a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Physics
Circuit Problems

Science and Mathematics
Education Research Group

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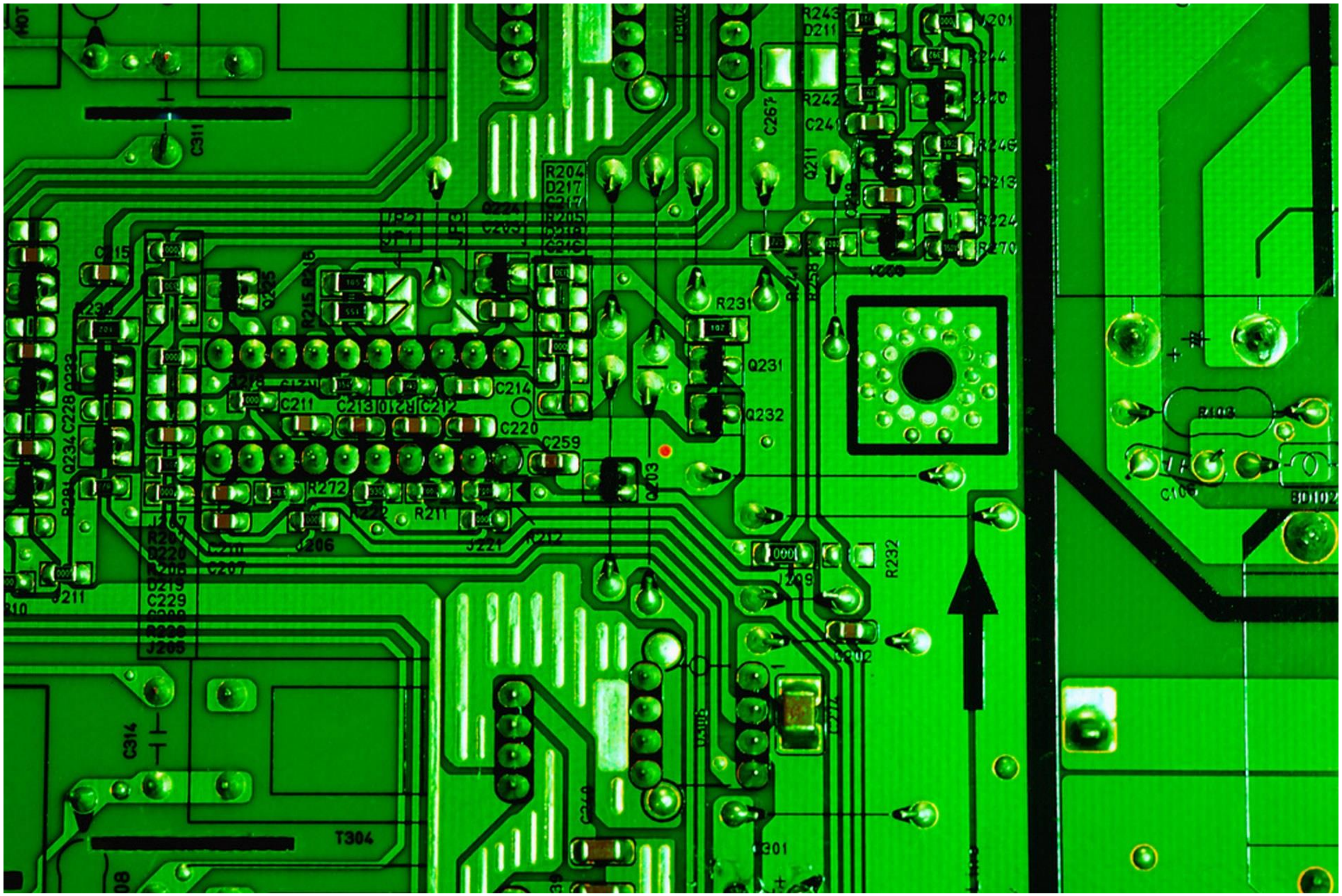
Circuit Problems



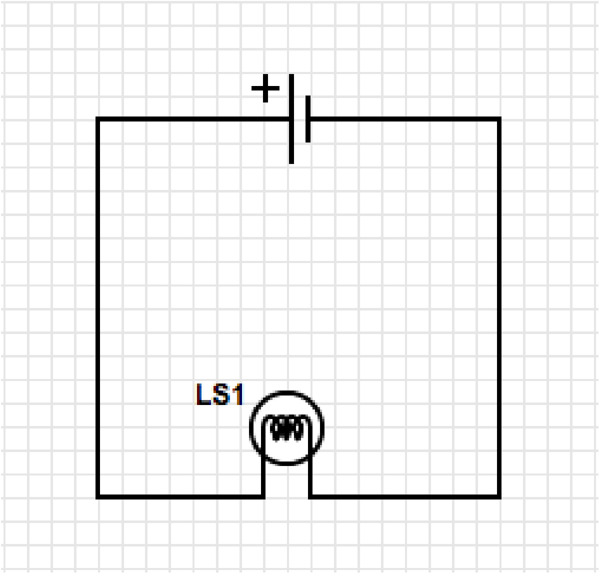
Retrieved from: http://cdni.wired.co.uk/1920x1280/a_c/Circuit-board.jpg

Circuit Problems

The following questions have been compiled from a collection of questions submitted on PeerWise (https://peerwise.cs.auckland.ac.nz/) by teacher candidates as part of the EDCP 357 physics methods courses at UBC.

Circuit Problems I

Which direction do both the conventional current and the electrons in this circuit flow?



A. The current and the electrons both flow in the same direction, clockwise.
B. The current and the electrons both flow in the same direction, counterclockwise.
C. The current and electrons flow in different directions. The current flows clockwise while the electrons flow counterclockwise.
D. The current and electrons flow in different directions. The current flows counterclockwise while the electrons flow clockwise.

Solution

Answer: D

Justification: A current is defined as the movement of an electric charge. This charge does not have to be a positive or negative charge. A flow of positive charges creates the same current as a flow of negative charges moving in the opposite direction.

We now know that the charge carriers in an electric circuit are free electrons, which move from the negatively charged terminal of the power source towards the positively charged terminal (**clockwise** in the diagram on the previous page).

However, when Benjamin Franklin was conducting experiments in electricity, we did not know this fact about electric circuits. He imagined positive charges were the carriers of electric current, and defined electric current as flowing from a positive terminal to a negative one (**counterclockwise** in the diagram).

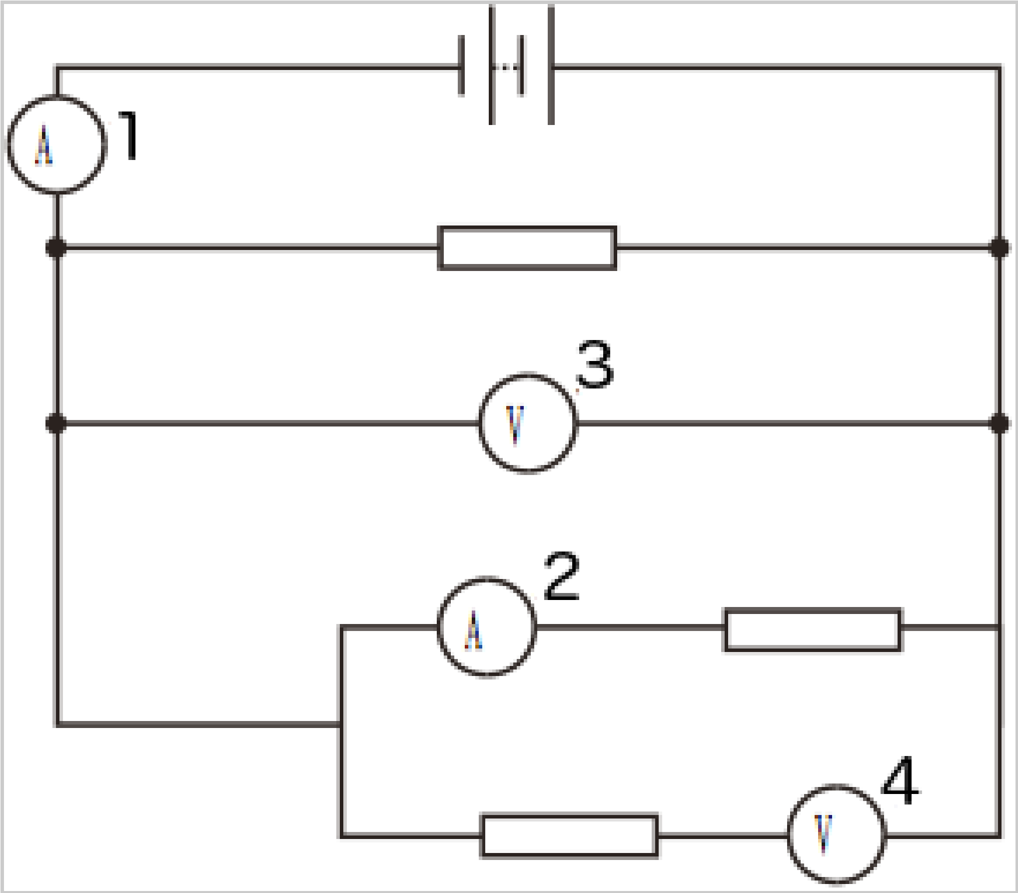
Solution continued

Even though we now know that this is not the reality, to this day we still define the conventional electric current direction as the direction in which a positive charge would move. So even though the electrons are flowing from negative to positive (clockwise in the diagram), we describe current as flowing from positive to negative (counterclockwise in the diagram).

Therefore the answer is **D**.

Circuit Problems II

In the diagram below, which meter is not connected correctly?



A. 1
B. 2
C. 3
D. 4

Solution

Answer: D

Justification: When using voltmeter and ammeters, ammeters always need to be in series with the current they are measuring, and voltmeters need to be in parallel.

This is because of the nature of series and parallel circuits.

In a parallel circuit, the potential difference is always the same, but the current of the circuit is split between the multiple paths. Thus, if we were to try to connect an ammeter in parallel, its presence would in fact reduce the amount of current received by both it and the circuit it was trying to measure.

In a series circuit, the current is always the same, but the potential difference across components varies between the components in series. Thus, if we were to connect a voltmeter in series, it would further split up the potential difference it was trying to measure, and the reading would be lower than the actual value in the circuit.

Solution continued

Looking at the previous diagram, meters 1 and 2 are ammeters, and 3 and 4 are voltmeters. We are looking for the meter that is **NOT** connected correctly.

Meter 1 is in series with the circuit, so it is hooked up correctly. So **A** is incorrect.

Meter 2 is in series with one of the parallel resistors, so it is also hooked up correctly. So **B** is also incorrect.

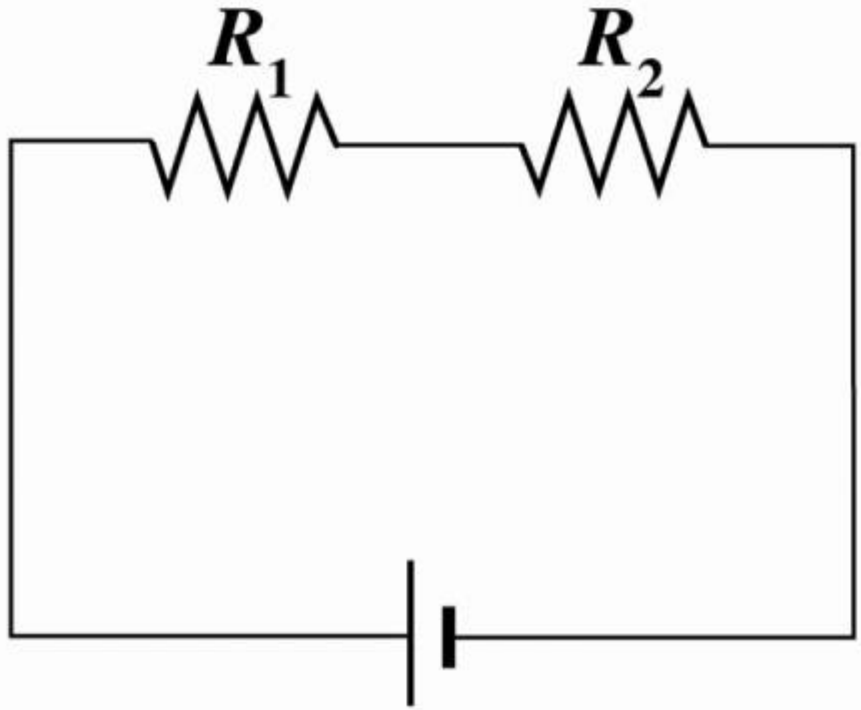
Meter 3 is in parallel with the first parallel resistor branch, so it is hooked up correctly. So **C** is incorrect.

