a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Physics
Momentum Problems

Science and Mathematics
Education Research Group

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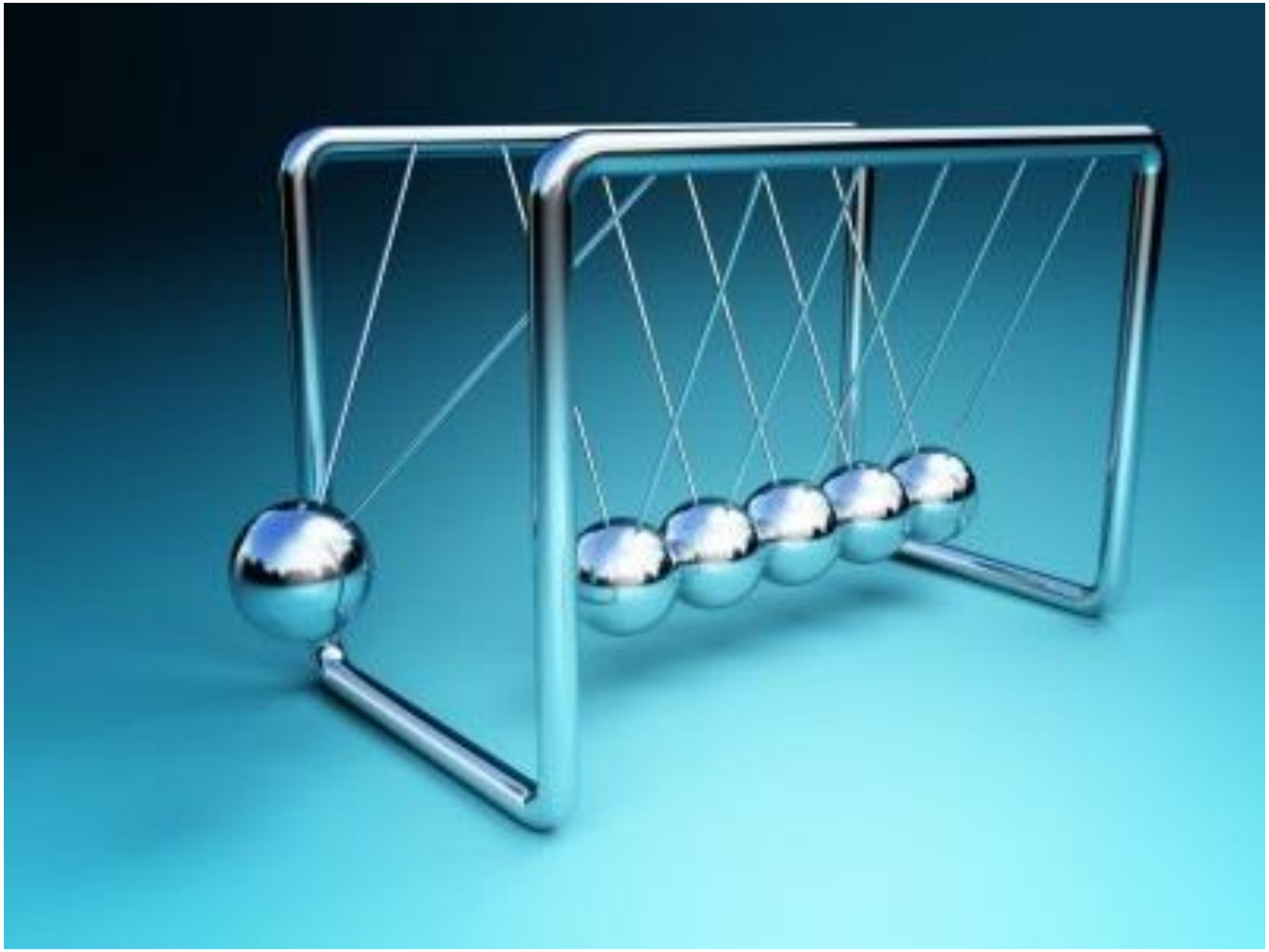
Momentum Problems



Retrieved from: http://www.two-it.com/the-momentum-is-building/

Momentum Problems

The following questions have been compiled from a collection of questions submitted on PeerWise (https://peerwise.cs.auckland.ac.nz/) by teacher candidates as part of the EDCP 357 physics methods courses at UBC.

Momentum Problems I

Two cars of equal mass (1100 kg) and speed (36 km/h) collide head on in a completely inelastic collision.

What is the vector sum of the momentum of the system of two cars after the collision?

A. 0 kg.m/s
B. 11,000 kg.m/s
C. 22,000 kg.m/s
D. 39,600 kg.m/s
E. 79,200 kg.m/s

Solution

Answer: A

Justification: This is an example of an inelastic collision. In these types of collisions kinetic energy is not conserved (it is converted into some other form of energy, such as heat energy or energy of deformation). However, no matter if it is an elastic collision (kinetic energy is conserved) or an inelastic collision, **momentum** would still be conserved.

Momentum, being a vector quantity, has both a magnitude and a direction. Momentum is defined as “mass in motion” and thus depends on the mass and velocity of the object. Since the cars were of equal mass and travelling towards each other at the same speed when they collided we know their momentums were equal in magnitude and opposite in direction. Therefore the vector sum of their momentum of the system **before** the collision would be zero.

