a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

# Physics
# More Vector Problems

Science and Mathematics
Education Research Group

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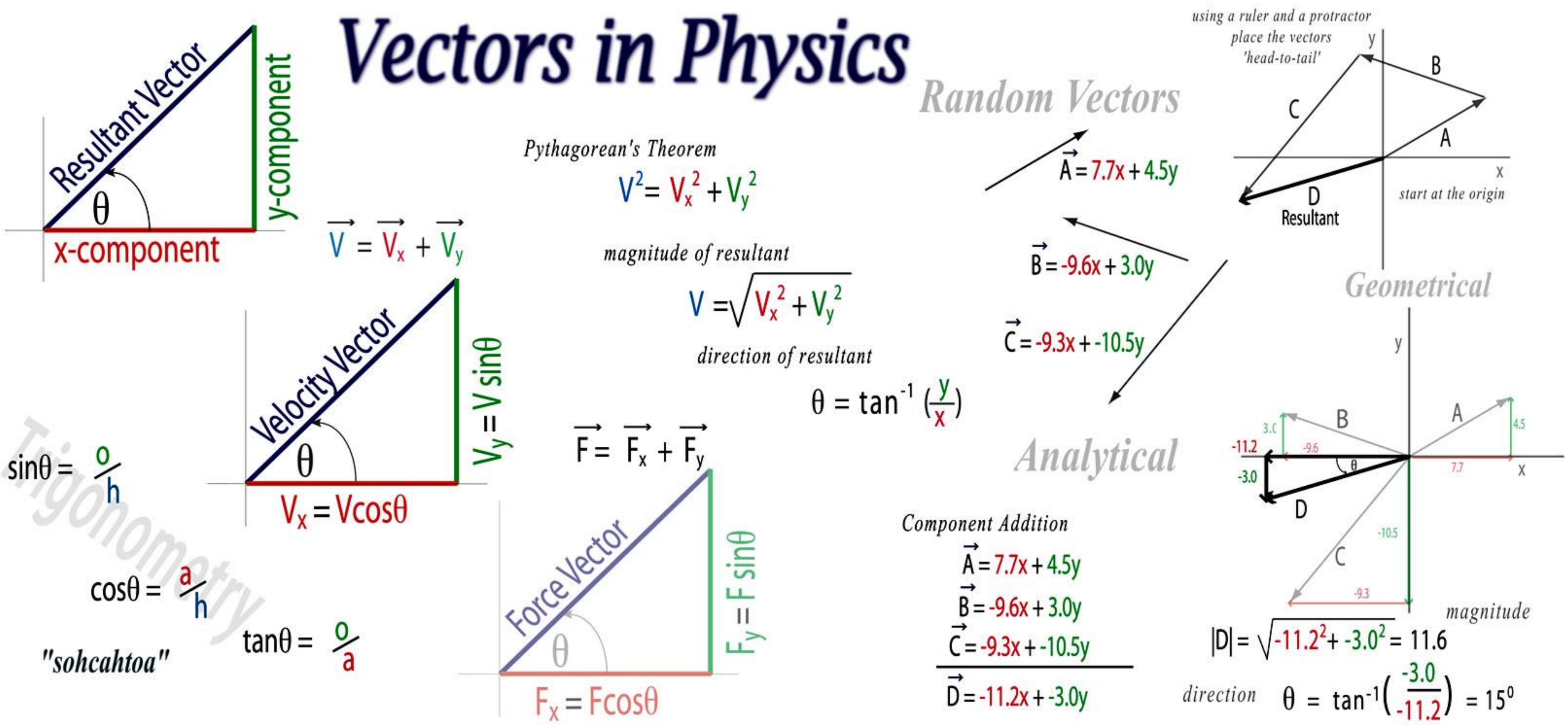
# More Vector Problems

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# More Vector Problems

The following questions have been compiled from a collection of questions submitted on PeerWise (https://peerwise.cs.auckland.ac.nz/) by teacher candidates as part of the EDCP 357 physics methods courses at UBC.