Meter 4 is hooked up in series with the last resistor. Since meter 4 is a voltmeter, this means it won't be able to accurately measure the potential difference across the resistor, and is hooked up incorrectly. **D** is the correct answer.

Circuit Problems III

Two resistors are wired in series. The second resistor has twice the resistance as the first. Current passes through the combination. Compared to the current through the first resistor, the current through the second resistor is:

A. Twice the magnitude
B. The same
C. Half the magnitude
D. Quarter of the magnitude



Solution

Answer: B

Justification: Since the two resistors are in series, the current is the same everywhere in the circuit.

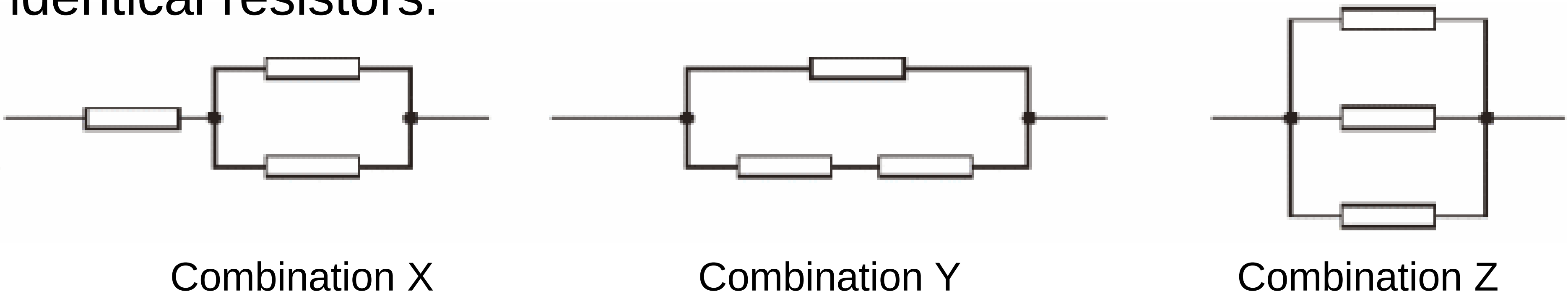
Charge does NOT pile up and begin to accumulate at any given location such that the current at one location is more than at other locations.

Charge does NOT become used up by resistors such that there is less of it at one location compared to another. The charges can be thought of as marching together through the wires of an electric circuit, everywhere marching at the same rate. Current is the rate at which charge flows and is the same everywhere in a series circuit.

Therefore, option **B** is the correct answer.

Circuit Problems IV

The diagram below shows combinations, X, Y and Z of three identical resistors.



What is the correct order of the **total** resistance of the combinations, going from LOWEST resistance to HIGHEST resistance?

A. Y, X, Z
B. Z, X, Y
C. X, Y, Z
D. Z, Y, X
E. Y, Z, X

Solution

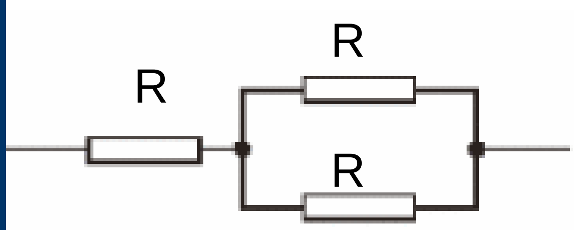
Answer: D

Justification: When resistors are in series, we simply add up the resistances to get the total resistance.

When resistors are in parallel however, we add up the inverse of their resistances, and then invert that sum. What this means is that in any parallel circuit, the sum of the resistances will always be less than the value of the smallest resistor.

So with combination X, we have one resistor in series with two parallel resistors. The resistance of the parallel resistors will be half the resistance of either one:

$$\frac{1}{R_{Parallel}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \quad \rightarrow \quad R_{Parallel} = \frac{1}{2}R$$

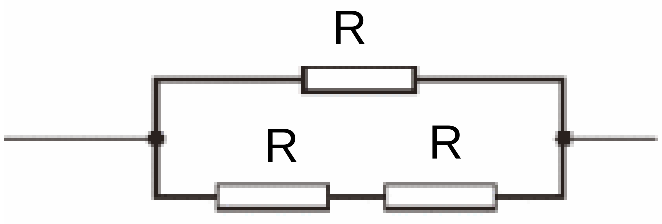


So the total resistance will be one and a half the resistance of one resistor:

$$R_{Total} = R + \frac{1}{2}R = 1\frac{1}{2}R$$

Solution continued

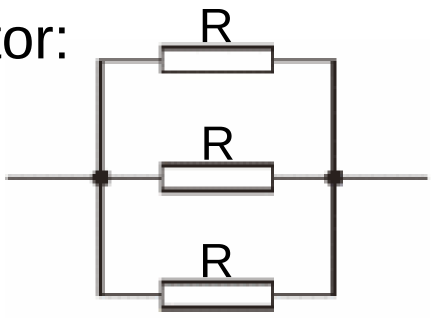
With combination Y, we have two resistors in series, in parallel with a third resistor. The two resistors in series will add up to double the resistance, but being in parallel with the third one means that the total resistance will be lower than a single resistor. The total resistance works out to two thirds the resistance of any one resistor:



$$R_{Series} = R + R = 2R$$

$$\frac{1}{R_{Total}} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R} \quad \rightarrow \quad R_{Total} = \frac{2}{3}R$$

With combination Z, all three resistors are in parallel, which means that their total resistance will only be a third of the resistance of a single resistor:

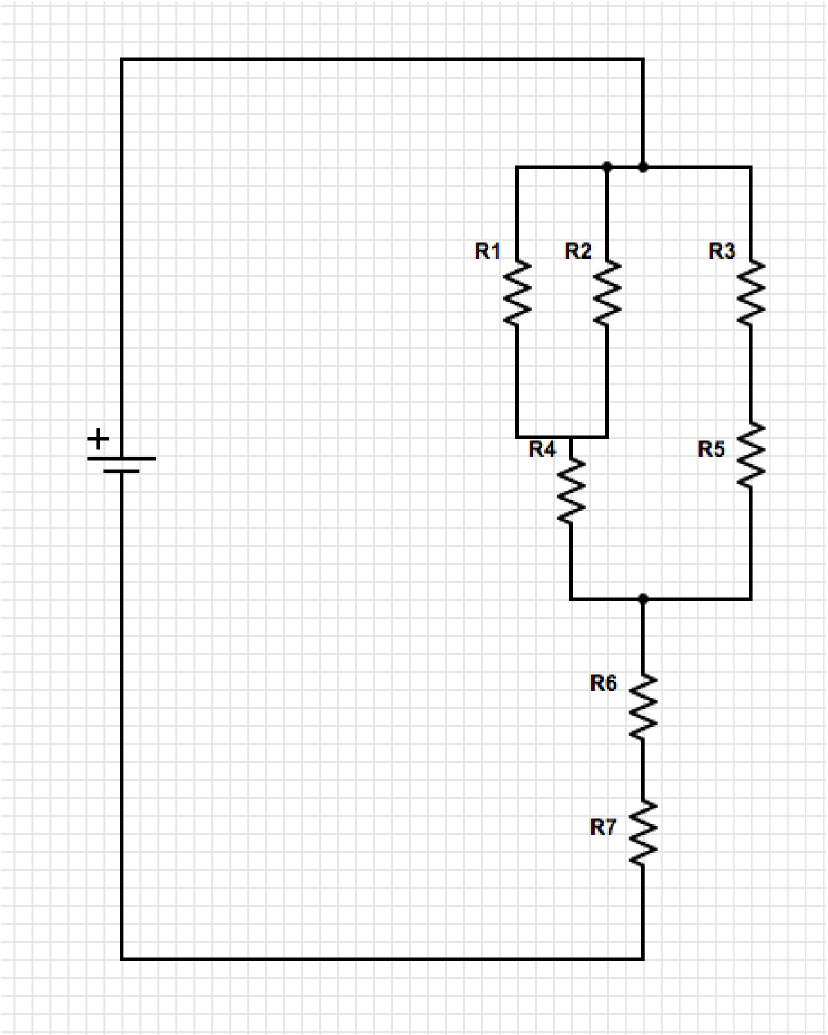


$$\frac{1}{R_{Total}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R} \quad \rightarrow \quad R_{Total} = \frac{1}{3}R$$

Therefore Z has the smallest resistance, followed by Y, and then X (Answer **D**).

Circuit Problems V

If we have a number of identical 7 Ω resistors set up as described in the diagram, what is the total resistance of that circuit, rounded to the nearest ohm?



A. 14 Ω
B. 20 Ω
C. 28 Ω
D. 35 Ω

Solution

Answer: B

Justification: We know that resistors in series are described with the relation: $R_{eq} = R_a + R_b + R_c + \ldots$

Resistors in parallel are describe with the relation:

$$\frac{1}{R_{eq}} = \frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_c} + \ldots$$

We know that we have R_6 and R_7 in series with a resistance of 14 Ω, and we can think of the rest of the resistors as making up an equivalent resistor that is in series with R_6 and R_7. So the total resistance would be the resistance of this equivalent resistor added to the resistance of R_6 and R_7. Therefore the total resistance has to be more than 14 Ω. Therefore answer A is incorrect.

Solution continued

If we look at the parallel branches which make up the equivalent resistor, we can further eliminate answers. We know that R_3 and R_5 are similarly in series with a resistance of 14 Ω. And we know that R_1 and R_2 are in parallel, so their equivalent resistance has to be less than 7 Ω (because for resistors in parallel, the equivalent resistance is always less than the branch with the smallest resistance). In series with $R_{1,2}$ is R_4, so we know the total resistance of $R_{1,2,4}$ has to be less than 14 Ω, but more than 7 Ω.

Since we have two parallel branches ($R_{3,5}$ and $R_{1,2,4}$) that are respectively 14 Ω and somewhere between 14 Ω and 7 Ω, the equivalent resistance has to be below 14 Ω as well (the branch with the lowest resistance). We have to add this to the series resistors R_6 and R_7, so we have 14 Ω being added to something less than 14 Ω, so the total has to be below 28 Ω.

Thus, Answers C and D cannot be correct because they are both above 28 Ω, leaving **B** as the correct answer.

Solution continued 2

If you want to see the detailed calculation explaining why the answer is exactly 20 Ω, then read the section below.

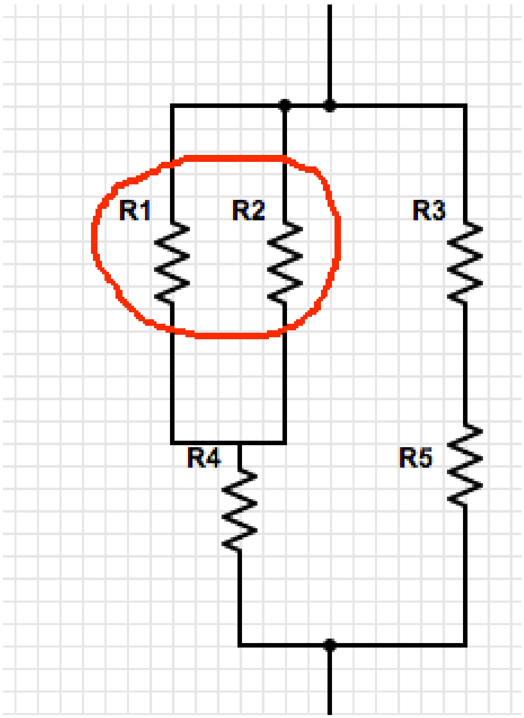
To determine the total resistance of the circuit, we need to look at the combination of the resistors in series and in parallel.

R_1 and R_2 are parallel resistors that act as a single resistor in series with R_4.