Solution continued

Due to **conservation of momentum**, we know that the momentum of the system must be equal before and after the collision, therefore the vector sum of the momentum of the system **after** the collision would also be zero (answer **A**).

We can solve this without performing any calculations, but here is the solution using numbers:

First we convert km/h to m/s: $36km/h \times \frac{1000}{3600} = 100m/s$

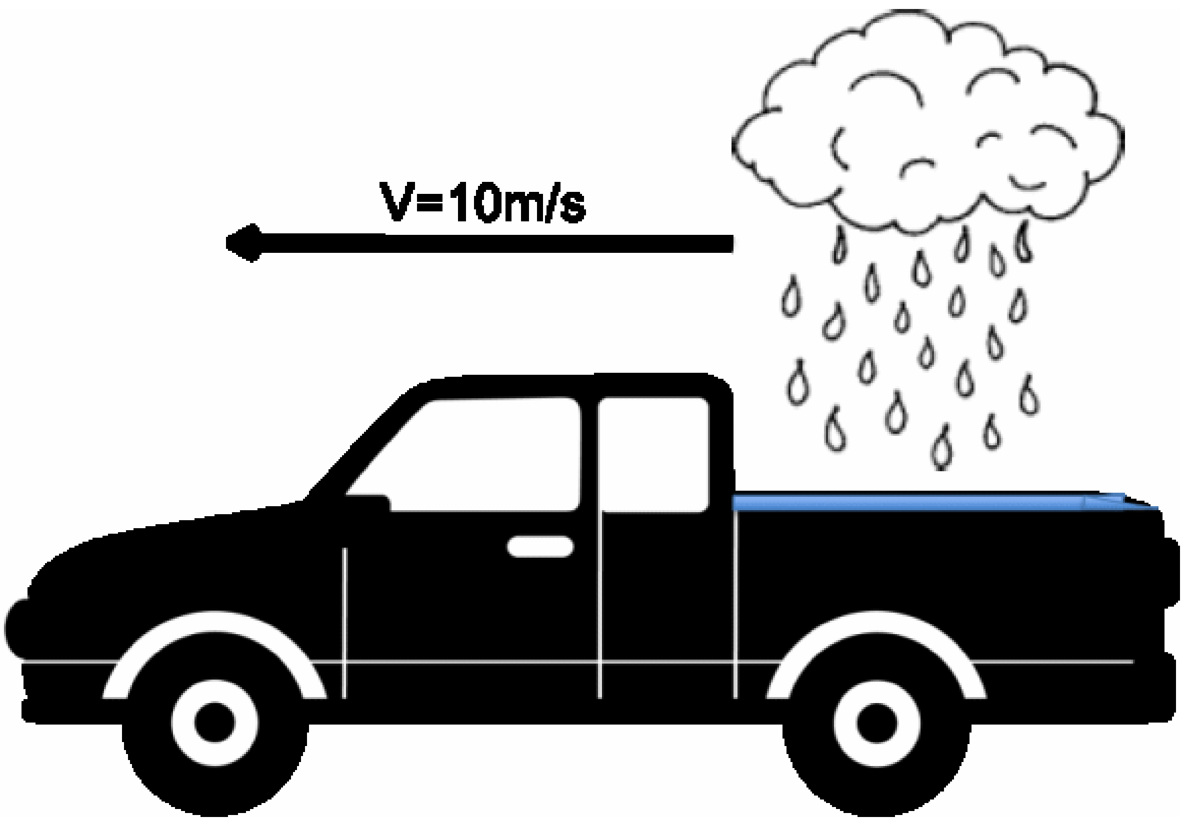
$$\begin{aligned} \boldsymbol{p_{car1}} &= m_{car1} \times \boldsymbol{v_{car1}} \\ &= 1100kg \times 100m/s \\ &= 11000kg.m/s \end{aligned}$$

$$\begin{aligned} \boldsymbol{p_{car2}} &= m_{car2} \times \boldsymbol{v_{car2}} \\ &= 1100kg \times -100m/s \\ &= -11000kg.m/s \end{aligned}$$

$$\boldsymbol{p_{system}} = 11000kg.\frac{m}{s} - 11000kg.m/s = 0kg.m/s$$

Momentum Problems II

Suppose you are driving your pick up truck to the store when rain begins to fall vertically from the sky into the bed of your truck.



Momentum Problems II continued

If you have set Cruise Control to 10 m/s what will happen to the kinetic energy and momentum of the truck as a result of the accumulating rain in the bed of the truck? You can assume that the road you are driving on has negligible friction and that you can ignore the effect of the rain droplets hitting the front of the vehicle.

A. The momentum of the truck increases, and the kinetic energy of the truck does not change.
B. The momentum of the truck does not change , and the kinetic energy of the truck decreases.
C. The momentum of the truck increases, and the kinetic energy of the truck increases.
D. The momentum of the truck increases, and the kinetic energy of the truck decreases.
E. The momentum of the truck does not change , and the kinetic energy of the truck does not change.

Solution

Answer: C

Justification: To answer this question we need to know that "Cruise Control" is a setting on most modern vehicles that allows the driver to maintain a **constant speed** without adjusting the gas pedal manually to account for going up or down small hills.

The question describes a truck moving along at a constant velocity (due to Cruise Control) with an increasing mass (increasing mass of water in the bed of the truck).