# More Vector Problems I

To get to work, Hamid drives northwest for 1.5hrs at 80km/h, and then east for 45 minutes at 60km/h.

What is Hamid's average speed, rounded to the nearest second?

A. 42 km/h

B. 60 km/h

C. 70 km/h

D. 73 km/h

E. 80 km/h

# Solution

**Answer:** D

**Justification:** Speed is a scalar and only has a magnitude, and no direction. Here we are looking for Hamid's **average speed**, which is defined as the total distance travelled divided by the total time taken.

So we first need to find out how far Hamid drives in each direction:

NorthWest: 1.5 hours × 80 km/h = 120 km

East: 0.75 hours × 60 km/h = 45 km (Note: 45 min = 0.75 hrs)

Therefore Hamid travels for a total of 165 km in 2.25 hours

So his average speed is: 165/2.25 = 73.333 ≈ 73 km/h

Therefore the answer is **D**

# Solution continued

If you just took the average of the two speeds (80 km/h and 60 km/h) you would get 70 km/h. This is incorrect because Hamid travelled twice as long at 80 km/h than he did at 60 km/h.

If you worked out the magnitude of the displacement vector of Hamid's trip, and used this value in your calculation you would get 42 km/h. This is actually the magnitude of the average velocity of Hamid's trip.

Note: This question can also be worked out conceptually. We know that the average speed must be somewhere between 60 and 80 km/h, so this eliminates options A, B and E. We also know that Hamid travelled longer at 80 km/h than at 60 km/h, so we would expect our answer to be closer to 80 than 60. Therefore our answer is D.

# More Vector Problems II

Once again, Hamid drives northwest for 1.5hrs at 80km/h, and then east for 45 minutes at 60km/h to get to work. However, this time Hamid decides that he will fly his helicopter straight home from work.

How far (**d**) would Hamid have to fly in his helicopter to get back home?

How long (**t**) would it take if his helicopter flies 200km/h?

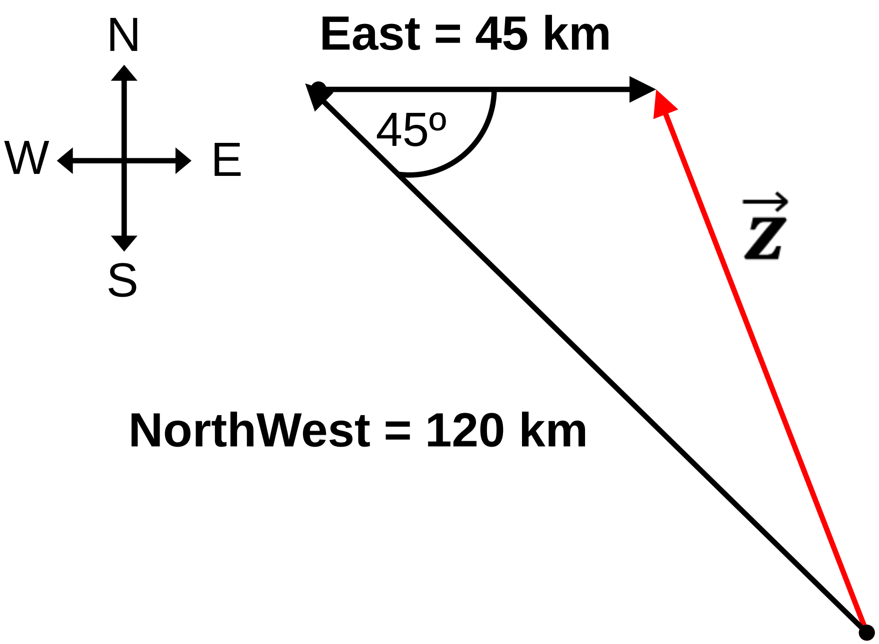
| | |
|---|---|
| A. d = 165 km | t = 1 hour 13 minutes |
| B. d = 155 km | t = 47 minutes |
| C. d = 94 km | t = 28 minutes |
| D. d = 165 km | t = 50 minutes |
| E. d = 94 km | t = 2 hours 8 minutes |