The resistance of this equivalent resistor $R_{1,2}$ is found with:

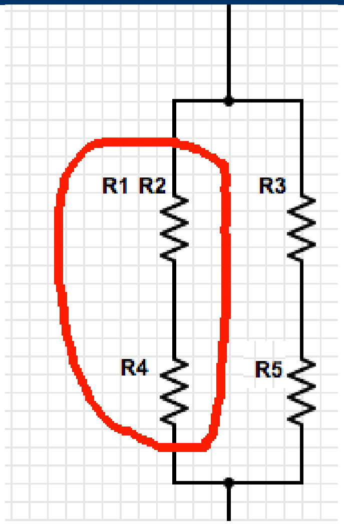
$$\frac{1}{R_{1,2}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{7\Omega} + \frac{1}{7\Omega} = \frac{2}{7\Omega}$$

$$Therefore\ R_{1,2} = \frac{7}{2}\ \Omega$$



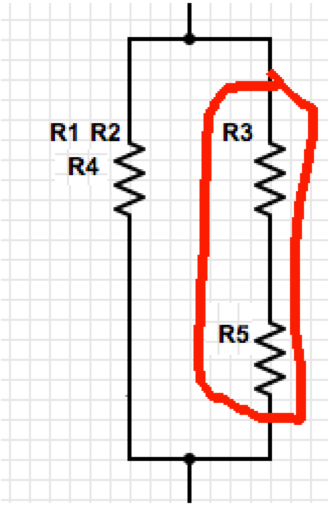
Solution continued 3

The equivalent resistor of R_1, R_2, and R_4 has a resistance of:



$$R_{1,2,4} = R_{1,2} + R_4 = \frac{7}{2}\,\Omega + 7\ \Omega = \frac{21}{2}\,\Omega$$

Resistors R_3 and R_5 are acting in series, so we add their resistances up to find:



$$R_{3,5} = R_3 + R_5 = 7\ \Omega + 7\ \Omega\ = 14\ \Omega$$

Solution continued 4

The equivalent resistor $R_{3,5}$ is acting in parallel with the equivalent resistor $R_{1,2,4}$ and to find the equivalent resistor $R_{1,2,3,4,5}$ we use:

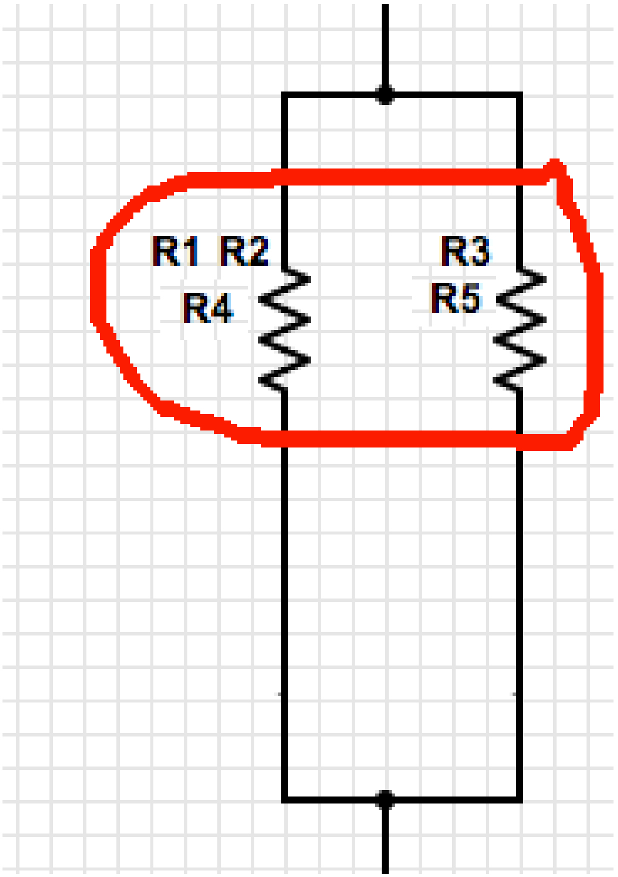
$$\frac{1}{R_{1,2,3,4,5}} = \frac{1}{R_{1,2,4}} + \frac{1}{R_{3,5}}$$

$$\frac{1}{R_{1,2,3,4,5}} = \frac{1}{\frac{21}{2}\Omega} + \frac{1}{14\Omega}$$

$$\frac{1}{R_{1,2,3,4,5}} = \frac{2}{21\Omega} + \frac{1}{14\Omega}$$

$$\frac{1}{R_{1,2,3,4,5}} = \frac{4}{42\Omega} + \frac{3}{42\Omega} = \frac{7}{42\Omega}$$

$$Therefore\ R_{1,2,3,4,5} = \frac{42}{7}\Omega = 6\ \Omega$$



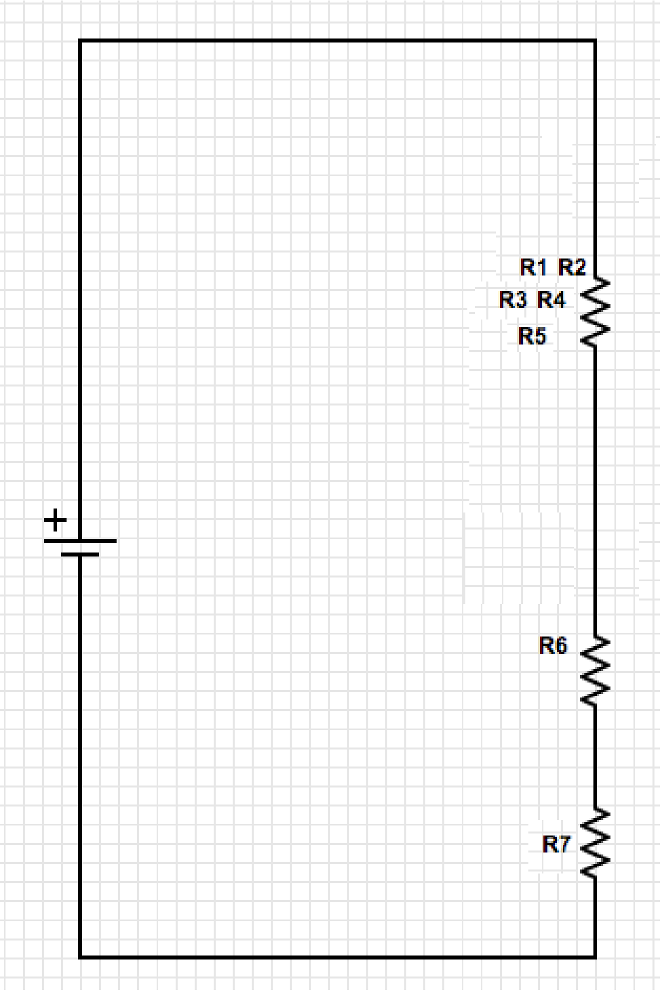
Solution continued 5

Now, we have this equivalent resistor acting in series with R_6 and R_7 so all we need to do is add up these resistances to find the total resistance:

$$R_{Total} = R_{1,2,3,4,5} + R_6 + R_7$$

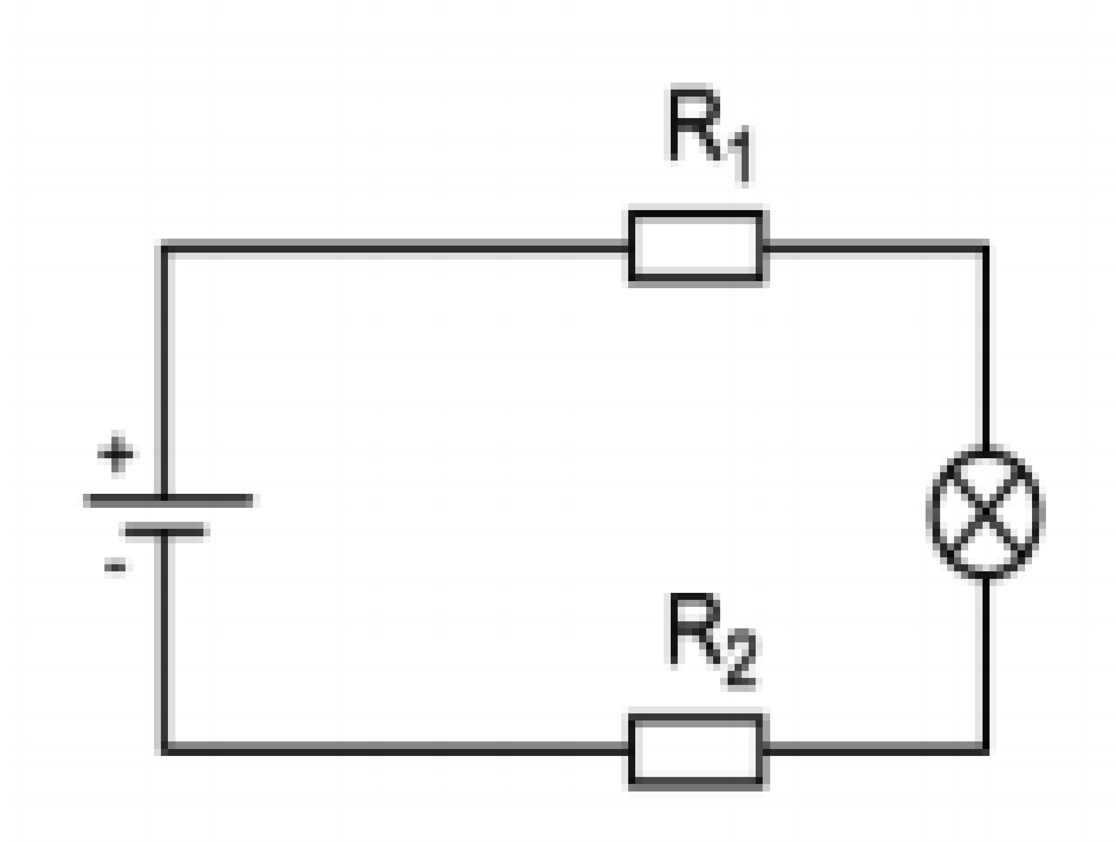
$$R_{Total} = 6\ \Omega + 7\ \Omega + 7\ \Omega = 20\ \Omega$$

Therefore the answer is **B**



Circuit Problems VI

Please use this diagram to answer the question on the following page:



Circuit Problems VI continued

What will happen to the brightness of the bulb if the resistance of R_2 is increased and R_1 remains constant?

A. The brightness of the lamp remains the same, because changing a resistor only influences the brightness of the bulb if the bulb comes after the resistor.

B. The brightness of the lamp remains the same, because the battery is the same and thus the same current is delivered.

C. The brightness of the lamp decreases, because the battery is the same and thus the voltage is the same across the lightbulb.

D. The brightness of the bulb decreases, because the increased total resistance lowers the current of the circuit.

E. The brightness of the bulb increases, because the increased total resistance increases the current of the circuit.

Solution

Answer: D

Justification: The brightness of a bulb is dependent on the power the bulb consumes.

Power is given by $P = IV$, where I is the current passing through the bulb and V is the voltage drop over the bulb.

The current (I) drawn from a battery is dependent on the potential difference ($V_{battery}$) of the battery and the total resistance of the circuit.

The formula that relates these three variables is $V_{battery} = IR_{total}$.

R_{total} is calculated by adding up the resistances of each element in the circuit.

In this case $R_{total} = R_1 + R_2 + R_{bulb}$

In this problem we are increasing R_2 and thus increasing the total resistance of the circuit.

Solution continued

From the equation $V_{battery} = IR_{total}$ we can rearrange to see that $I = V_{battery} / R_{total}$. Since $V_{battery}$ is not changing (the battery remains the same) and R_{total} has increased we know that I must decrease.

Looking back we have that the power consumed by the lightbulb is given by $P = IV$. We now know how the current behaves, but how do we determine what happens to V? (Remember that V in this case refers to the voltage across the bulb, and not the total voltage of the circuit)

The voltage drop across the lightbulb is given by $V = IR_{bulb}$ where I is the current passing through the bulb and R_{bulb} is the resistance of the bulb. We are not changing the lightbulb so we know that its resistance remains constant, however we do know that the current passing through the bulb will decrease due to the increasing R_{total}. Since I is decreasing and R_{bulb} is constant we know that V must also be decreasing.

