a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Mathematics
Relations and Functions: Multiplying Polynomials

Science and Mathematics
Education Research Group

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Factoring Polynomials II

$$a(b+c) = ab + ac$$ *Multiplying*

$$ab + ac = a(b+c)$$ *Factoring*

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Multiplying Polynomials I

Multiply completely:

$$(2x^2)(6x^3)$$

A. $8x^5$

B. $8x^6$

C. $12x^5$

D. $12x^6$

E. $26x^5$

Solution

Answer: C

Justification:

Multiplying polynomials is to multiply factors in a division statement into one single expression.

In order to multiply polynomials, we multiply coefficients with coefficients, and variables with variables.

For coefficients, we multiply 2 with 6, which is 12.

For variables, we multiply x^2 with x^3, which is $x^{2+3} = x^5$

$$(2x^2)(6\boldsymbol{x}^{\mathbf{3}}) = (2 \times 6) \times (x^2 \times x^3)$$

Multiplying them back together, we get $12x^5$. Our answer is C.

Multiplying Polynomials II

Multiply completely:

$$(2x^2y^2)(-3x^3y^{-2}z)$$

A. $-6x^5z$

B. $-6x^6z$

C. $-6x^5y^{-4}z$

D. $-6x^6y^{-4}z$

E. $-x^5z$

Solution

Answer: A

Justification:

In order to multiply polynomials, we multiply coefficients with coefficients, and variables with variables

For coefficients, we multiply 2 with -3, which is -6.

For x, we multiply x^2 with x^3, which is $x^{2+3} = x^5$

For y, we multiply y^2 with y^{-2}, which is $y^{2-2} = y^0 = 1$

For z, there is only 1 z; thus it is just z

$(2x^2y^2)\left(-3\boldsymbol{x}^{\boldsymbol{3}}\boldsymbol{y}^{\boldsymbol{-2}}\boldsymbol{z}\right) = (2 \times 6) \times (x^2 \times x^3) \times (y^2 \times y^{-2}) \times z$

Multiplying them back together, we get $-6x^5z$. Our answer is A.

Multiplying Polynomials III

Expand:

$$(x-3)(x+4)$$

A. $(x^2 - 1)$

B. $(x^2 - 12)$

C. $(x^2 + x - 12)$

D. $(x^2 - x - 12)$

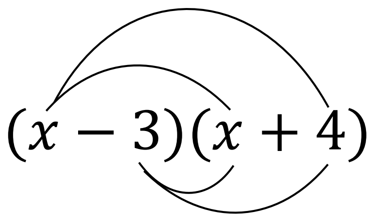
E. $(x^2 - 7x)$

Solution

Answer: C

Justification:

In order to multiply polynomials with more than 1 term, we have to “distribute” our multiplication to each term as such!



Multiplying each term over and under, we get:

$(x-3)(x+4) = x^2 + 4x - 3x - 12 = x^2 + x - 12$

Our answer is C.

Multiplying Polynomials IV

Expand:

$$(3x - 2)^2$$

A. $(9x^2 - 4)$

B. $(9x^2 + 12x + 4)$

C. $(9x^2 + 12x - 4)$

D. $(9x^2 - 12x - 4)$

E. $(9x^2 - 12x + 4)$

Solution

Answer: E

$$(3x-2)^2 \neq (3x)^2-(2)^2$$

Justification:

Notice that this question can also be written as a multiple of two factors as such!

$$(3x-2)(3x-2)$$

Multiplying each term over and under, we get:

$(3x-2)(3x-2) = 9x^2 - 6x - 6x + 4 = 9x^2 - 12x + 4$

Our answer is E.

Multiplying Polynomials V

Expand:

$$-(2x-1)(x+1)^2$$

A. $(-2x^3+2x^2+x+1)$

B. $-(2x^2+x-1)^2$

C. $-(2x^3+3x^2-1)$

D. $(-2x^3-3x^2+1)$

E. $-3x^2$

Solution

Answer: D

$$(x+1)^2 \neq (x^2+1^2)$$

Justification:

Notice that this question cannot be written as a multiple of two factors as such!

$$-(2x-1)(x+1)^2$$

Due to the order of operation, we must evaluate the exponents first!

$$(x+1)^2 = (x+1)(x+1) = x^2 + x + x + 1 = x^2 + 2x + 1$$

Solution Continued

Now, this question can be written as multiple of two factors as such!

$$-(2x-1)(x^2+2x+1)$$

$$(2x-1)(x^2+2x+1) = 2x^3 + 4x^2 + 2x - x^2 - 2x - 1$$
$$= 2x^3 + 3x^2 - 1$$

Since there is a negative sign in front of it, we multiply -1 to all terms.

$$-(2x^3+3x^2-1) = -2x^3 - 3x^2 + 1$$

Thus, our answer is D.