Since momentum is defined as $\boldsymbol{p} = m \times \boldsymbol{v}$ and the velocity is constant, increasing the mass of the truck will result in an increase to the momentum of the truck (it is a heavier object moving at the same speed).

Similarly, if we remember that the kinetic energy of the truck is written as $E_K = \frac{1}{2}m\boldsymbol{v}^2$ we can see that since the mass is growing larger and the velocity is constant the kinetic energy of the truck must increase as well.

Solution continued

Answer A is **incorrect** because it implies that the kinetic energy of the truck is not dependent on the truck’s mass.

Answer B is **incorrect** because it implies that the momentum of the truck is not dependent on the truck’s mass and that the truck’s kinetic energy is somehow inversely related to the mass.

Answer **C** is the **correct** answer.

Answer D is **incorrect** because it implies that the kinetic energy of the truck is somehow inversely related to the mass.

Answer E is **incorrect** because it implies that neither kinetic energy nor momentum is dependent on the mass of the truck.

Momentum Problems III

After finishing shopping at the store you head back outside to your truck and discover that the rain has stopped falling. After letting all the accumulated water out of the bed of your truck you start heading home.

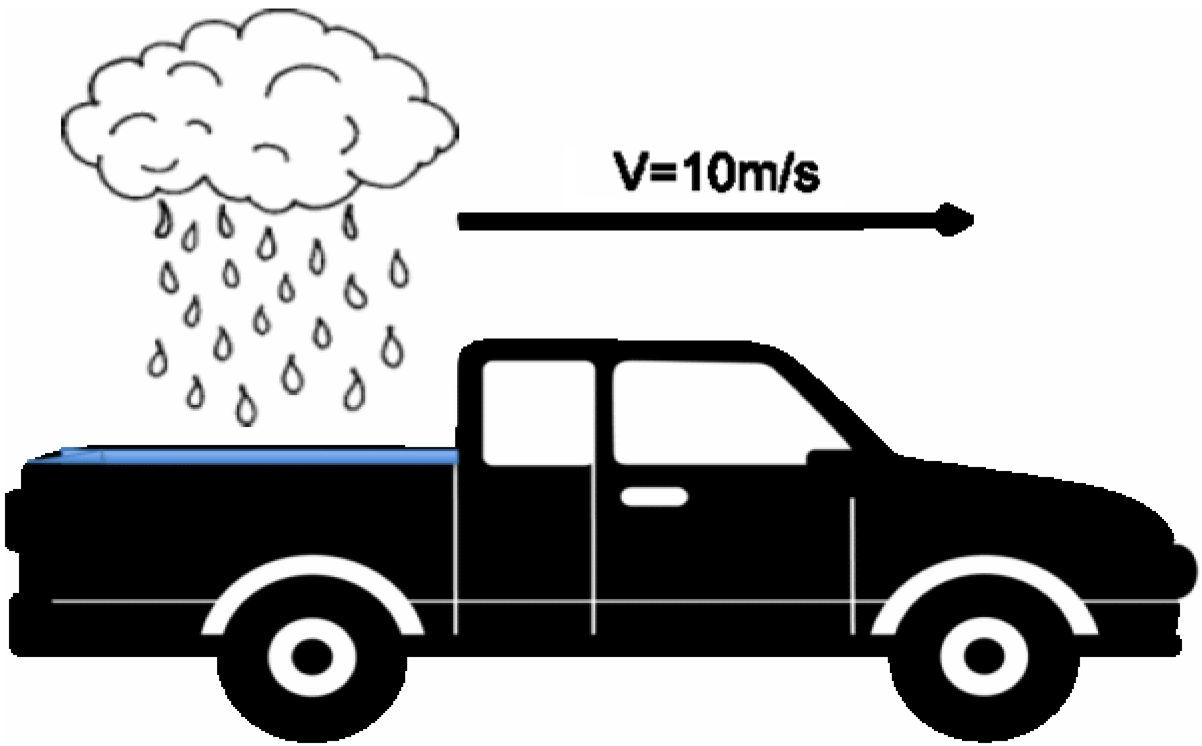
Suddenly, as you are driving along at 10 m/s, two things happen:

1. The vertical rain starts falling again.
2. The truck's engine suddenly dies.

You are almost home and decide to roll in neutral the rest of the distance to avoid getting wet.

Momentum Problems III continued

The rain begins accumulating again in the water-tight bed of the pick up truck.



Momentum Problems III continued

What effect will the accumulating water have on the momentum and kinetic energy of the truck?

You can assume that the road you are driving on has negligible friction and that you can ignore the effect of the rain droplets hitting the front of the vehicle.

A. The momentum of the truck increases, and the kinetic energy of the truck does not change.
B. The momentum of the truck does not change , and the kinetic energy of the truck decreases.
C. The momentum of the truck decreases, and the kinetic energy of the truck decreases.
D. The momentum of the truck does not change, and the kinetic energy of the truck does not change.
E. The momentum of the truck increases, and the kinetic energy of the truck increases.

