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FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

# Mathematics
# Relations and Functions: Factoring Polynomials

Science and Mathematics
Education Research Group

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# Factoring Polynomials I

$a(b+c) = ab + ac$ *Multiplying*

$ab + ac = a(b+c)$ *Factoring*

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# Multiplying Polynomials I

Factor completely:

$$2x^2y - 6x$$

A. $2(x^2y - 3x)$

B. $2(x^2y - 6x)$

C. $2x(xy - 3)$

D. $2x(xy - 6)$

E. $6x^2y$

# Solution

**Answer:** C

**Justification:**

In order to factor polynomials you need to write the polynomials into a multiplication statement of their factors. In order to find the factored form, we have to find the Greatest Common Factor (GCF) of the terms first.

In order to find the GCF between $2x^2y$ and $6x$, these are the rules we should follow:

1. For the coefficients, we can simply find the GCF between them.
2. For the variables, the GCF is the multiple of any common variables with the lower exponent.

# Solution Cont'd

Thus, following the procedure:

1. The GCF between two coefficients of 2 and 6 is 2.
2. The GCF between $x^2y$ and $x$ is $x$.

Multiplying these two together, the final GCF we want to factor from $2x^2y - 6x$ is $2x$.

Thus, our factored form must be $2x(xy - 3)$. Our answer is C.

For option A, notice that the expression can be factored further. Thus, it cannot be our ideal solution!

Other options are incorrect, because they are not equivalent to the original expression.

# Factoring Polynomials II

Factor completely:

$$6x^3y^5 - 9x^4y^2$$

A. $3x^{-1}y^{-3}(2y^3 - 3x)$

B. $-3x^4y^5(-2x^3 + 3y)$

C. $3x^3y^2(2x^3 - 3y)$

D. $3x^3y^2(2y^3 - 3x)$

E. $-3x^2y^3(-2y^3 + 3x)$

# Solution

**Answer:** D

**Justification:**

In order to find the GCF between $6x^3y^5$ and $9x^4y^2$, we follow the procedure from the previous question:

1. For the coefficients, the GCF between 6 and 9 is 3 (not -3 because both 6 and 9 are both positive integers).

2. For the variables, the GCF between $x^3$ and $x^4$ is $x^3$, whereas the GCF between $y^5$ and $y^2$ is $y^2$. Therefore, the GCF between $x^3y^5$ and $x^4y^2$ is $x^3y^2$.

Multiplying the results from 1 and 2 together, the final GCF is $3x^3y^2$. Factoring out this from both terms, we get $3x^3y^2(2y^3 - 3x)$. Our answer is D.

# Factoring Polynomials III

Factor completely:

$$12x^4y^7z - 18x^{10}y^2 - 36x^7y^5z$$

A. $3x^4y^2z(4y^5 - 6x^6 - 12x^3y^3)$

B. $3x^4y^2(4y^5z - 6x^6 - 12x^3y^3z)$

C. $4x^4y^2(3y^5z - 4x^6 - 6x^3y^3z)$

D. $6x^4y^2z(2y^5 - 3x^6 - 4x^3y^3)$

E. $6x^4y^2(2y^5z - 3x^6 - 4x^3y^3z)$

# Solution

**Answer:** E

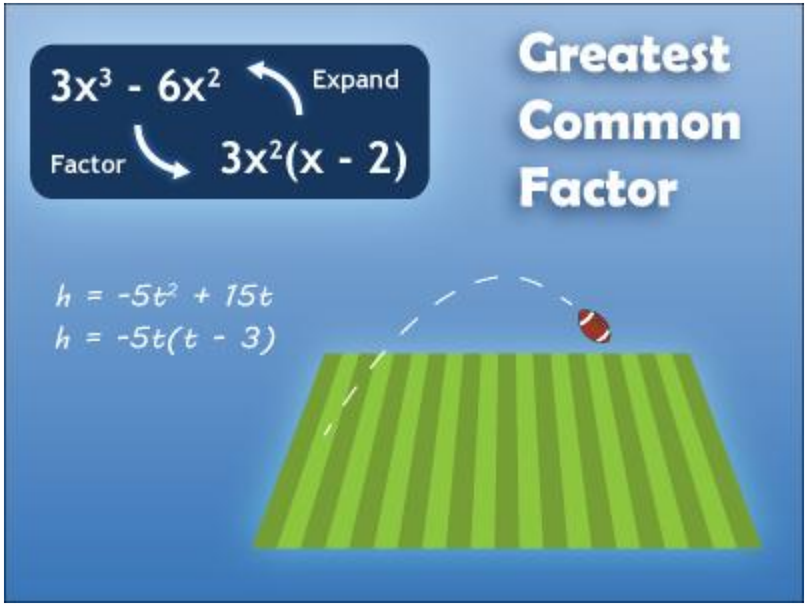
**Justification:**

In order to find the GCF of $12x^4y^7z, 18x^{10}y^2$, and $36x^7y^5z$, we follow the procedure from previous questions:

1. For the coefficients, the GCF between 12 and 18 and 36 is 6 (3 is not the greatest, 12 cannot be divided into 18).
2. For the variables, the GCF between $x^4$, $x^{10}$ and $x^7$ is $x^4$; the GCF between $y^7$, $y^2$ and $y^5$ is $y^2$. However, for z, the second term ($18x^{10}y^2$) does not have any z. Thus, z cannot be factored out. Our GCF between variables is $x^4y^2$.

# Solution Cont'd

Multiplying the results from 1 and 2 together, the final GCF is $6x^4y^2$. Factoring out this from three terms, we get $6x^4y^2(2y^5z - 3x^6 - 4x^3y^3z)$. Our answer is E.

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# Factoring Polynomials IV

Factor completely:

$$x^2 + 5x - 6$$

A. $(x-6)(x+1)$

B. $(x+6)(x-1)$

C. $(x-2)(x+3)$

D. $(x+2)(x-3)$

E. $(x-5)(x-1)$

# Solution

**Answer:** B

**Justification:**

For this equation, there is no GCF. Thus, in order to factor this expression of $ax^2 + bx + c$, where a = 1, we have to do the following:

1. Since a = 1, we know that the front variable of the factors will be x. → (x +  ) (x +  )
2. Find two numbers that add up to b and multiply into c.

   ☆ + ◇ = b

   ☆ × ◇ = c
3. Once we find those two numbers, we can simply write them beside the factors → (x + ☆) (x + ◇)

# Solution Cont'd

Following the procedure, in order to find the factored form of $x^2 + 5x - 6$, we have to find two numbers that :

- Add up to 5
- Multiply into -6

| | | | |
|---|---|---|---|
| 6 | + | -1 | = 5 |
| 6 | × | -1 | = -6 |

For factors of 6, we have ±1, ± 2, ± 3, and ± 6.

In order to get two numbers that multiply into a negative number, these two numbers must have different signs!

Then, for our two numbers, we can have 6 and -1, which satisfy both requirements

Consequently, our factored form will look like (x + 6) (x - 1). Our answer is B.

# Factoring Polynomials V

Factor completely:

$$x^2 - 7x - 6$$

A. $(x-6)(x+1)$

B. $(x+6)(x-1)$

C. $(x-2)(x+3)$

D. $(x+2)(x-3)$

E. $(x-6)(x-1)$

# Solution

**Answer:** No Solution

**Justification:**

For this equation, there is no GCF. Thus, in order to factor this expression, we have to follow the procedure from the previous question. In order to find the factored form of $x^2 - 7x - 6$, we have to find two numbers that :

- Add up to -7
- Multiply into -6

$$\_\_\_\_ + \_\_\_\_ = -7$$

$$\_\_\_\_ \times \_\_\_\_ = -6$$

For factors of 6, we have:

±1, ± 2, ± 3, and ± 6.

# Solution Cont'd

In order to get two numbers that multiply into a negative number, these two numbers must have different sign (+,- or -, +) !

Then, for our two numbers, we have (+1, -6), (-1, +6), (+2, -3), or (-2, +3),

However, when they are added, we get -6, 6, -1, or 1, respectively . Since our b is -7, we do not have any choice!.

$$\underline{\quad ? \quad} + \underline{\quad ? \quad} = -7$$
$$\underline{\quad ? \quad} \times \underline{\quad ? \quad} = -6$$

Notice that option E does add up to -7, but it multiplies into +6.

Thus, we have no solution, and this example cannot be factored further!

# Factoring Polynomials VI

Factor completely:

$$x^2 + 12x + 36$$

A. $(x+12)(x+3)$

B. $(x+4)(x+8)$

C. $(x+4)(x+9)$

D. $(x+6)(x-6)$

E. $(x+6)(x+6)$

# Solution

**Answer:** E

**Justification:**

For this equation, there is no GCF. Thus, in order to factor this expression, we have to follow the procedure from the previous question. In order to find the factored form of $x^2 + 12 + 36$, we have to find two numbers that :

- Add up to 12
- Multiply into 36

$$\_\_\_\_ + \_\_\_\_ = +12$$

$$\_\_\_\_ \times \_\_\_\_ = +36$$

For factors of 36, we have:

±1, ± 2, ± 3, ± 4, ± 6, ± 9, ± 12, ± 18, and ± 36.

# Solution Cont'd

In order to get two numbers that multiply into a positive number, these two numbers must have same sign (+,+ or -, -) !