Multiplying Polynomials VI

Expand:

$$(x+y)^3$$

A. $x^3 + y^3$

B. $x^3 + 2x^2y^2 + y^3$

C. $x^3 + x^2y + xy^2 + y^3$

D. $x^3 + 3x^2y + 3xy^2 + y^3$

E. $x^3 + 6x^2y^2 + y^3$

Solution

Answer: D

$$(x + y)^3 \neq (x^3 + y^3)$$

Justification:

Notice that this question can be written as a multiple of three factors as such!

$$(x + y)\ (x + y)\ (x + y)$$

Due to the order of operation, we must evaluate the first exponents first!

$$(x + y)(x + y) = x^2 + xy + xy + y^2 = x^2 + 2xy + y^2$$

Then, we multiply this result with the last $(x + y)$ factor

$$(x^2 + 2xy + y^2)(x + y) = x^3 + x^2y + 2x^2y + 2xy^2 + xy^2 + 1$$
$$= x^3 + 3x^2y + 3xy^2 + 1$$

Thus, the answer is D.

Multiplying Polynomials VII

Expand:

$$(x-y)^5$$

A. $x^5 + 5x^4 - 5y^4 - y^5$

B. $x^5 + 5x^4 + 10x^3y^2 - 5y^4 - y^5$

C. $-x^5 - 5x^4y - 10x^3y^2 - 10x^2y^3 - 5x^1y^4 - y^5$

D. $x^5 - 5x^4y + 10x^3y^2 + 10x^2y^3 - 5x^1y^4 - y^5$

E. $x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

Solution

Answer: E

Justification:

We can solve this problem by using Pascal’s triangle

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

Pascal’s triangle is a triangular model that starts with 1 at the top, continues to place number below in a triangular shape, and has pattern that each number of a row is the two numbers above added together excluding the edges.

It is found that $(n+1)^{th}$ row of the Pascal’s triangle represents the coefficients of the terms in a polynomial $(x+y)^n$, where n is a positive integer.

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Solution Continued

Since n = 5 for our example, we know that the coefficients of the terms of $(x-y)^5$ are at the 6th row of the Pascal's triangle. (1)

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

However, we have a negative sign between x and y: $(x-y)^5$

This means that each term will have an alternating sign in a descending order from the first term. (2)

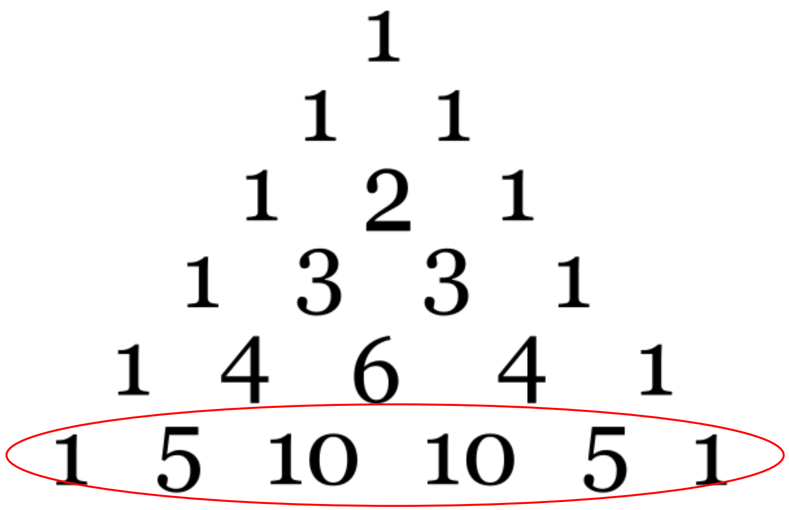
For example, when we have $(x-y)^2$, we should get: $x^2-2xy+y^2$. As you can see, the coefficients of these terms are +1, – 2, and +1, respectively. They alternate sign as they progress.

Solution Continued

Thus, incorporating both (1) and (2), our solution will look as below:

$$x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$$

Thus, our answer is E.



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Alternate signs

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