Solution

Answer: B

Justification: The question describes a truck rolling along with an initial velocity of 10 m/s with a steadily increasing mass. Similarly to the last question we are going to need remember how momentum and kinetic energy depend on mass and relate to one another:

1) $\boldsymbol{p} = m \times \boldsymbol{v}$ 2) $E_K = ½m\boldsymbol{v}^2$

Unlike the last question, momentum is going to be conserved since there is no net external force acting on the system (i.e. the truck). This is because last time the Cruise Control setting meant that the truck maintained a constant speed even though its mass increased, whereas in this case the engine has cut out and cannot impart any force on the truck. With this bit of information we can immediately disregard any answer that suggests the momentum changes.

Solution continued

For the sake of clarity lets imagine that the mass of the truck doubles due to the accumulating rain. Although this is not realistic it will make the math easier to understand and the results will hold for any smaller increase in the mass of the truck.

If the mass of the truck doubles we can use the conservation of momentum to figure out the change in the truck's velocity:

Momentum before: $\boldsymbol{p} = m \times \boldsymbol{v_i}$ Momentum after: $\boldsymbol{p} = 2m \times \boldsymbol{v_f}$

Since momentum is conserved: $m \times \boldsymbol{v_i} = 2m \times \boldsymbol{v_f}$

Therefore: $\boldsymbol{v_f} = \frac{1}{2}\boldsymbol{v_i}$

Thus we can see that in this case, the velocity of the truck will be reduced by half.

Solution continued

Now let us look at how these changes will impact the kinetic energy of the system.

Kinetic energy before:

$$E_{Kinital} = \frac{1}{2}m \times \boldsymbol{v_i}^2$$

Kinetic energy after:

$$E_{Kfinal} = \frac{1}{2}(2m) \times \boldsymbol{v_f}^2$$
$$= m \times (\frac{1}{2}\boldsymbol{v_i})^2$$
$$= \frac{1}{4}m \times \boldsymbol{v_i}^2$$

Thus we can see that the final kinetic energy is less than the initial kinetic energy by a factor of ½ (the amount here is not important, it is more important to see that the kinetic energy of the truck **decreases**).

Therefore the momentum of the system will remain constant while the kinetic energy of the system will decrease (answer **B**).

Momentum Problems IV

Andrew is driving his 2000 kg car at 50 km/h. He diverts his attention from the road to text a friend and drives into the back end of the car in front of him, moving them both at a speed of 40 km/h. If the second car has a mass of 1000 kg, how fast was it travelling prior to the collision?

A. 0 km/h
B. 10 km/h
C. 20 km/h
D. 30 km/h
E. 40 km/h

Solution

Answer: C

Justification: The principle of the conservation of momentum requires that the momentum of the system (the two cars together) before the collision be equivalent to the momentum of the system after the collision.

The momentum of Andrew's car before the collision is:

$$\mathbf{p}_{\mathrm{Ai}} = m_{\mathrm{A}} v_{\mathrm{Ai}} = (2000\ \mathrm{kg})(50\ \mathrm{km/h}) = 100\ 000\ \mathrm{kg.km/h}$$

The momentum of the second car before the collision is:

$$\mathbf{p}_{\mathrm{Ci}} = m_{\mathrm{C}} v_{\mathrm{Ci}} = (1000\ \mathrm{kg})(v_{\mathrm{Ci}})$$

Therefore the total momentum of the cars before the collision is:

$$\mathbf{p}_{\mathrm{Ti}} = \mathbf{p}_{\mathrm{Ai}} + \mathbf{p}_{\mathrm{Ci}} = 100\ 000\ \mathrm{kg.km/h} + (1000\ \mathrm{kg})(v_{\mathrm{Ci}})$$

Solution continued

After the collision, the two cars are joined, therefore the total momentum of the two cars after the collision is:

$$p_{Tf} = (m_A + m_C)v_f = (1000kg + 2000kg)(40km/h) = 120\ 000\ kg.km/h$$

Conservation of momentum requires that:

$$p_{Ti} = p_{Tf}$$

$$100\ 000\ kg.km/h + (1000\ kg)(v_{Ci}) = 120\ 000\ kg.km/h$$

$$(1000\ kg)(v_{Ci}) = 20\ 000\ kg.km/h$$

$$v_{Ci} = 20\ km/h$$

Therefore the second car was initially driving at 20 km/h (answer **C**).

Momentum Problems V

When the momentum of an object doubles and its mass remains constant, its kinetic energy:

A. is halved
B. doubles
C. quadruples
D. increases by $\sqrt{2}$
E. None of the above

Solution

Answer: C

Justification: Momentum of an object is defined as the product of its mass and its velocity: $\mathbf{p} = m\boldsymbol{v}$

If the momentum is doubled while the mass remains constant then the velocity must be doubled: $2\mathbf{p} = m(2\boldsymbol{v})$

The kinetic energy of an object is defined by the formula: $E_K = \frac{1}{2}m\boldsymbol{v}^2$

Substituting the new velocity, 2***v***, into this equation:

$$E_K = \frac{1}{2}m(2\boldsymbol{v})^2$$
$$E_K = \frac{1}{2}m(4\boldsymbol{v}^2)$$
$$E_K = 4(\frac{1}{2}m\boldsymbol{v}^2)$$

Therefore, by doubling the momentum of an object we quadruple its kinetic energy (answer **C**).

Momentum Problems VI

If the system pictured below is an isolated system and has an initial momentum of zero, what can be said about the initial kinetic energy (E_K) of the system?