Then, for our two numbers, we can have ± 6 and ±6, which must have identical signs (+6 with +6, -6 with -6).

However, when they are added, we get +12 or – 12. Since our b is +12, our two numbers are +6 and +6.

$$\underline{\ 6\ } + \underline{\ 6\ } = +12$$

$$\underline{\ 6\ } \times \underline{\ 6\ } = +36$$

Then, our factored form will look like (x + 6) (x + 6). Notice that this answer can be written as a complete square form $(x+6)^2$

Our answer is E.

# Factoring Polynomials VII

Factor completely:

$$3x^2 - 30x + 72$$

A. $3(x-8)(x-9)$

B. $3(x+8)(x+9)$

C. $3(x-4)(x-6)$

D. $3(x+4)(x+6)$

E. $3(x-18)(x-12)$

# Solution

**Answer:** C

**Justification:**

For this equation, there is a GCF of 3. Thus, in order to factor this expression, we have to factor out the GCF first, then follow the procedure from the previous question. Thus, we have the factored form of $3(x^2 - 10x + 24)$.

In order to find the factored form of $x^2 - 10x + 24$, we have to find two numbers that :

- Add up to -10
- Multiply into 24

$$\_\_\_\_ + \_\_\_\_ = -10$$

$$\_\_\_\_ \times \_\_\_\_ = +24$$

.

# Solution Cont'd

In order to get two numbers that multiply into a positive number, these two numbers must have the same sign (+,+ or -, -) !

So, looking at the factors of 24, we can have ± 4 and ±6, which must have identical signs (+4 with +6, -4 with -6).

However, when they are added together, we get +10 or – 10. Since our b is -10, our two numbers are -4 and -6.

$$\underline{-4} + \underline{-6} = -10$$

$$\underline{-4} \times \underline{-6} = +24$$

Then, our factored form will look like (x - 4)(x - 6). Multiplying with the GCF, we get 3(x - 4)(x - 6).

Our answer is C.

# Factoring Polynomials VIII

Factor completely:

$$2x^2 + 7x + 3$$

A. $2(x^2 + 3.5x + 1.5)$

B. $(2x + 1)(x + 3)$

C. $(2x + 3)(x + 1)$

D. $(2x - 1)(x - 3)$

E. $(2x - 3)(x - 1)$

# Solution

**Answer:** B

**Justification:**

For this equation, there is no GCF. Thus, in order to factor this expression of $ax^2+bx+\mathrm{c}$, where a ≠ 1 , we have to do the following the “decomposition” method. For this procedure, we will use the example of: $2x^2+7x+3$

1. Find two numbers that add up to 7 and multiply into the product of 2 and 3, which is 6.

| | | | | |
|---|---|---|---|---|
| 6 | + | 1 | = | 7 |
| 6 | × | 1 | = | 6 |

2. Once we find those two numbers (6 and 1 for this example), we can simply “decompose” $7x$ into x with these two numbers as coefficients: $(2x^2+6x+x+3)$

# Solution Cont'd

3. Draw brackets to separate the expression into two expressions: $(2x^2 + 6x + x + 3) \longrightarrow (2x^2 + 6x) + (x + 3)$

4. For each of these factors, factor out the GCF of each factor. For the first factor, the GCF would be 2x, and for the second factor, the GCF would be 1:

$$(2x^2 + 6x) + (x + 3) \longrightarrow 2x(x + 3) + 1(x + 3)$$

5. Now, you can factor out (x+3) from both of them.

$$2x(x + 3) + 1(x + 3) \longrightarrow (x + 3)(2x + 1)$$

Rewriting this, we get (2x + 1)(x + 3).

Thus, our answer is B.

# Factoring Polynomials IX

Factor completely:

$$5x^2 - 26x - 24$$

A. $(5x+4)(x-6)$

B. $(5x-3)(x+8)$

C. $(5x+12)(x-2)$

D. $(5x+8)(x-3)$

E. $(5x-4)(x+6)$

# Solution

**Answer:** A

**Justification:**

For this equation, there is no GCF. Thus, in order to factor this expression $5x^2 - 26x - 24$, we have to follow the "decomposition" method from the previous example.

1. Find two numbers that add up to -26 and multiply into the product of 5 and -24, which is -120.

Careful! It cannot be -6 and -20 (if we multiply them we get +120)

-30 + 4 $= -26$

-30 × 4 $= -120$

2. "Decompose" $-26x$ into $-30x$ and $4x$:

$(5x^2 - 30x + 4x - 24)$

The order doesn't matter as $4x$ could come first before $-30x$ !

