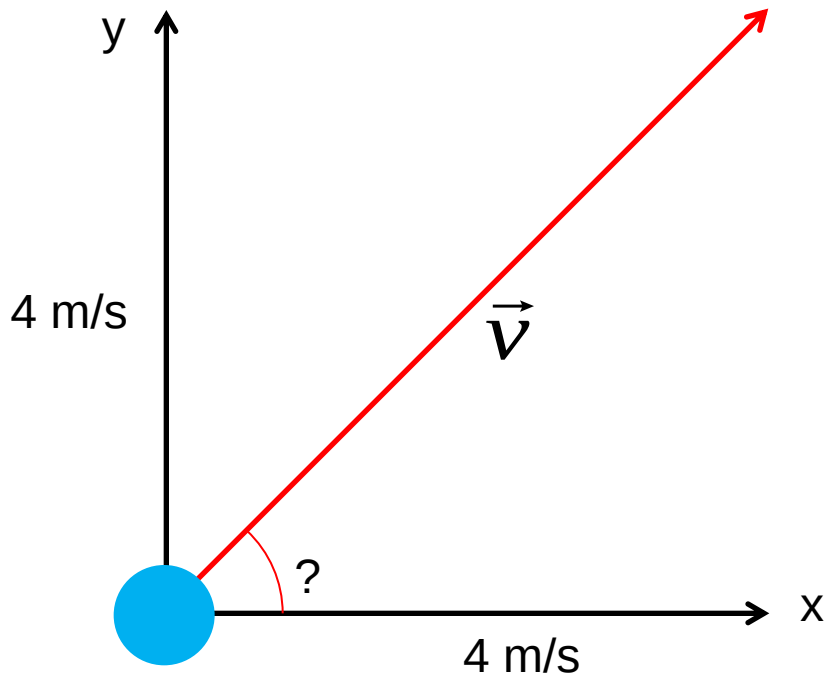
A ball is travelling with velocity **v**. The x-component of the velocity is 4 m/s and the y-component is 4 m/s. What is the magnitude of the velocity of the ball?

A. 2.8 m/s

B. 5 m/s

C. 5.7 m/s

D. 8 m/s



Solution

Answer: C, 5.7 m/s

Justification: The vector components form a right angle triangle. Pythagorean Theorem can be applied to obtain the length of the resultant vector.

$v^2 = (4\ m/s)^2 + (4\ m/s)^2$

$v^2 = 32\ m^2/s^2$

$v = 5.7\ m/s$

Vector Components IV

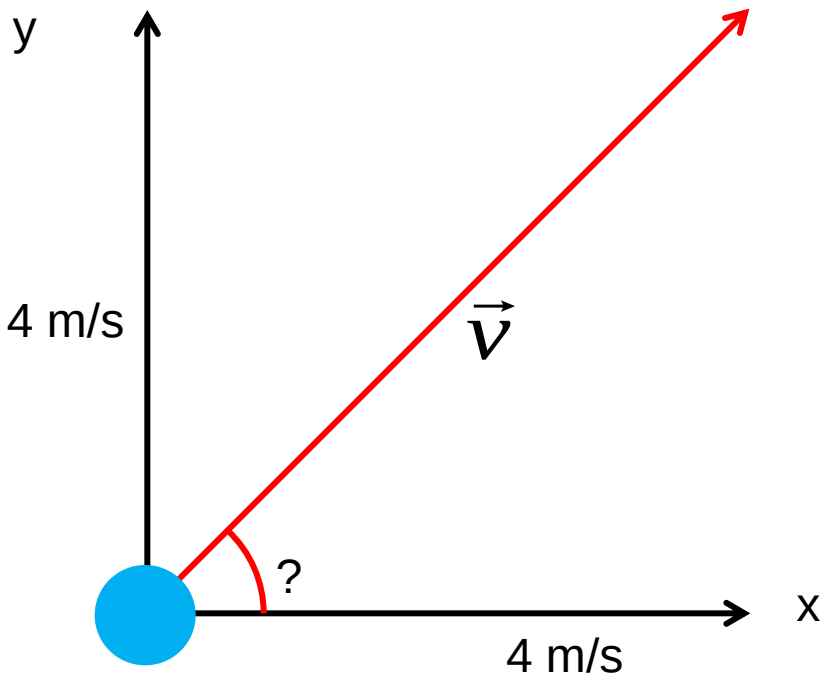
A ball is travelling 4 m/s in the x direction and 4 m/s in the y direction. At what angle is the ball moving above the x-axis?

A. 15°

B. 30°

C. 45°

D. 60°



Solution

Answer: C

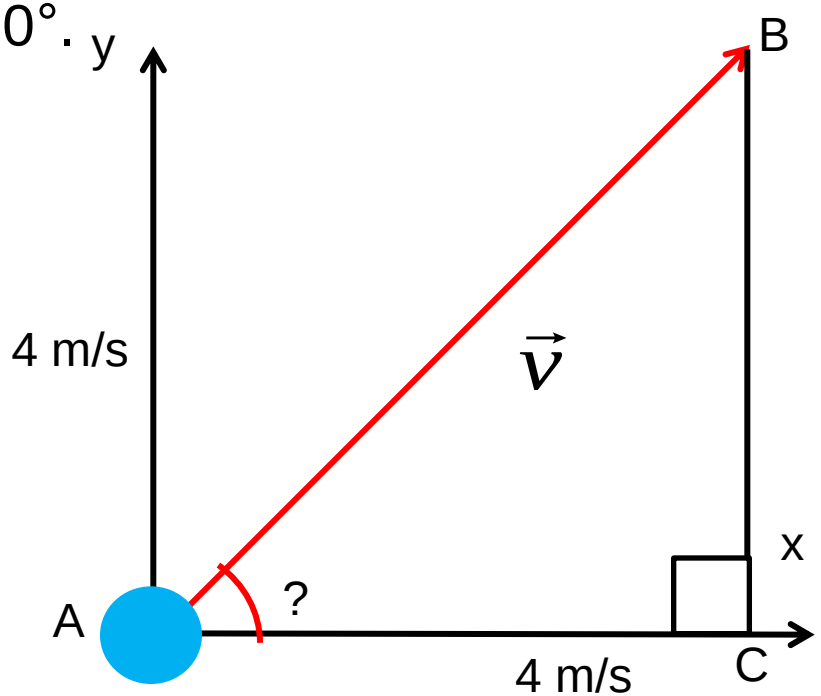
Justification: The vector components form an isosceles, right angle triangle. In an isosceles triangle, the angles across from the equal sides are also equal. The angles in a triangle must always add up to 180°.