Note that M_A and M_B are the masses of Block A and B respectively. Similarly, $\mathbf{V_A}$ and $\mathbf{V_B}$ are the velocities of Block A and B respectively.

A. $E_K > 0$ J
B. $E_K < 0$ J
C. $E_K \geq 0$ J
D. $E_K = 0$ J
E. $E_K \leq 0$ J

Solution

Answer: C

Justification: To assess this question it is important to understand that momentum is a **vector** quantity and kinetic energy is a **scalar** quantity. Since the initial momentum of the system is zero this means that the sum of the momentum vectors is zero. This can happen when:

1)the momentum of block A and block B are in equal and opposite directions or

2)the momentum of each block is zero, which occurs when $\mathbf{V_A} = \mathbf{V_B} = 0$ m/s.

The corresponding kinetic energies to each situation are as follows:

1)$E_K > 0$ J since $E_k = \frac{1}{2}m\mathbf{v}^2$ and each of the blocks has a velocity that is non-zero

2)$E_K = 0$ J since $E_k = \frac{1}{2}m\mathbf{v}^2$ and both of the blocks have a velocity of zero

Solution continued

Therefore, by putting the two possible options together, we get the answer of $E_K \geq 0$ J (answer **C**).

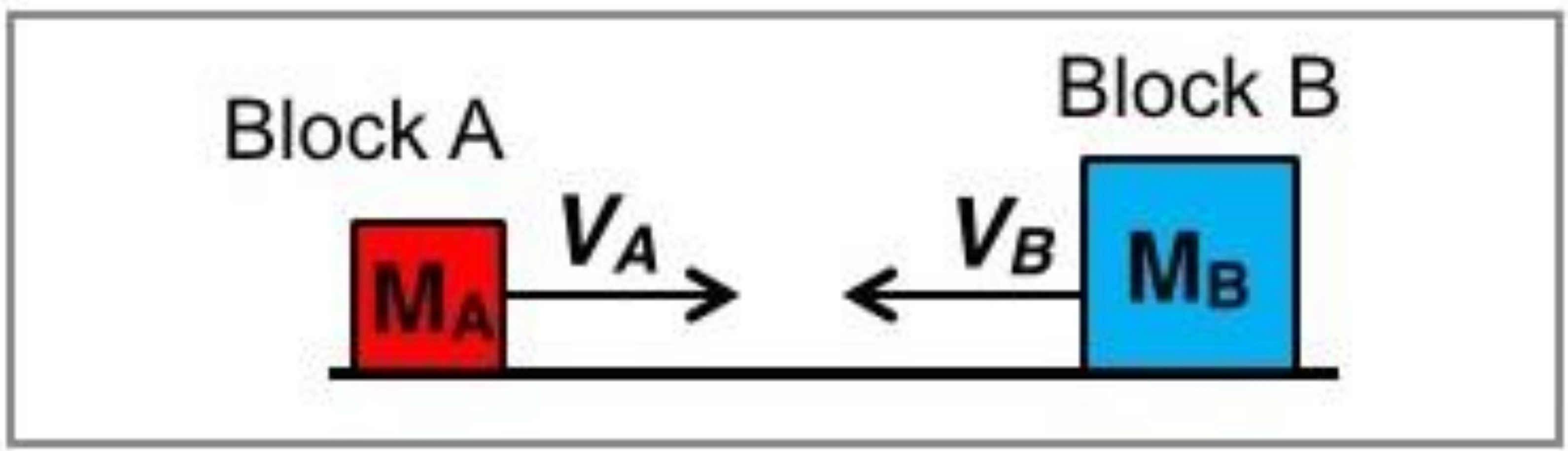
It is important to note that answers **B** and **E** could never be true, since the kinetic energy of a system can never be less than zero.

Momentum Problems VII

The system pictured below is an isolated system and it is known that the kinetic energy of Block B is four times that of Block A.

Note that M_A and M_B are the masses of Block A and B respectively.

Similarly, $\mathbf{V_A}$ and $\mathbf{V_B}$ are the velocities of Block A and B respectively.



Momentum Problems VII continued

Which of the following statements is true?

Choose the **best** answer.

i. $|\mathbf{V_A}| = |\mathbf{V_B}|$ and $M_A = 4M_B$

ii. $|\mathbf{V_A}| = |\mathbf{V_B}|$ and $4M_A = M_B$

iii. $|2\mathbf{V_A}| = |\mathbf{V_B}|$ and $M_A = M_B$

iv. $|\mathbf{V_A}| = |2\mathbf{V_B}|$ and $M_A = M_B$

A. i, iii

B. ii, iii

C. i, iv

D. ii, iv

E. ii only

Solution

Answer: B

Justification: To figure out this question you need to know that

$E_K = \frac{1}{2} m\boldsymbol{v}^2$

Since the question says "the kinetic energy of Block B is four times that of Block A" this means we can write:

$4E_{KA} = E_{KB}$ where the A and B represent blocks A and B respectively

Now we can assess each of the scenarios (i to iv) using the formula for kinetic energy of each block:

$E_{KA} = \frac{1}{2} M_A \boldsymbol{V_A}^2$ and $E_{KB} = \frac{1}{2} M_B \boldsymbol{V_B}^2$

To assess each scenario we need to substitute the values provided into one of the two equations above so that we have both equations written in terms of $\mathbf{V_A}$ and M_A OR $\mathbf{V_B}$ and M_B. Once we have done this we can check to see if the equality $4E_{KA} = E_{KB}$ holds up in each situation.