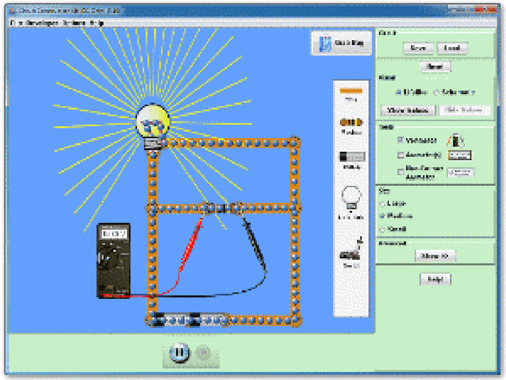
Solution continued 2

Finally, what is happening to the power? Since $P = IV$ and we know that by increasing R_2 we are decreasing the current running through the lightbulb, and we are also decreasing the voltage drop across the lightbulb, therefore the power must also be decreased. As a result of the decreasing power consumption the bulb will be less bright.

It is important to note that it wouldn't matter if R_1 or R_2 were increased (before or after the bulb), since increasing the resistance of one resistor in series will decrease the current of the whole circuit (because it increases the total resistance). Therefore the answer is **D**.

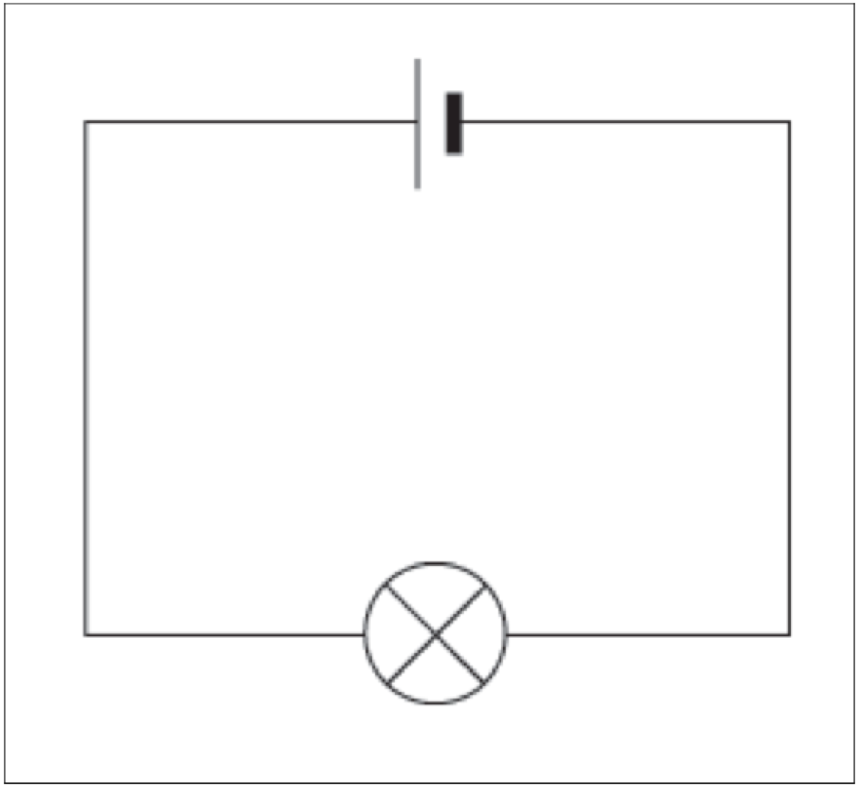
Try it out in this circuit simulator to see for yourself!

Circuit Construction Kit (DC Only)



Circuit Problems VII

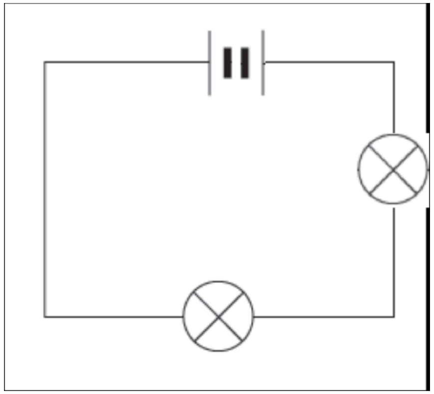
Please use this diagram to answer the question on the following page:



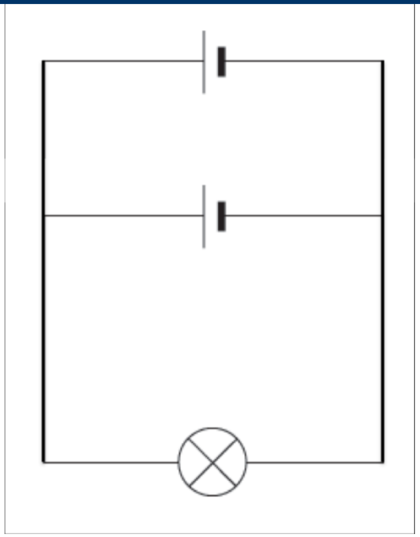
Circuit Problems VII continued

Which of the following diagrams contains a lightbulb that will shine as bright as the one on the previous page? Each lightbulb has the same resistance.

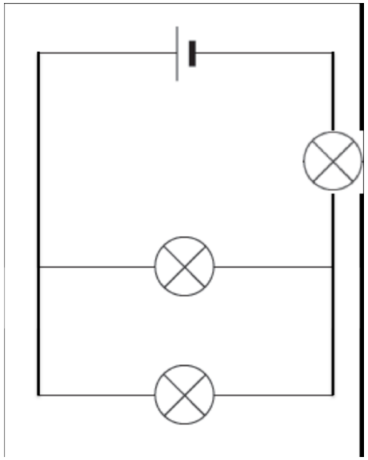
A.



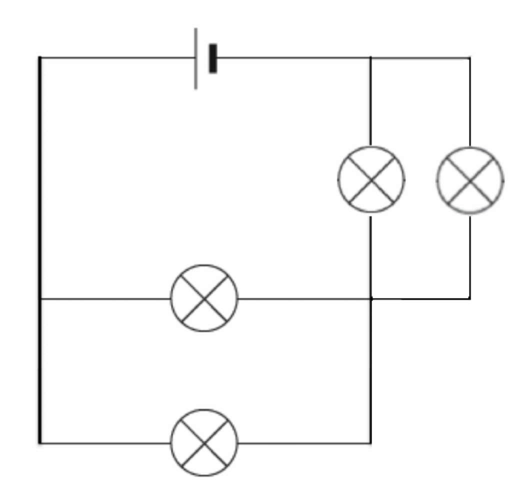
B.



C.



D.



Solution

Answer: B

Justification: The brightness of a lightbulb depends on the power that the lightbulb dissipates. This power is determined by the current that passes through the lightbulb, by using the relationship $P = I^2R$.

A) Two batteries connected backwards will produce **no current**, no matter if you have 2 batteries in a circuit connected to the 2 lightbulbs.

B) Correct! Parallel batteries will produce the same voltage as the single battery in the original diagram. The only difference is that the system will run longer with twice as much energy.

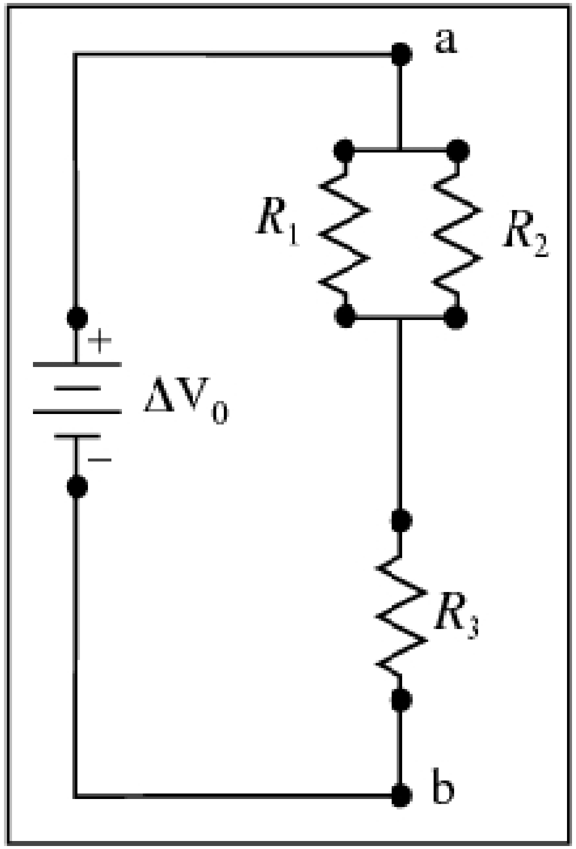
C) The circuit has two lightbulbs in parallel connected with one lightbulb in series. We can use the parallel/series equations for adding up resistance to find out that the equivalent resistance of these lightbulbs is 1.5 times greater than the resistance of the original circuit. Since the voltage is the same as in the original circuit, the total current of the circuit will decrease (using the relation $I = V/R$). Therefore none of the lightbulbs will shine equally as the one in the question.

Solution continued

D) The circuit has two lightbulbs in parallel connected with another two lightbulbs in parallel. The equivalent resistance of this combination is the same as one light bulb connected to the battery, so it is the same as the resistance of the original circuit. Therefore, the same current will be drawn from the battery initially. However, the current splits off as it passes the first parallel junction, and therefore the current (and thus the power) will be lower for each parallel branch. Thus the lightbulbs will shine less brightly than the one in the question. The current meets but splits up again in the next junction. Therefore, none of the lightbulbs will shine as brightly as the one in the question.

Circuit Problems VIII

For the following circuit, if a resistor is added in parallel with R_1, what happens to the current delivered by the battery?



A. It will decrease.
B. It will stay the same.
C. It will increase.

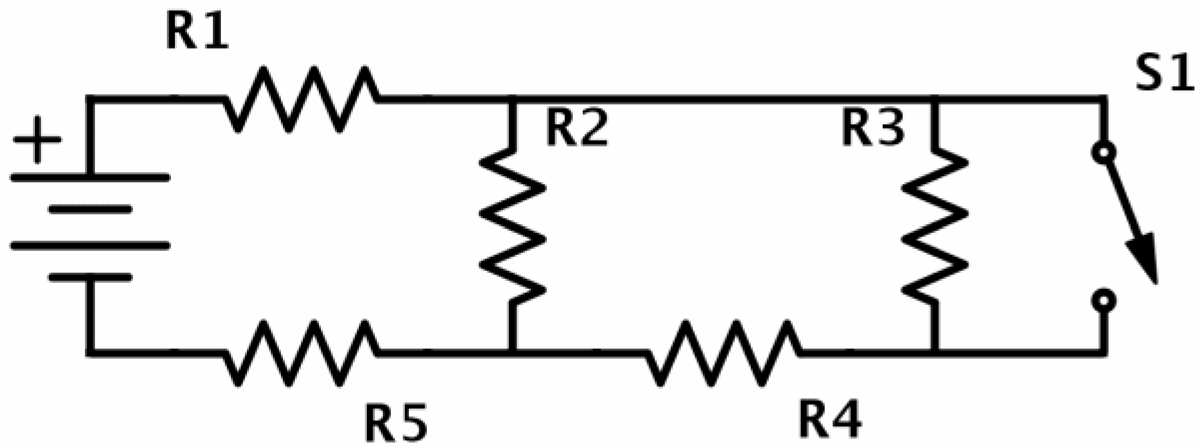
Solution

Answer: C

Justification: If we add another resistor (let us call it R_4) in parallel with R_1, there would be three resistors in parallel (R_1, R_2 and R_4), and these resistors would be in series with resistor R_3. Adding another resistor will **decrease** the equivalent resistance of the resistors in parallel and eventually decrease the overall equivalent resistance of the circuit. Since $V = IR$ and V stays the same because we are using the same battery, the current I will increase due to the decrease in R.

Circuit Problems IX

The circuit below consists of a battery, five identical resistors and a switch.



With the switch open, which resistor(s) has the least current flowing through it?

A. R1 & R5

B. R2

C. R3 & R4

D. R3

E. R5

Solution

Answer: C

Justification: As R1 and R5 are connected in series to the battery, the full current of the circuit will flow through both, so neither answer **A** or **E** can be correct.

R2 is in parallel with R3 & R4 (combined). It may look like only R3 is in parallel to R2, as they are literally parallel in the diagram, but if you follow the path of the current, it flows through R3 & R4 before returning to a junction connecting it to R2. R3 & R4 are in series. Since the total resistance of the R2 branch (1 resistor) is less than the total resistance of the R3 & R4 branch (2 resistors in series), the R2 branch will have a greater current flowing through it than the R3 & R4 branch, thus answer **B** is incorrect.