# Solution Cont'd

3. Draw brackets to separate the expression into two expressions:

$$(5x^2 - 30x + 4x - 24) \longrightarrow (5x^2 - 30x) + (4x - 24)$$

4. For each of those factors, factor out the GCF of each factor. For first factor, the GCF would be 5x, and for the second factor, the GCF would be 4:

$$(5x^2 - 30x) + (4x - 24) \longrightarrow 5x(x - 6) + 4(x - 6)$$

5. Now, you can factor out (x – 6) from both of them.

$$5x(x - 6) + 4x(x - 6) \longrightarrow (x - 6)(5x + 4)$$

Rewriting this, we get (5x + 4)(x – 6).

Thus, our answer is A.

# Factoring Polynomials X

Factor completely:

$$x^2 - 225$$

A. $(x+1)(x-225)$

B. $(x-5)(x+45)$

C. $(x+5)(x-45)$

D. $(x-15)(x-15)$

E. $(x+15)(x-15)$

# Solution

**Answer:** E

**Justification:**

For this equation, there is no GCF. Also, there is no $\mathrm{b}x$ term ("the middle term"), and both $x^2$ and 225 are perfect squares, and there is a negative sign between them. Thus, in order to factor this expression $x^2 - 225,$ we have to follow this method:

1. Find two same numbers that multiply into the constant. In our case, it would be 15.

   15 × 15 = 225

2. Since a = 1, we know that the front variable of the factors will be x. → (x )(x )

# Solution Cont'd

3. Alternate the sign ⟶ (x + )(x – )

4. Plug in the number we got from step 1.

$$(x + 15)(x - 15)$$

Notice that if we expand this, we get

$$x^2 - \cancel{15x} + \cancel{15x} - 225 = x^2 - 225$$

"Difference of squares"

Thus, our answer is E.

# Factoring Polynomials XI

Factor completely:

$$9x^2 + 225$$

A. $(3x+1)(3x+225)$

B. $(3x+5)(3x+45)$

C. $(3x+15)(3x+15)$

D. $9(x+5)(x+5)$

E. $9(x+5)(x-5)$

# Solution

**Answer:** No solution

**Justification:**

For this equation, there is a GCF, which is 9 (notice 225 is divisible by 9). Factoring out 9, we have $9(x^2 + 25)$.

Also, there is no $\mathrm{b}x$ term (“the middle term”), and both $x^2$ and 25 are perfect squares.

However, notice that there is no negative sign between $x^2$ and 25. Thus, we need to have factors with the same sign.

$$(x + 5)(x + 5) \text{ or } (x - 5)(x - 5)$$

But, these answers will either have the middle term +10x or –10x. Thus, we have no solution.

# Factoring Polynomials XII

Factor completely:

$$4x^4 - 64$$

A. $4(x^4 - 16)$

B. $(2x^2 - 8)(2x^2 + 8)$

C. $4(x^2 + 4)(x^2 - 4)$

D. $4(x + 2)(x - 2)(x + 2)^2$

E. $4(x + 2)(x - 2)(x^2 + 4)$

# Solution

**Answer**: E

**Justification:**

For this equation, there is a GCF, which is 4. Factoring out 4, we have $4(x^4 - 16)$.

Also, there is no $\mathrm{b}x$ term ("the middle term"), and both $x^4 (x^2 \times x^2)$ and 16 are perfect squares, with a negative sign between them.

Thus, we can write $x^4 - 16$ as $(x^2 + 4)(x^2 - 4)$.

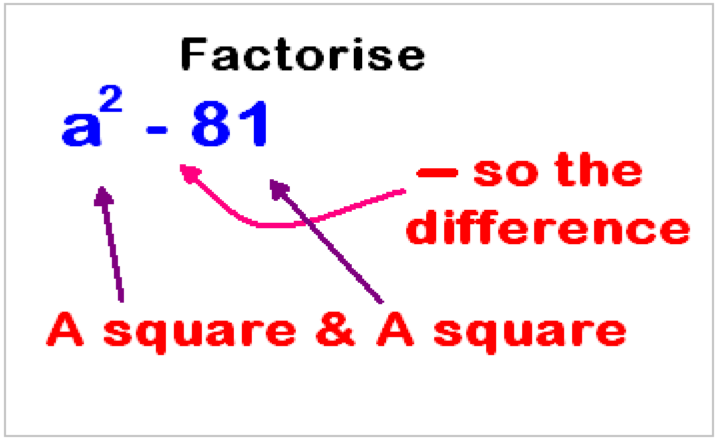
Moreover, notice that $x^2 - 4$ is also a difference of squares! Thus, we can factor $x^2 - 4$ further by $(x + 2)(x - 2)$

# Solution Cont'd

We cannot factor $x^2 + 4$ as it is not a difference of squares.

In conclusion, we can write $4(x+2)(x-2)(x^2+4)$.

Our answer is D.

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