Solution continued

i) $|\mathbf{V_A}| = |\mathbf{V_B}|$ and $M_A = 4M_B$

$$E_{KA} = ½M_A\boldsymbol{V_A}^2 = ½(4M_B)\boldsymbol{V_B}^2 = 4(½M_B\boldsymbol{V_B}^2) = 4E_{KB}$$

This equality is the opposite of what is required (in this case the kinetic energy of block A is four times that of B), therefore (i) is **incorrect**.

ii) $|\mathbf{V_A}| = |\mathbf{V_B}|$ and $4M_A = M_B$

$$E_{KB} = ½M_B\boldsymbol{V_B}^2 = ½M_A\boldsymbol{V_A}^2 = 4(½M_A\boldsymbol{V_A}^2) = 4E_{KA}$$

In this case we get the correct equality, therefore (ii) is **correct**.

Solution continued

iii. $|2\mathbf{V_A}| = |\mathbf{V_B}|$ and $M_A = M_B$

$$E_{KB} = ½M_B\boldsymbol{V_B}^2 = ½M_A(2\boldsymbol{V_A})^2 = ½M_A(4\boldsymbol{V_A}^2) = 4(½M_A\boldsymbol{V_A}^2) = 4E_{KA}$$

In this case we get the correct equality, therefore (iii) is **correct**.

iv. $|\mathbf{V_A}| = |2\mathbf{V_B}|$ and $M_A = M_B$

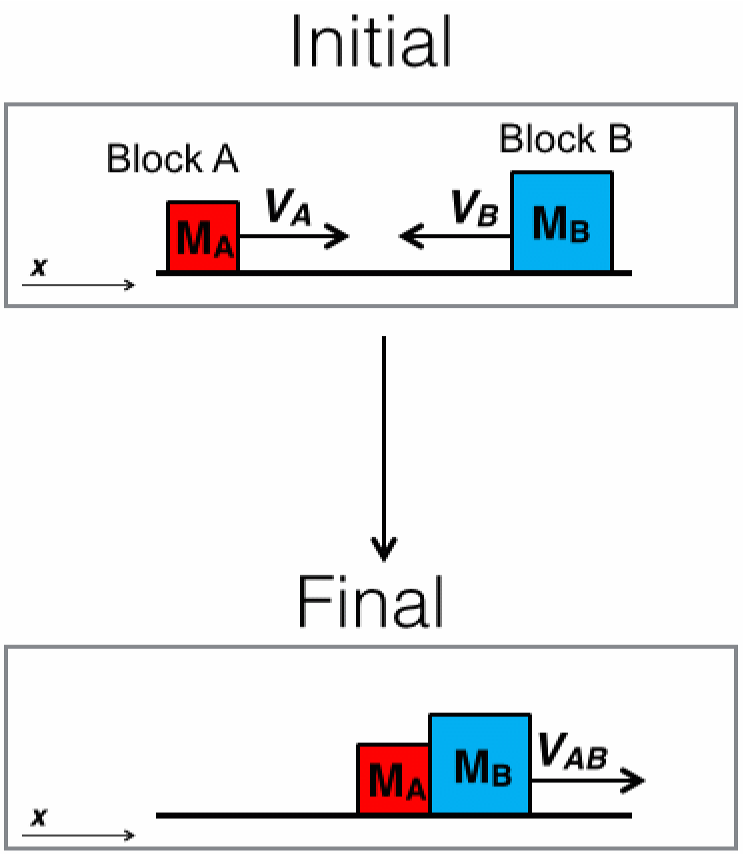
$$E_{KA} = ½M_A\boldsymbol{V_A}^2 = ½M_B(2\boldsymbol{V_B})^2 = ½M_B(4\boldsymbol{V_B}^2) = 4(½M_B\boldsymbol{V_B}^2) = 4E_{KB}$$

This equality is the opposite of what is required (in this case the kinetic energy of block A is four times that of B), therefore (iv) is **incorrect**.

The correct answers were (ii) and (iii), therefore the answer is **B**.

Momentum Problems VIII

Consider the isolated system shown to the right. When Blocks A and B collide it is an inelastic collision where momentum is conserved, but some kinetic energy is converted to sound and heat upon the collision. The moment when the two blocks collide the velocity of Block B is ½$\boldsymbol{V}$ in the direction shown in the "Initial" diagram. After the collision the velocity of the two blocks "stuck together" is 2$\boldsymbol{V}$ in the direction shown in the "Final" diagram. The positive direction is indicated by the *x* arrow.



Momentum Problems VIII continued

If the mass of block B is **two times** the mass of Block A, what must the velocity of Block A be?

Note: M_A and M_B are the masses of Block A and B respectively. Similarly, $\mathbf{V_A}$ and $\mathbf{V_B}$ are the velocities of Block A and B respectively. M_{AB} is the combined mass after the collision and $\mathbf{V_{AB}}$ is the velocity of this mass after the collision.