As R3 & R4 are in series, they will have the same current flowing through them. Thus the correct answer is **C**.

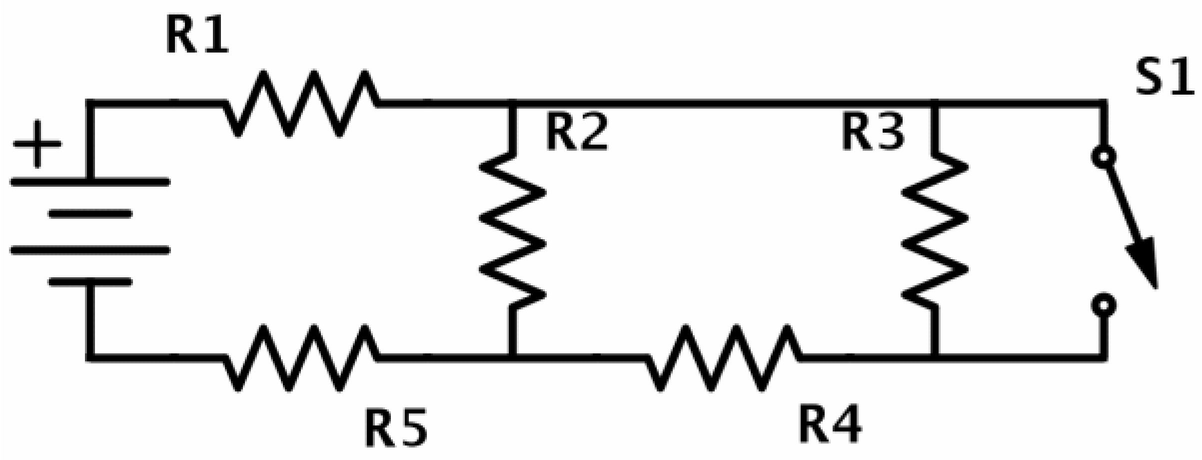
To further investigate these concepts, go to:

Circuit Construction Kit (DC Only)

and build a circuit.

Circuit Problems X

If the switch in the circuit below is closed, what happens to the current passing through R3? Assume all resistors are identical and that it is an ideal system.

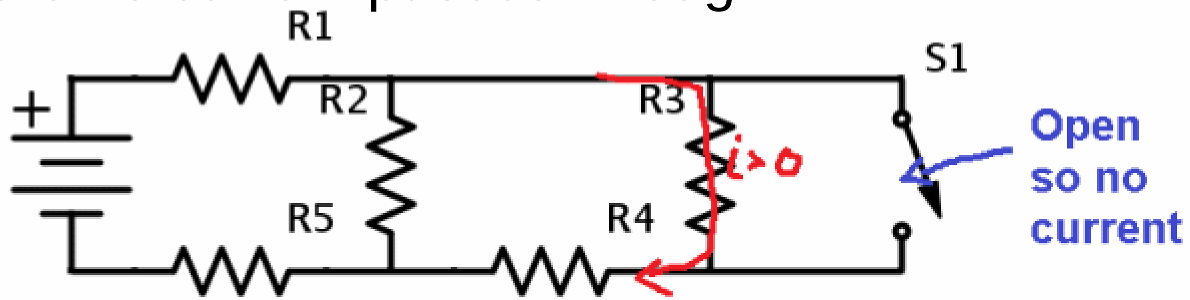


A. It increases
B. It decreases
C. It stays the same
D. It becomes 0
E. Not enough information

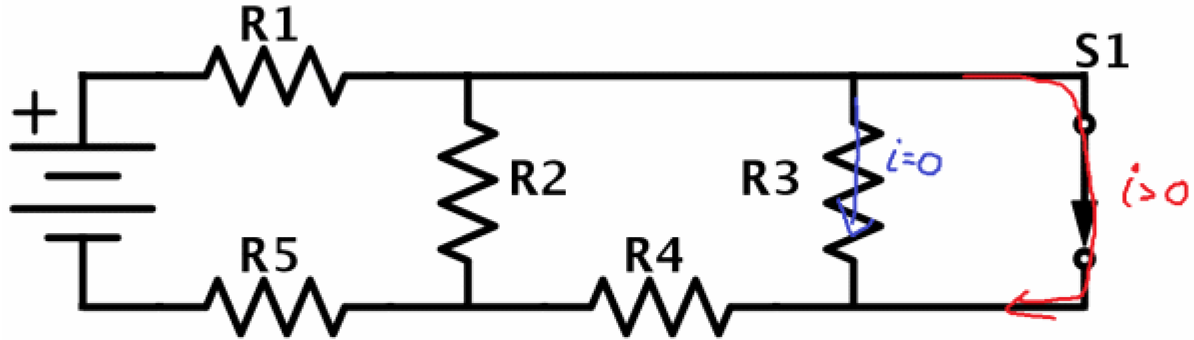
Solution

Answer: D

Justification: The switch is in a parallel circuit loop to R3. Before the switch is closed, the switch is in an open (incomplete) loop, and therefore no current passes through it.



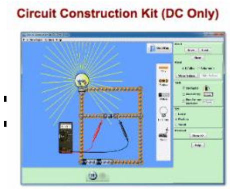
When the switch is closed, it creates a closed loop in the circuit with 0 resistance (ideal wire) that is parallel to R3. The current will all flow through the switch, and none will flow through R3.



Solution continued

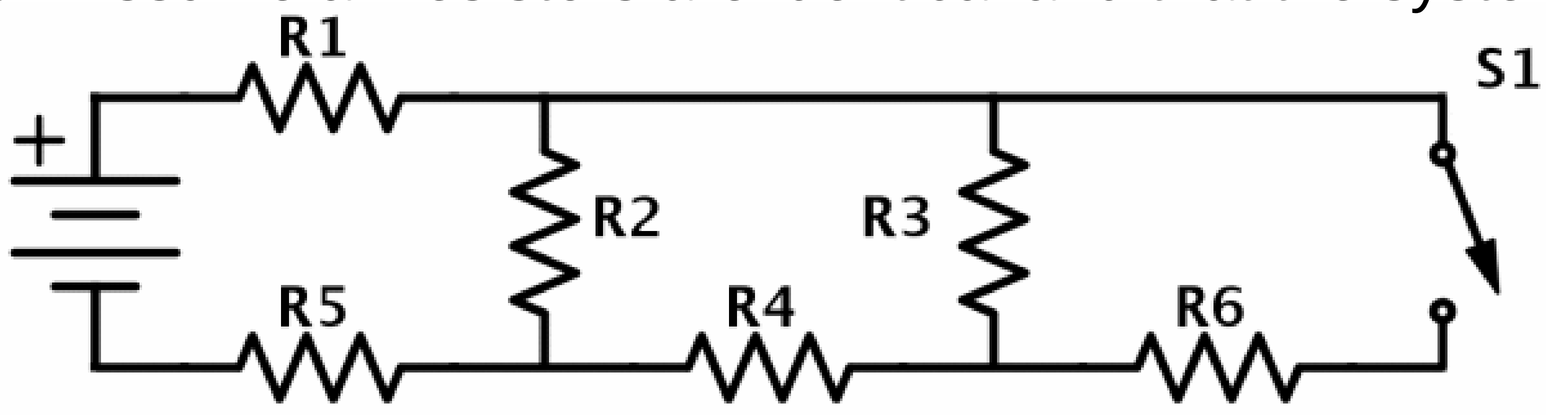
Remember that current flowing through parallel branches is inversely proportional to the ratio of the branches’ resistances. Since the switch has infinitely less resistance (0 ohms vs. >0 ohms), it will have infinitely more current (>0 amps vs. 0 amps).

Therefore **D** is correct.

To further investigate these concepts, go to:  and build a circuit.

Circuit Problems XI

If another identical resistor (R6) is added to the circuit, what happens to the total resistance of the system when the switch is closed? Assume all resistors are identical and that the system is ideal.



A. It stays the same

B. It increases

C. It decreases

D. It decreases to 0

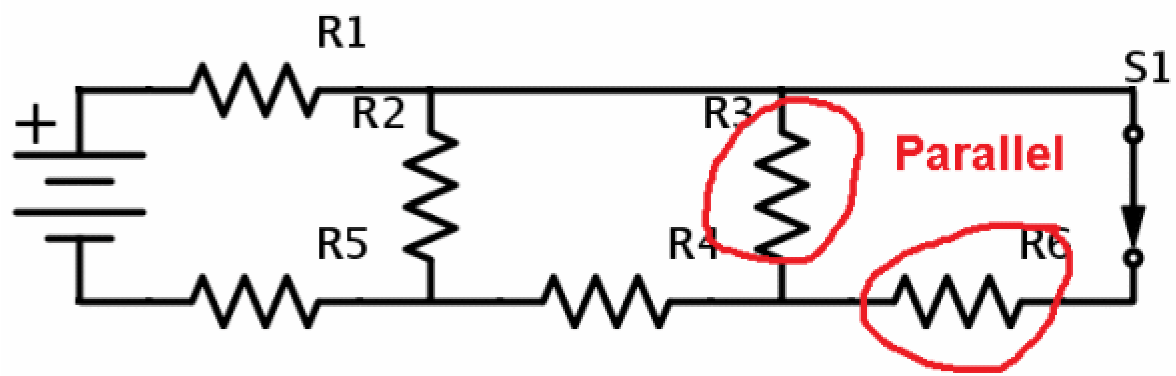
E. We need to know the resistance of R1 - R6

Solution

Answer: C

Justification: This problem can be solved conceptually using knowledge of parallel and series circuits.

When the switch is closed, R6 becomes parallel to R3 (but not R4!).



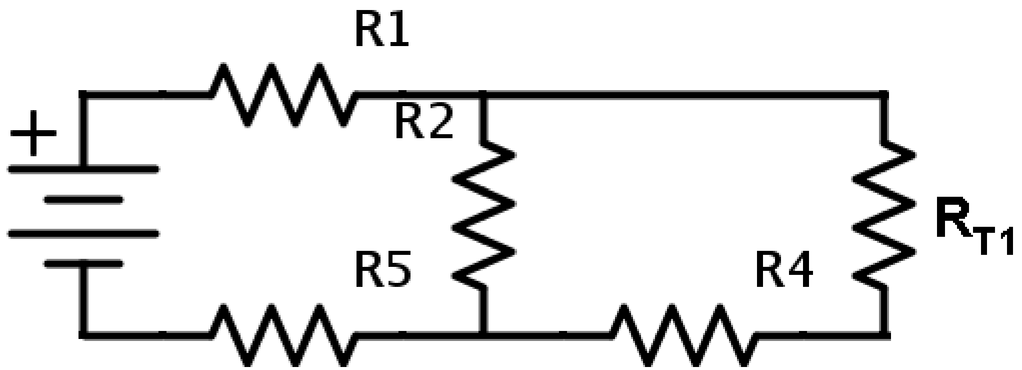
The resistances of R3 and R6 can be added to find the total resistance R_{T1} using:

$$\frac{1}{R_{T1}} = \frac{1}{R3} + \frac{1}{R6}$$

Since we know that the total resistance of parallel resistors is less than the smallest of the resistors, and since R3 and R6 are identical, we know that R_{T1} is less than R3 or R6.

Solution continued

We can redraw the circuit as below.



This circuit is the same as the circuit before the switch was closed, with R_{T1} replacing R3. We have essentially just adjusted the resistance of one of the resistors in the circuit.

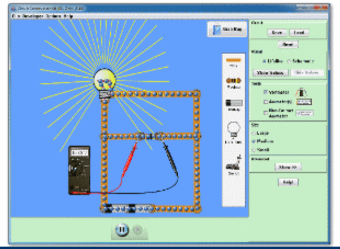
Since we know that $R_{T1} < R3$, we therefore know that the total resistance of the new circuit is less than that of the old circuit.

Therefore **C** is the correct answer.