$180° = 90° + A + B$

$180° = 90° + 2A$

$90° = 2A$

$45° = A$



Vector Components V

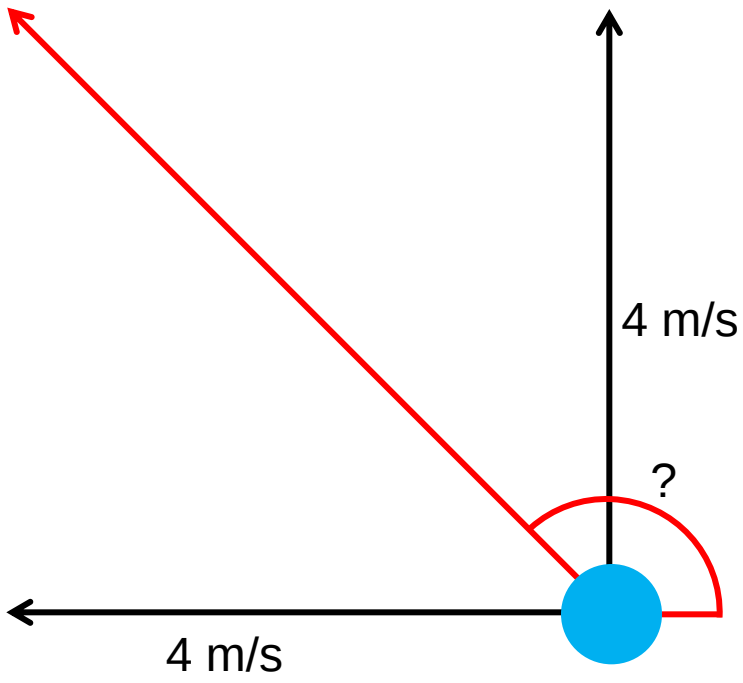
A. 45°

B. 135°

C. 225°

D. 315°

A ball is travelling 4 m/s in the -x direction and 4 m/s in the y direction. At what angle is the ball moving relative to the positive side of the x-axis?



Solution

Answer: B

Justification: Similar to the previous question, the vector components form a right angle isosceles triangle. This means that the angles between the vector and the negative x-axis and positive y-axis are both 45°. Because we are looking for the angle between the vector and the positive x-axis, we need to consider the distance between the positive y-axis and positive x-axis, which is 90°. Adding the two angles together gives 135°.

If the vector were in the third quadrant, the answer would be 225°, and if the vector were in the fourth quadrant the answer would be 315°.

Vector Components VI

A ball is travelling with the velocity *v*. What are the x- and y- components of the velocity vector?

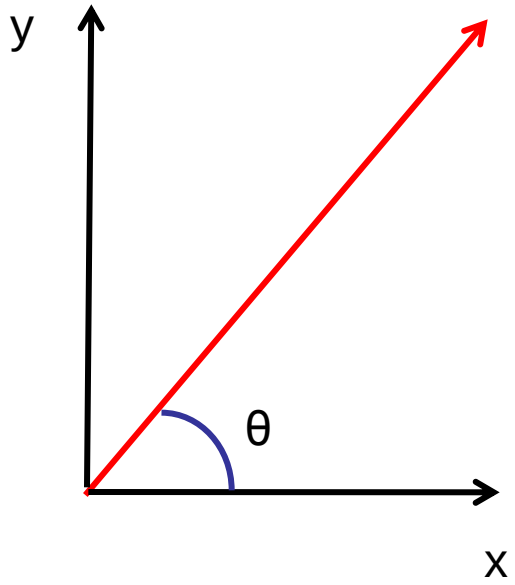
A. $v_x = v\sin\theta;\ v_y = v\tan\theta$

B. $v_x = v\sin\theta;\ v_y = v\cos\theta$

C. $v_x = v\cos\theta;\ v_y = v\sin\theta$

D. $v_x = v\tan\theta;\ v_y = v\cos\theta$

E. $v_x = v\tan\theta;\ v_y = v\sin\theta;$



Solution

Answer: C

Justification: In this scenario, $\mathbf{v}_x$ is the vector adjacent to the angle and $\mathbf{v}_y$ is the vector opposite to the angle.

sin θ = opposite/hypotenuse

cos θ = adjacent/hypotenuse

tan θ = opposite/adjacent

Since tan θ involves both $\mathbf{v}_y$ and $\mathbf{v}_x$, it will give us no information about the individual components.

sin θ = $\mathbf{v}_x$ /**v**, rearrange to solve for $\mathbf{v}_x$ = **v**sinθ

cos θ = $\mathbf{v}_y$/**v**, and rearranging this gives $\mathbf{v}_y$ = **v**cosθ