A. $V_A = 7\mathbf{V}$

B. $V_A = 5\mathbf{V}$

C. $V_A = 3.4\mathbf{V}$

D. $V_A = -3.4\mathbf{V}$

E. $V_A = -\frac{11}{2}\mathbf{V}$

Solution

Answer: A

Justification: First of all, since this is an inelastic collision, we know that kinetic energy is **not** conserved. Therefore we cannot use conservation of kinetic energy to solve this problem. However, we can use the conservation of momentum.

To begin, we need to write down all of the information that has been provided in the text of the question. We need to **keep in mind the positive *x* direction** as indicated in the question.

Initially we know:

$\boldsymbol{V_B} = -\frac{1}{2}\boldsymbol{V}$
$M_B = 2M_A$
$\boldsymbol{V_A} = ?$

Finally we know:

$\boldsymbol{V_{AB}} = 2\boldsymbol{V}$
$M_{AB} = M_A + M_B = M_A + 2M_A = 3M_A$

Solution continued

Since momentum is conserved this means that the momentum of the system in the initial and final diagrams must be equal to one another. Using this information in combination with the equation for momentum, $\boldsymbol{p} = m\boldsymbol{v}$, and keeping in mind that momentum is a vector quantity we can say:

$$\boldsymbol{p_i} = \boldsymbol{p_f}$$

$$\boldsymbol{p_A} + \boldsymbol{p_B} = \boldsymbol{p_{AB}}$$

$$M_A\boldsymbol{V_A} + M_B\boldsymbol{V_B} = M_{AB}\boldsymbol{V_{AB}}$$

$$M_A\boldsymbol{V_A} + (2M_A)(-\tfrac{1}{2}\boldsymbol{V}) = (3M_A)(2\boldsymbol{V})$$

Cancel out all M_A's because they are in all terms and simplify:

$\boldsymbol{V_A} - \boldsymbol{V} = 6\boldsymbol{V}$

Therefore $\boldsymbol{V_A} = 7\boldsymbol{V}$ (answer **A**)

Momentum Problems IX

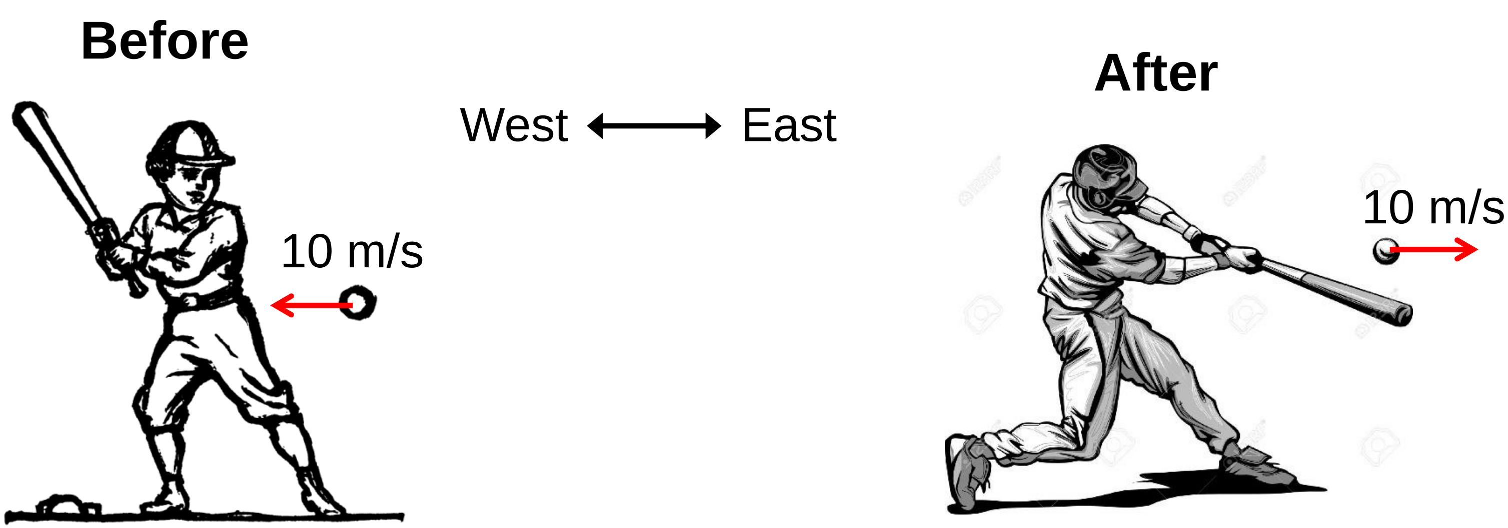
A baseball (1 kg) coming **from** the East is approaching Barry at 10 m/s. Barry hits the baseball back to where it came from and the baseball leaves his bat at 10 m/s. What is the impulse Barry delivered to the baseball?