You can use the PhET Circuit Construction simulation:

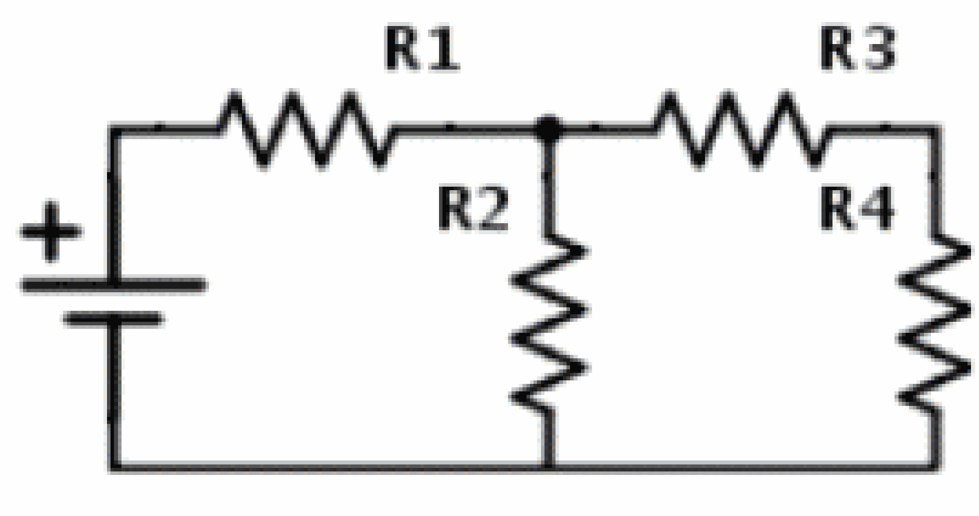
to test such circuits out and see how adding/removing resistors affects the overall resistance and the current.

Circuit Construction Kit (DC Only)



Circuit Problems XII

In the circuit diagram R1, R2, R3 and R4 are all lightbulbs with a resistance of 2 Ω. The battery has a voltage of 12 V. Use the diagram to answer the following question.



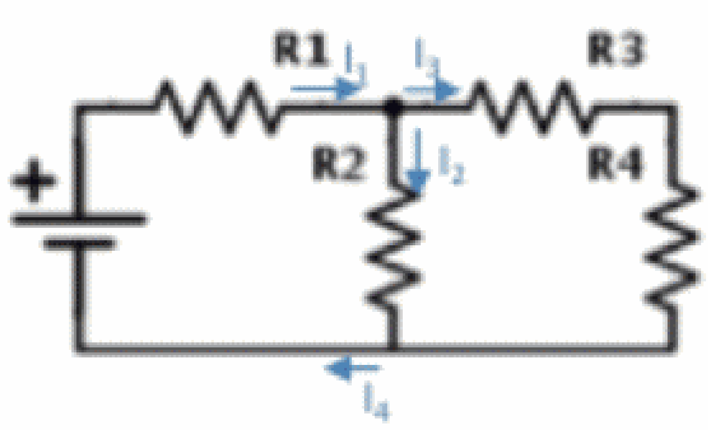
Which bulb is the brightest?

A. R1
B. R2
C. R3
D. R4
E. All bulbs will have the same brightness because they all have the same resistance.

Solution

Answer: A

Justification: To begin it can be useful to draw how the current gets divided up in this circuit as shown below:



From this labelling we can make the following claims:

$I_1 = I_2 + I_3$

$I_4 = I_2 + I_3$

Therefore $I_1 = I_4$ (this is not needed for this question, but it is useful to understand!).

Solution continued

Since $I_1 = I_2 + I_3$ and we cannot have a negative current, this means that $I_1 \geq I_2$ and $I_1 \geq I_3$. The brightness of a bulb is determined by the power that the lightbulb dissipates, given by the formula $P = I^2R$. Since all of the lightbulbs have the same resistance, this means that the determining factor for their power output is the current (I) passing through them. Specifically, the bulb with the greatest amount of current running through it will shine the brightest. Therefore, R1 shines the brightest (since the current passing though R1 (I_1) is greater than the current passing through any of the other bulbs).

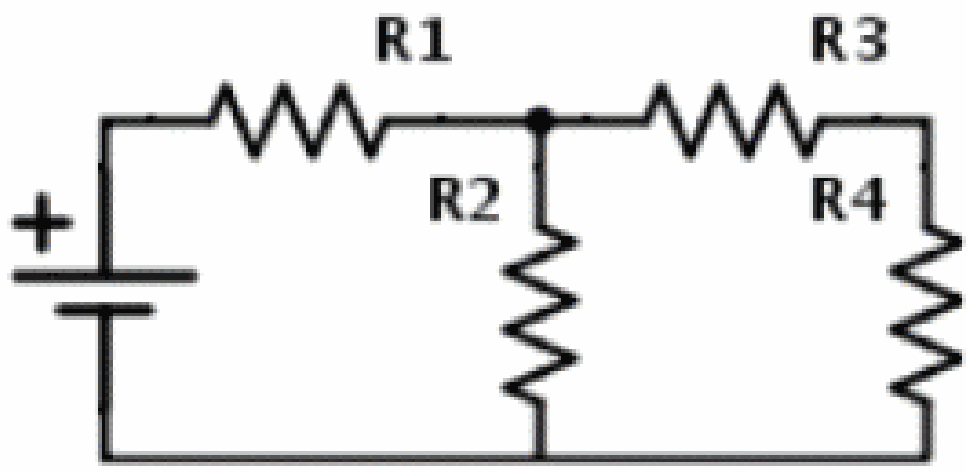
Another way to think about it is that R1 is brightest because all of the current from the battery **must** go through R1. The current going through the other bulbs is only a fraction of the current going through R1 and since they all have the same resistance, this means R1 is the brightest.

Therefore the answer is **A**.

Circuit Problems XIII

In the circuit diagram R1, R2, R3 and R4 are all lightbulbs with a resistance of 2 Ω. The battery has a voltage of 12 V. Use the diagram to answer the following question.

What is the total effective resistance of the circuit?



A. 8 Ω
B. 1 Ω
C. $\frac{10}{3}$ Ω
D. $\frac{13}{3}$ Ω

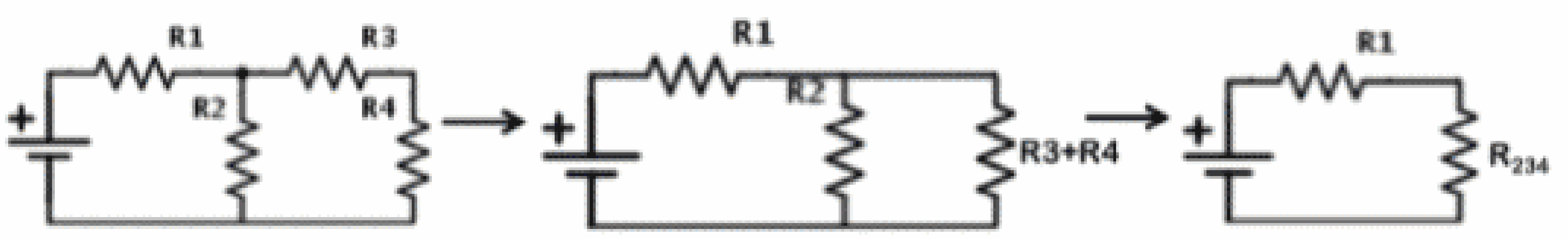
Solution

Answer: C

Justification: Here the first thing to notice is that R3 and R4 are actually in series with one another. Therefore the effective resistance of these two bulbs can simply be added. R2 and (R3 + R4) are in parallel, which means that the effective resistance of these three bulbs R_{234} can be found using:

$$\frac{1}{R_{234}} = \frac{1}{R2} + \frac{1}{(R3 + R4)} = \frac{1}{2} + \frac{1}{(2+2)} = \frac{3}{4} \quad \rightarrow \quad R_{234} = \frac{4}{3}\Omega$$

This process above can be thought of as reducing the circuit to a simpler and simpler circuit. This is shown schematically below:



Solution continued

Now it is as if there are two bulbs in this circuit R_{234} and R1 that are in series with one another. Therefore the total resistance is:

$$R_{total} = R1 + R_{234} = 2 + \frac{4}{3} = \frac{10}{3}\Omega$$ (answer **C**)

This answer can be arrived at conceptually by analyzing the provided answers to see which answers are actually possible:

A) 8 Ω – this is R_{total} when all bulbs are in series with one another, but we know that in parallel we do not simply add the resistances together, therefore this is **incorrect**.

B) 1 Ω – this cannot be the total resistance of the circuit because it is less than the resistance of R1, and all the current must go through R1, therefore the total resistance needs to be at least the value of R1. Therefore this answer is **incorrect**.

Solution continued 2

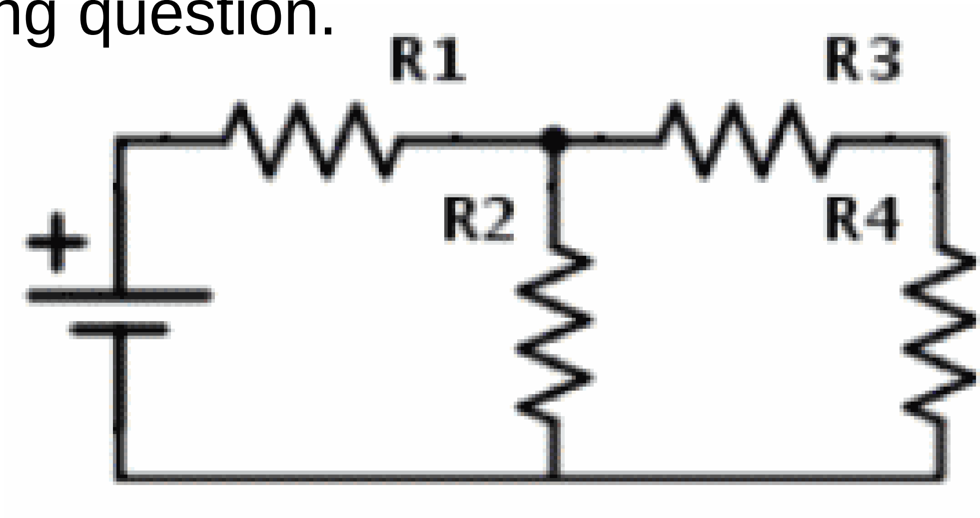
C) $\frac{10}{3}\,\Omega$ – this can be thought of as $2\Omega + \frac{4}{3}\Omega$ = R1 + all other resistances. Here we can see that "all other resistances" will be **less** than the resistance of each bulb on its own (i.e. less than 2 Ω). Since the other bulbs are in parallel, we know this should be true. Without crunching the numbers we can say this answer is a **possibility**.

D) $\frac{13}{3}\,\Omega$ – this can be thought of as $2\Omega + \frac{7}{3}\Omega$ = R1 + all other resistances. Here, "all other resistances" is **greater** than the resistance of each bulb on its own (i.e. greater than 2 Ω), which is not possible for a parallel circuit. Therefore this answer is **incorrect**.

After going through all of the possible answers, we can see that **C** is the only one that is not incorrect, therefore it must be the correct answer.

Circuit Problems XIV

In the circuit diagram R1, R2, R3 and R4 are all lightbulbs with a resistance of 2 Ω. The battery has a voltage of 12 V. Use the diagram to answer the following question.



What will happen if R2 → 0 Ω (i.e. the bulb is replaced with a wire)?

i. R1 will get brighter
ii. R1 will get dimmer
iii. R3 and R4 will both get brighter
iv. R3 and R4 will both get dimmer
v. R3 and R4 will both go out

A. i & iii
B. iii only
C. i & v
D. ii & iii
E. i &iv

Solution

Answer: C

Justification: First of all, lets figure out what happens when R2 → 0 Ω. When R2 → 0 Ω this means that there will be no resistance along the path where R2 is, which means that instead of the current splitting into the two different branches (R2 branch and R3+R4 branch), all of the current will only along the R2 branch (of zero resistance). This means that no current will pass through R3 and R4. This will decrease the effective resistance of the whole circuit (since R1 is the only resistor with current passing through it), and since the battery remains the same we can use the relationship $V = IR$ to see that the current produced by the battery will increase. We know that an increase in current corresponds to an increase in power, from the relation $P = I^2R$. Therefore the power output of R1 will increase, and since power output is directly related to brightness, the brightness of R1 will increase.