# Solution

**Answer:** C

**Justification:** From the previous question, we know how far Hamid travelled in each direction. We can calculate the magnitude of the resultant vector in two different ways:

**1. Cosine rule:**

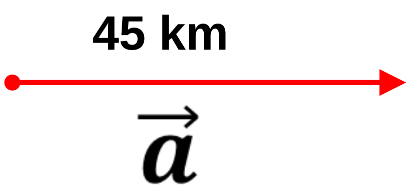



We can use the cosine rule to calculate vector **z**:

$$\boldsymbol{z}^2 = 45^2 + 120^2 - 2(45)(120)\cos 45$$
$$\boldsymbol{z}^2 = 2025 + 14400 - 7636.75$$
$$\boldsymbol{z}^2 = 8788.25$$
$$\boldsymbol{z} = \sqrt{8788.25}$$
$$\boldsymbol{z} = 93.7 \approx 94\ km$$

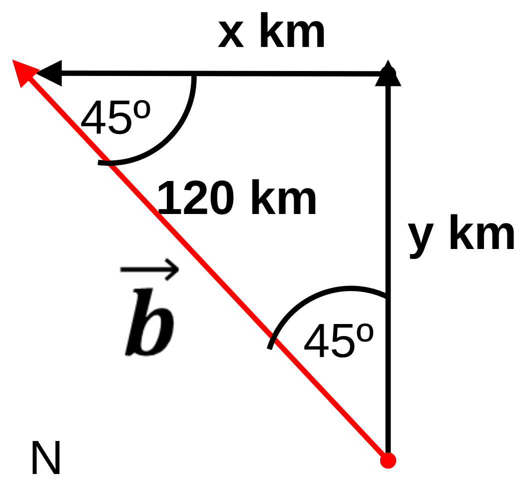
# Solution continued

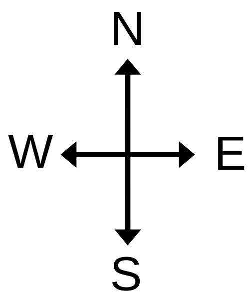
**<u>2. Adding vectors:</u>** First, we describe the two sections of the journey as two vectors, **a** and **b**. If we define West-East as the x-coordinates (East is positive) and North-South as the y-coordinates (North is positive), we can calculate the x and y components of each vector:




Since vector **a** only has an x-component,

**a** = (45, 0)




We can use the Sine rule to find the components of **b**:

$$\frac{|\vec{b}|}{\sin 90°} = \frac{|\vec{x}|}{\sin 45°} = \frac{|\vec{y}|}{\sin 45°}$$

Therefore:

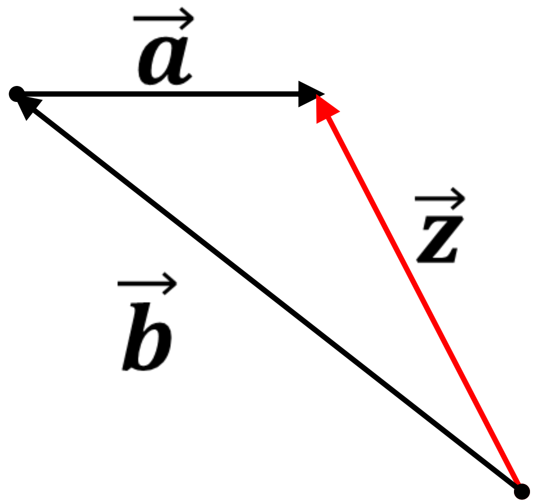
$$|\vec{x}| = |\vec{y}| = \frac{120 \times \sin 45°}{\sin 90°}$$

$$= 84.8528 \approx 85\ km$$

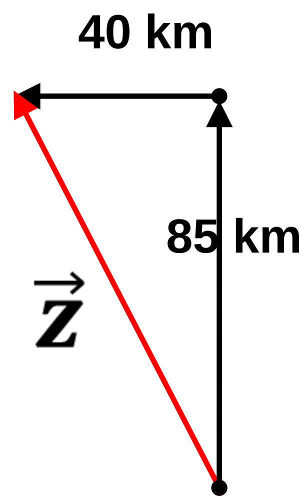
So **b** = (–85, 85)