A. 0 N.s to the East
B. 0 N.s to the West
C. 10 N.s to the East
D. 20 N.s to the East
E. 20 N.s to the West

Solution

Answer: D

Justification: In a collision, an object experiences a force for a specific amount of time that results in a change in momentum. The result of the force acting for the given amount of time is that the object's mass either speeds up or slows down (or changes direction). The impulse experienced by the object equals the **change in momentum** of the object. To answer this question, let us first draw a diagram:



Solution continued

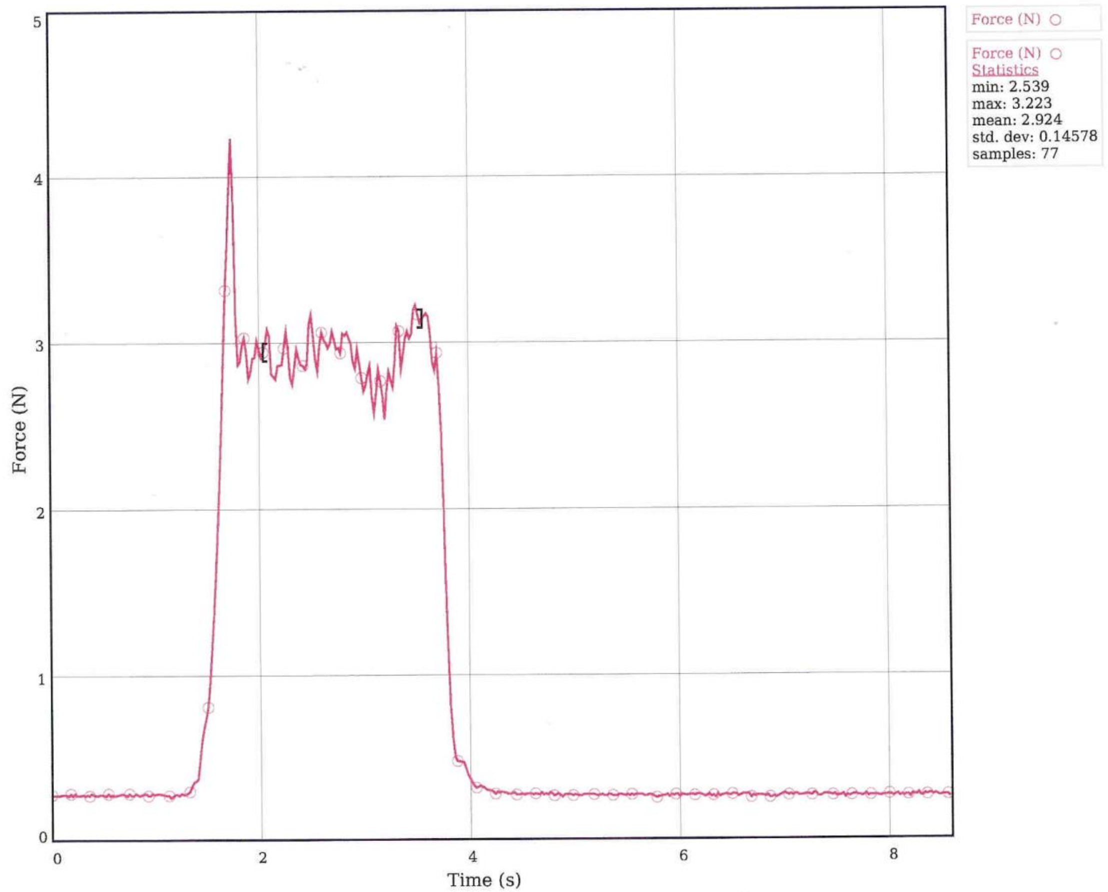
The ball is coming **from** the East, therefore it must be heading West. If we take East as the positive direction, we have:

$$\begin{aligned} Impulse = m\Delta\boldsymbol{v} &= m\left(\boldsymbol{v}_{\boldsymbol{final}} - \boldsymbol{v}_{\boldsymbol{initial}}\right) \\ &= 1kg\left((10m/s) - (-10m/s)\right) \\ &= 1kg(20m/s) = 20\ kg.m/s = 20\ N.s \end{aligned}$$

Since the final answer is positive, the change in momentum must be directed towards the East. Therefore the answer is 20 N.s to the East (answer **D**).

Momentum Problems X

Below is a graph of the impulse experienced by a driver without his seat belt and air bag during a car crash.



Momentum Problems X continued

What is **NOT** true if he was wearing his seat beat and his air bag was working?

A. The maximum force would be lower
B. The graph would be more spread out
C. The area under the graph would be less
D. The peak of the graph may be slightly shifted

Solution

Answer: C

Justification: Impulse can be delivered in one big force for a short duration or distributed over a longer time frame at a lower force. We can see this in one of the equations used to define impulse:

Impulse = Force × Time

We can now analyze each answer. Remember that we are looking for the answer which is **NOT** true:

A) Both the air bag and the seat belt would help dampen the crash for the driver. They would distribute the force of the crash over a longer period of time, and therefore the maximum force experienced by the driver would be decreased. Therefore this answer is true.

Solution continued

B) The air bag and the seat belt would dampen the force and spread this out over a longer period of time. Since the graph is one of Force vs. Time, if the time is increased, the graph would be more spread out. Therefore this answer is true.

C) Since we are dealing with the same collision, we know that the total change in momentum should be the same for both the situation with and without a seatbelt and airbag. Therefore the impulse delivered is the same. Since the area of the graph is *Force* × *Time* = *Impulse*, the area should not be lower. Therefore this answer is not true, and is the answer we are looking for.

D) That the peak "MAY" be shifted doesn't necessarily mean it will, in some cases it will. As the graph spreads out, the peak may be slightly shifted. It will depend on the design of the seatbelt and airbag, and how exactly they spread out the force of the impact. See the next page for some examples of how the force can be distributed over time.

Solution continued 2