Solution continued

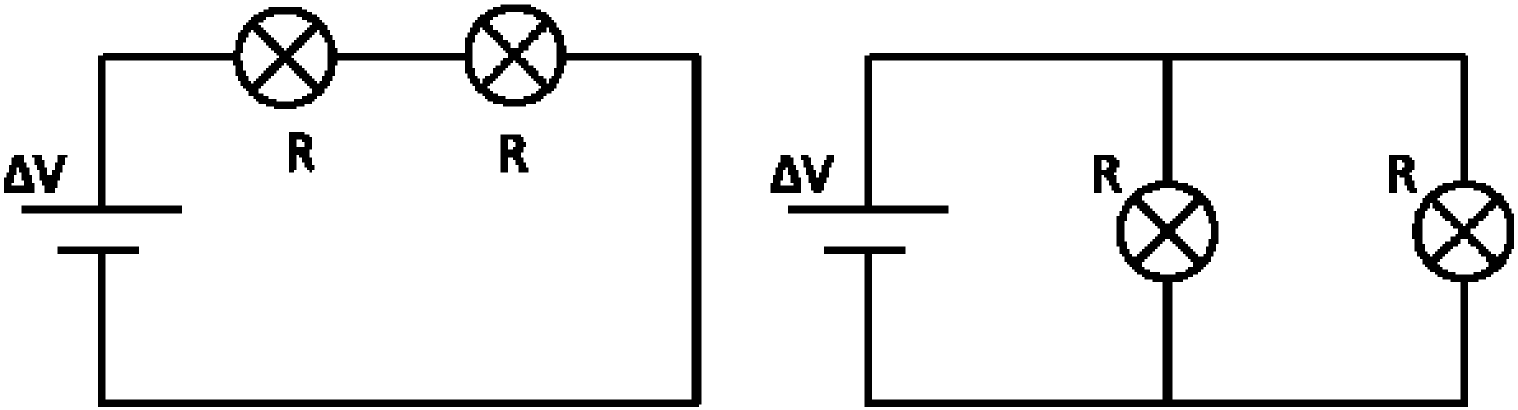
i) R1 will get brighter – this is **correct** because more current flows through R1
ii) R1 will get dimmer – this is **incorrect** because more current flowing through R1 will make it brighter
iii) R3 and R4 will both get brighter – this is **incorrect** because no current will be flowing through this part of the circuit
iv) R3 and R4 will both get dimmer – this is **incorrect** because no current will be flowing through this part of the circuit
v) R3 and R4 will both go out – this is **correct** because no current will be flowing through this part of the circuit

Therefore we can see that i and v are correct, therefore the correct answer is **C**.

Circuit Problems XV

Two identical light bulbs are connected first in a series circuit and then in a parallel circuit with the same battery.

In which circuit will the bulbs be brighter?



A. The bulbs will be brighter in the series circuit
B. The bulbs will be brighter in the parallel circuit
C. The bulbs will be equally bright in both circuits
D. There is not enough information provided to answer this question

Solution

Answer: B

Justification: If we simplify the circuit where the bulbs are in series we see that the total resistance of the circuit is: R_{total} = 1R + 1R = 2R.

If we simplify the circuit where the bulbs are in parallel we see that the total resistance of that circuit is:

$$\frac{1}{R_{total}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \quad \rightarrow \quad R_{total} = \frac{R}{2}$$

The total resistance when the bulbs are in series is higher than the total resistance when the bulbs are in parallel.

The power dissipated by a bulb is: $P = \frac{V^2}{R}$

So, as the total resistance of a circuit is reduced its corresponding power is increased. Therefore the circuit with the lowest total resistance would have the most power – thus the parallel circuit will have more power output, which means the bulbs will shine brighter.

Solution continued

Furthermore, if we look at the flow of current through the two systems we see that it decreases in a circuit wired in series, whereas it is split equally in the circuit wired in parallel (in this scenario, where the resistances are the same). This is because in the relationship $V = IR$, if R decreases then I must increase in order for voltage to stay constant (the battery stays the same). When resistance is lower (in the parallel circuit) the electrons flow more freely in the system.

We can put the power relationship in this way: $P = IV$

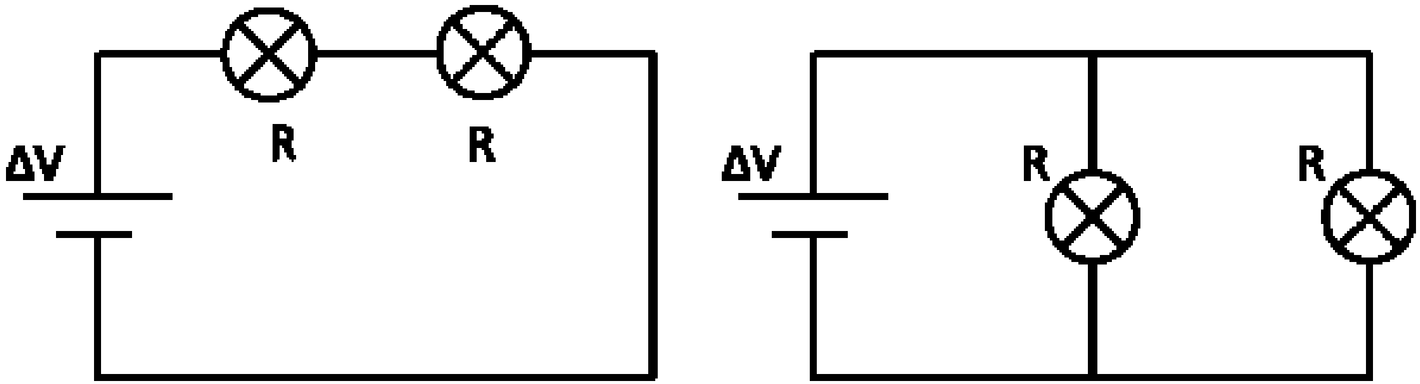
We can see here that if voltage is kept the same (the same battery), then a decrease in current would decrease the power output of the system. Therefore the series circuit (which has higher total resistance, and therefore lower current) will have a lower power output, whereas the parallel circuit (with lower total resistance and therefore higher current) will have a higher power output (brighter bulbs).

Therefore the answer is **B**.

Circuit Problems XVI

Two identical light bulbs are connected first in a series circuit and then in a parallel circuit with the same battery.

Which of the follow equations best describes the relative amount of power dissipated in each circuit?



A. $P_{series} = P_{parallel} / 4$
B. $P_{series} = P_{parallel} / 2$
C. $P_{series} = P_{parallel}$
D. $P_{series} = P_{parallel} \times 2$
E. $P_{series} = P_{parallel} \times 4$

Solution

Answer: A

Justification: If we simplify the circuit where the bulbs are in series we see that the total resistance of the circuit is: R_{total} = 1R + 1R = 2R.

If we simplify the circuit where the bulbs are in parallel we see that the total resistance of that circuit is:

$$\frac{1}{R_{total}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \quad \rightarrow \quad R_{total} = \frac{R}{2}$$

The power dissipated by a bulb is: $P = \frac{V^2}{R}$

Power for the circuit in series is:

Power for the circuit in parallel is:

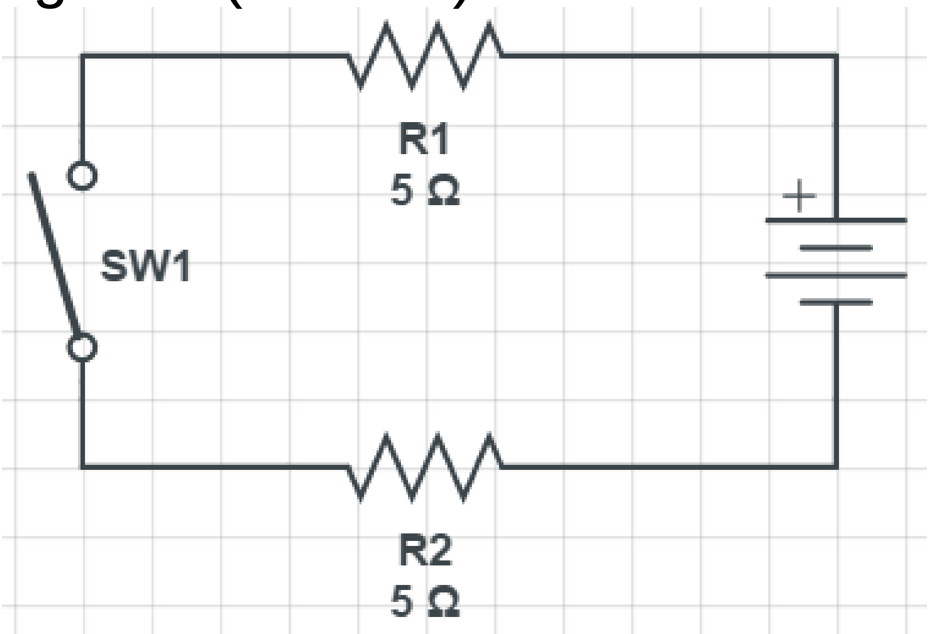
Solution continued

The relationship between the power of the two circuits is:

Therefore: (answer **A**)

Circuit Problems XVII

Compare the current passing through R1 (5 ohms) to the current passing through R2 (5 ohms) the moment after the switch is closed.



A. $I_1>I_2$
B. $I_1=I_2$
C. $I_1<I_2$
D. No current will flow through the circuit.

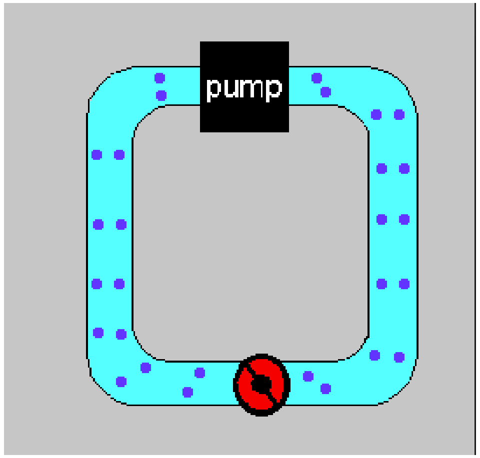
Solution

Answer: B

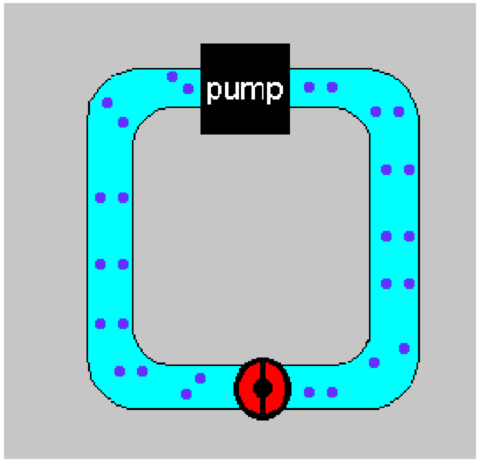
Justification: The moment the switch is closed the potential difference between the positive and negative ends of the batteries acts on the electrons in the wires, causing them to move from the negative end to the positive end of the battery.

By replacing the wires with pipes full of water, the battery with a pump and the resistors with turbines you can make a hydraulics analogy. When the switch is open the pump is off, and when the switch is closed the pump is on:

Pump off:



Pump on:



Note: To view animation, enter 'slide show' mode

Solution continued

Water is an incompressible fluid, which means if you push water at one end of a pipe the water throughout the remainder of the pipe has to move to make room for it. By turning on the pump the water inside the pump is propelled forward, which in turn pushes the water beyond that, resulting in movement of the water throughout the pipes.

Electrons behave in the same way. There are electrons throughout the wires before the switch is closed and since the electrons repel and are also incompressible, when some electrons move all electrons move. So when the switch is closed, current will immediately start flowing throughout the whole circuit.

Therefore as soon as the current starts flowing through resistor R1, it will also start flowing through resistor R2 (this is instantaneous).

Since this is a series circuit, the current will be the same throughout the circuit, therefore $I_1 = I_2$ (answer **B**).

Solution continued 2

Extended explanation:

What if R1 and R2 had different resistances? Would these differences affect the current passing through the resistors?