# Solution continued 2

**<u>2. Adding vectors continued:</u>** If we add vectors **a** and **b** together we get our final vector **z**:




**a** + **b** = (45, 0) + (–85, 85) = (–40, 85)




We can use Pythagoras's theorem to calculate **z**:

$$\vec{z}^2 = 40^2 + 85^2$$
$$\vec{z}^2 = 40^2 + 85^2$$
$$\vec{z}^2 = 1588 + 7200$$
$$z = \sqrt{8788}$$
$$z = 93.7 \approx 94\ km$$

**NOTE:** Remember when doing the calculations to use the **original value** of 84.8528… and **NOT** the rounded up version of 85 – otherwise you will get a slightly different answer

# Solution continued 3

Now that we know how far Hamid has to fly in his helicopter (94 km), we can calculate the time it takes him to get home.

We are given that the speed of his helicopter is 200 km/h, and so:

$$Time = \frac{Distance}{Speed} = \frac{94}{200} = 0.47\ hours$$

0.47 hours × 60 = **28 minutes**

Therefore the answer is **C**

If you added the two vectors as scalars (120 + 45) you would get 165 km (option **D**). If you added the x-components of the vectors (85 + 45 instead of –85 +40) you would get option **B**. Both options **A** and **E** divide the speed by the distance (i.e. use the incorrect equation to calculate the time).

# More Vector Problems III

An X-wing pilot is trying to escape the planet Hoth, flying at 20 m/s, when an asteroid comes crashing into his ship. Just before the collision, the pilot's instruments tell him that the asteroid had a mass of 100 kg and was flying towards him at a speed of 800 m/s.

He also notes that the asteroid flew directly towards his ship, in the opposite direction of where he was flying.

If the ship has a mass of 4000 kg, what is the total momentum of the ship and asteroid just before the collision?

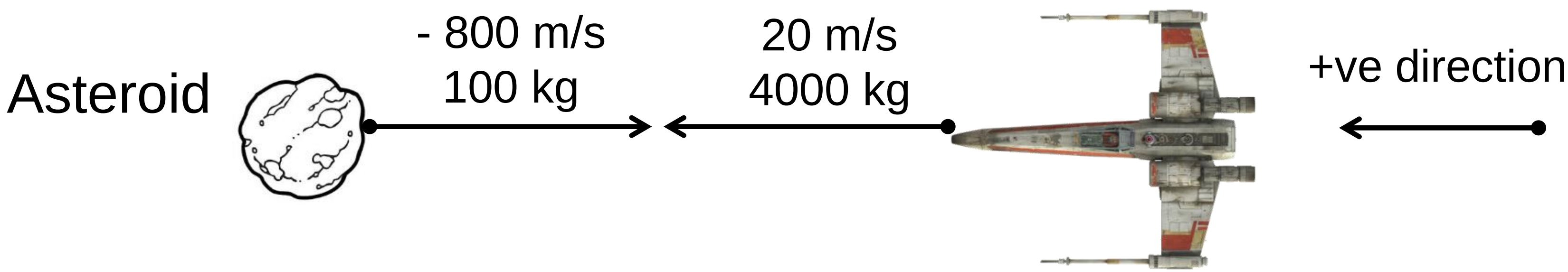
A. – 160 000 kg.m/s
B. – 80 000 kg.m/s
C. 0 kg.m/s
D. 80 000 kg.m/s
E. 160 000 kg.m/s

**Note:** Take the initial direction of the ship as the positive direction

# Solution

**Answer:** C

**Justification:** This question requires us to first draw a diagram:




We can then calculate the momentum of both the asteroid and the ship using the equation $\mathbf{p} = m\mathbf{v}$:

$\mathbf{p}_{\mathbf{asteroid}}$ = (- 800) (100) = – 80 000 kg.m/s

$\mathbf{p}_{\mathbf{ship}}$ = (20) (4000) = 80 000 kg.m/s

So we can see that the ship and the asteroid have the same magnitude of momentum, but in the opposite direction. Therefore their total momentum would be equal to **zero**. Therefore the answer is **C**.