The current flowing depends on the voltage and the **total resistance** of the circuit: $V=IR_{total}$, therefore $I=V/R_{total}$

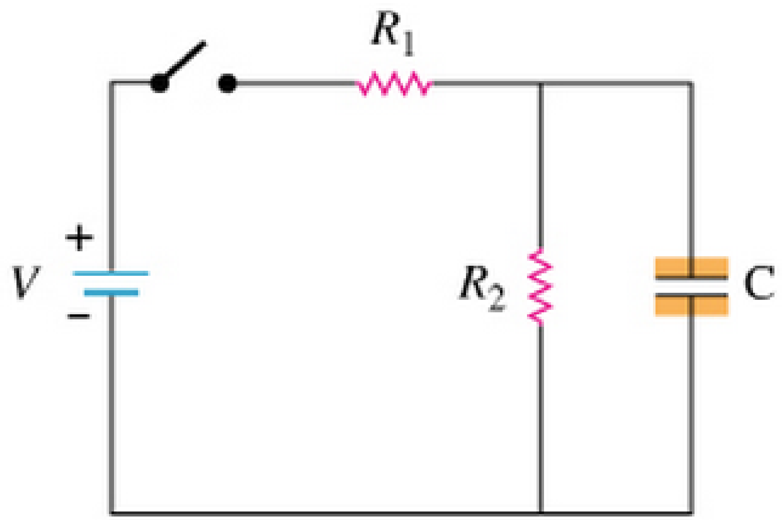
Therefore it doesn't matter if R1 and R2 have the same or different resistances, the current flowing through them will be the same (the current is dependent on the **combined total resistance** of the two).

In the hydraulic analogy we can visualize this by imagining a series of two turbines. If one turbine takes a huge amount of force to rotate and the other takes a small amount, the water will move at a fixed rate through both turbines. This is because the combined resistance sets the possible flow of water, given a constant pumping power (voltage).

Thus the current passing through R1 and R2 will be the same.

Circuit Problems XVIII

How does the current through R_1 compare with the current through R_2 immediately after the switch is first closed?



A. The current through R_1 is greater than the current through R_2.
B. The current through R_2 is greater than the current through R_1.
C. The current through R_1 is equal to the current through R_2.
D. The current flowing through both resistors will be zero.

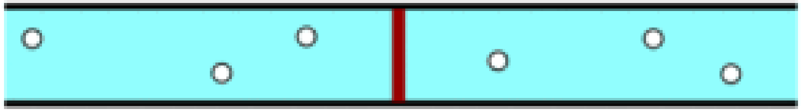
Solution

Answer: A

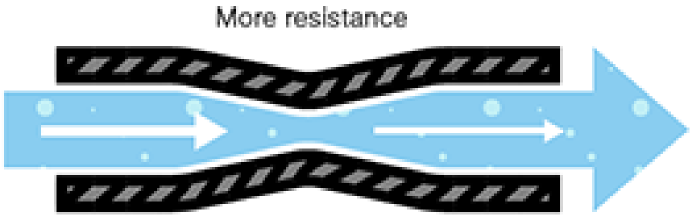
Justification: Immediately after the switch is closed there is no charge built up on the capacitor. An uncharged capacitor behaves like a short circuit in the sense that there is no voltage drop over the capacitor and thus current will flow with no resistance to charge the capacitor.

To create a hydraulic analogy to the situation imagine the same "circuit" of water pipes with resistor replaces by narrow segments of pipe and capacitors replaced with a stretchy water proof membrane.

Capacitor:



Resistor:



Note: To view animation, enter 'slide show' mode

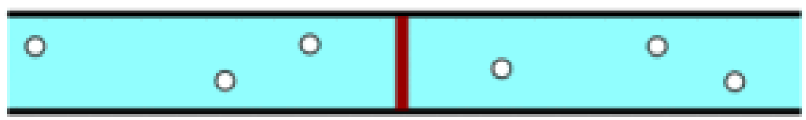
Solution continued

Immediately after the switch is closed we know water will flow through the first resistor because it is the only path the water can take. After the first resistor, the current has a choice between going through the second resistor (narrow pipe) or proceeding towards the membrane.

From the water's perspective the narrow pipe provides a resistance to the flow of water, while the membrane/capacitor hasn't been stretched/charged and thus provides no resistance to the water. Since water (and electrons) prefer the path of least resistance, all of the current will flow towards the capacitor and none will flow towards the second resistor. Thus the current through R_1 will initially be greater than the current through R_2.

Solution continued 2

From the analogy we can also see that the capacitor only has zero resistance for the **instant** the switch is flipped. As water stretches the rubber membrane the resistance to more water entering the pipe increases and thus the **moment after** the instant the switch is closed the resistance of the capacitor will be non-zero and thus some current will pass through R2.



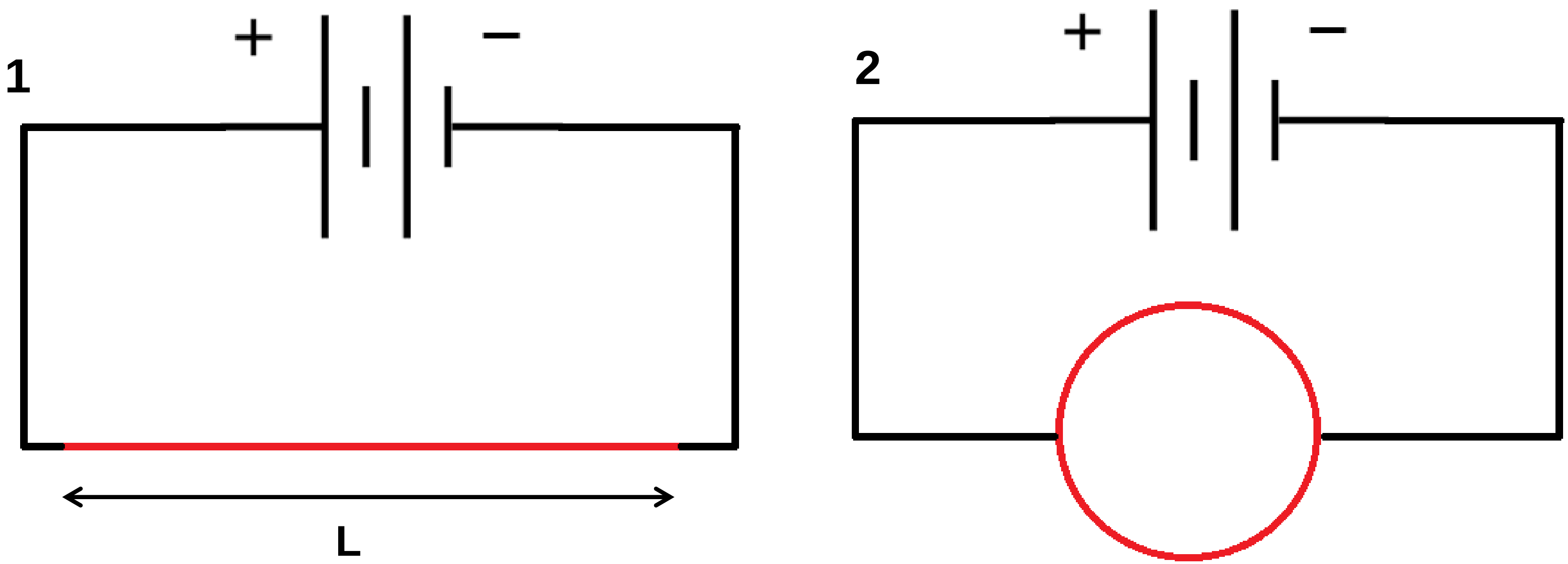
Returning to our electrical circuit we can now see that at the moment the switch is closed the capacitor has near zero resistance and thus "shorts" the second resistor.

This results in a current passing through R_1 but no current passing through R_2 the instant the switch is closed (answer **A**).

Try it for yourself using the following AC/DC circuit sim:

Circuit Problems XIX

A piece of Nichrome resistance wire of length L is connected up to a battery and has a resistance of R_L. This same wire is then shaped into a circle and connected up to the same battery by connecting two diametrically opposite points on the wire to the circuit.



Circuit Problems XIX continued

What is the equivalent resistance (R_E) of the wire in the second circuit?

A. R_E is infinite (an open circuit)
B. $R_E = R_L$
C. $R_E = ½ R_L$
D. $R_E = ¼ R_L$
E. $R_E = 0$ (a short circuit)

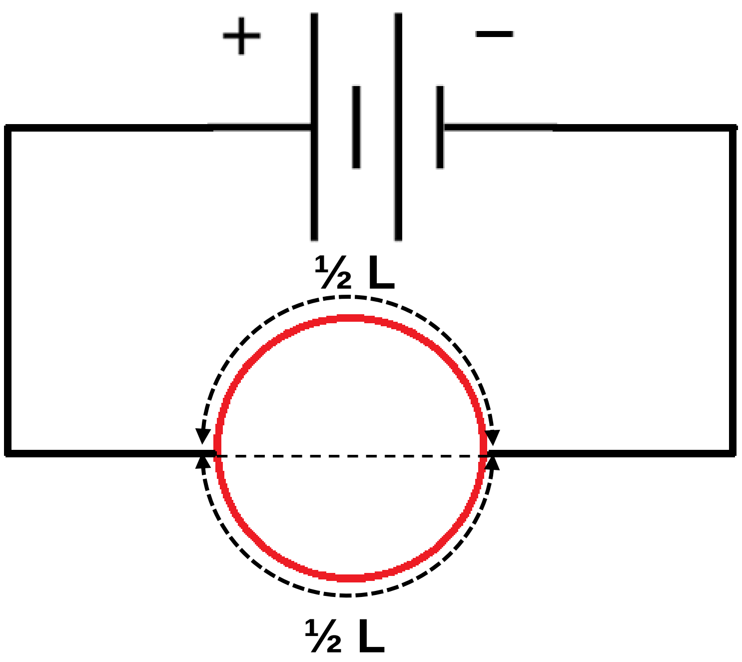
Solution

Answer: D

Justification: To answer this question we must know that the resistance of a wire is proportional to its length (in this case the cross sectional area of the wire is negligible compared to its length).

Since the same wire is used in both circuits, we know that the length of the wire (L) stays the same. Therefore, in the second diagram the circumference of the circle must be L.

Since the wires connected to the circle are diametrically opposite, this means that the circle is effectively split into two equal lengths that are half the circumference (½ L):

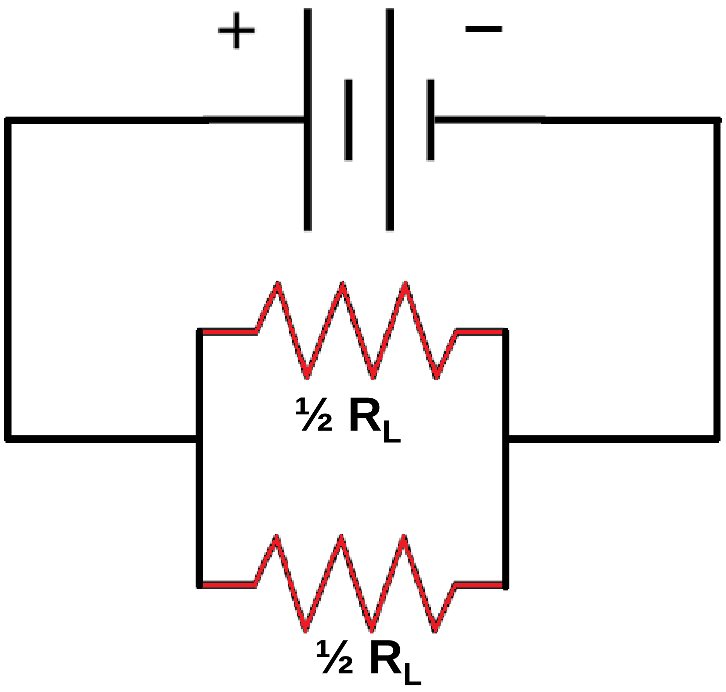


Solution

Since the length (L) of the wire is proportional to its resistance, then we can say that each branch (half of the circle) with length ½ L should have a resistance of ½ R_L.

So now we effectively have two resistors of resistance ½ R_L in parallel:

We can now use the formula for parallel resistors to work out the equivalent resistance:



$$\frac{1}{R_E} = \frac{1}{\frac{1}{2}R_L} + \frac{1}{\frac{1}{2}R_L} = \frac{2}{R_L} + \frac{2}{R_L} = \frac{4}{R_L}$$

Therefore: $R_E = \frac{1}{4}R_L$ (answer **D**)