# More Vector Problems IV

After the asteroid crashes into the ship, it breaks off one of the ship's wings and damages its navigation system. The pilot manages to catch a glimpse of where the wing has gone to. He thinks that he can still meet up with the rest of the fleet if he can figure out which direction to go.

The pilot thinks that if he can figure out the new momentum of the wing and the asteroid that hit him, he can tell how far off-course he was knocked by the asteroid. He sets up a coordinate system where the direction of the front of the X-Wing's nose is North, with East being off to his right, South behind him, and West to the left. Luckily for him, both the asteroid and the wing are travelling in the same plane that he is.

# More Vector Problems IV continued

His sensors tell him that the wing has a mass of 500 kg and is travelling away from him at 20 m/s. The direction of the wing's path is Southwest according to his makeshift coordinate system.

The asteroid (with a mass of 100 kg) is hurtling away at a speed of 100 m/s Northwest.

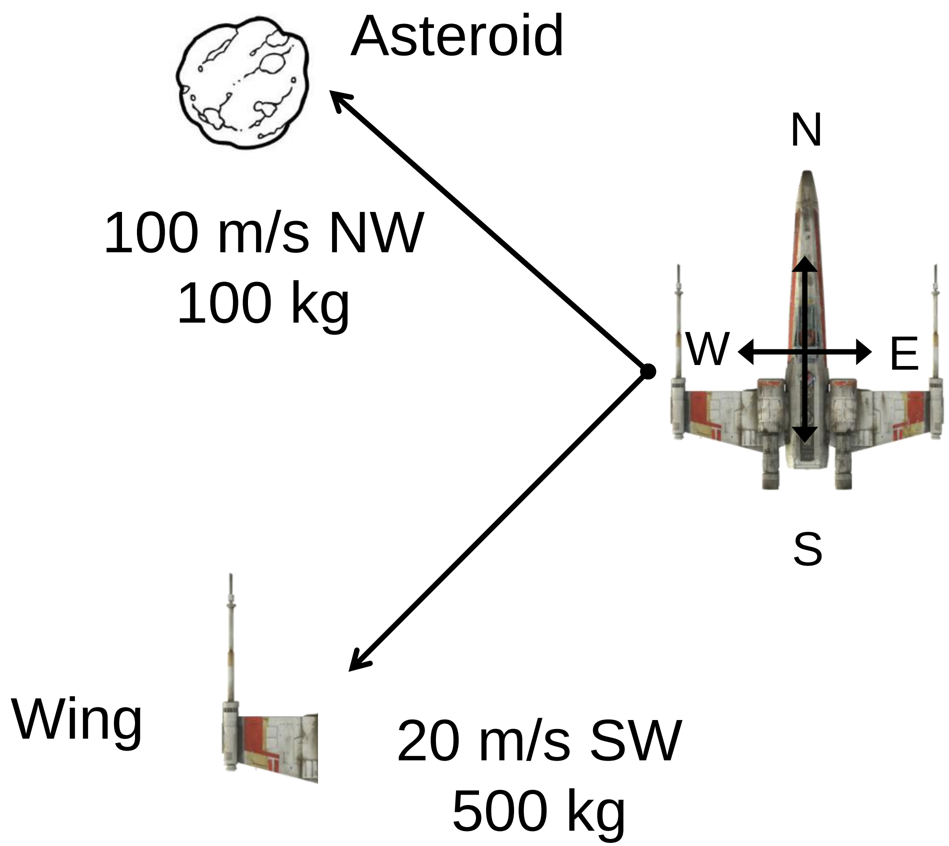
In which direction was he knocked off course?

A. Northeast
B. East-Northeast
C. East
D. East-Southeast
E. Southeast

# Solution

**Answer:** C

**Justification:** This question requires us to first draw a diagram:




We can calculate the momentum of both the wing and the asteroid using the equation **p** = m**v**:

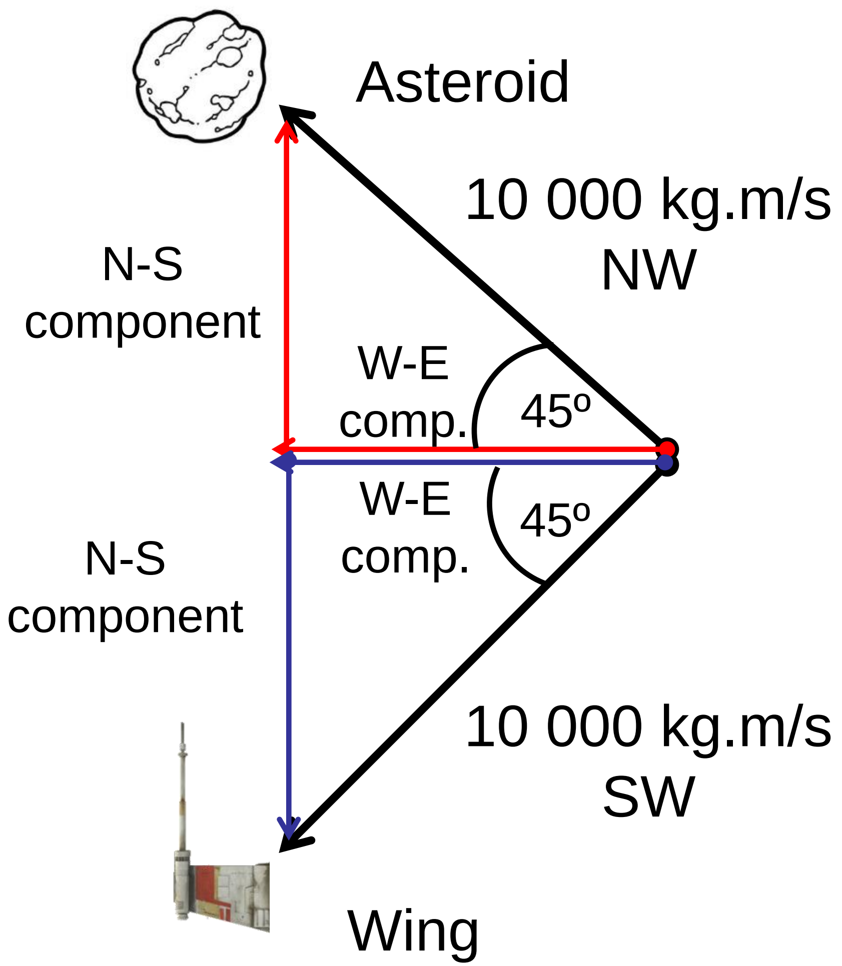
$$p_{asteroid} = 100 \times 100$$
$$= 10\,000 \text{ kg.m/s}$$

$$p_{wing} = 500 \times 20$$
$$= 10\,000 \text{ kg.m/s}$$

Therefore, the magnitude of the momentum of both the asteroid and the wing are the same

# Solution continued

If we take a closer look at the momentum vectors:




We can see from the diagram that since the magnitude of each of the momentum vectors is equal, then their West-East components and North-South components must also be of equal magnitude.

Therefore, since their N-S components are in **opposite** directions they cancel out, and since their W-E components are in the same direction, they can be added.

So we can see that the momentum of the wing and asteroid combined must be in the **Westward** direction only

# Solution continued 2

We know from the conservation of momentum that the total momentum before and after a collision must be conserved. Since we know from the previous question that the total momentum of the system **before** the collision was zero, then we know that the total momentum of the system **after** the collision must also be zero.

Since the wing and the asteroid have a combined momentum that is in the **Westward** direction, then in order for the total momentum of the system to be zero, the momentum of the ship must have the same magnitude, but in the opposite (**Eastward**) direction.

Therefore our answer is **C**