a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Physics
Vector Problems

Science and Mathematics Education Research Group

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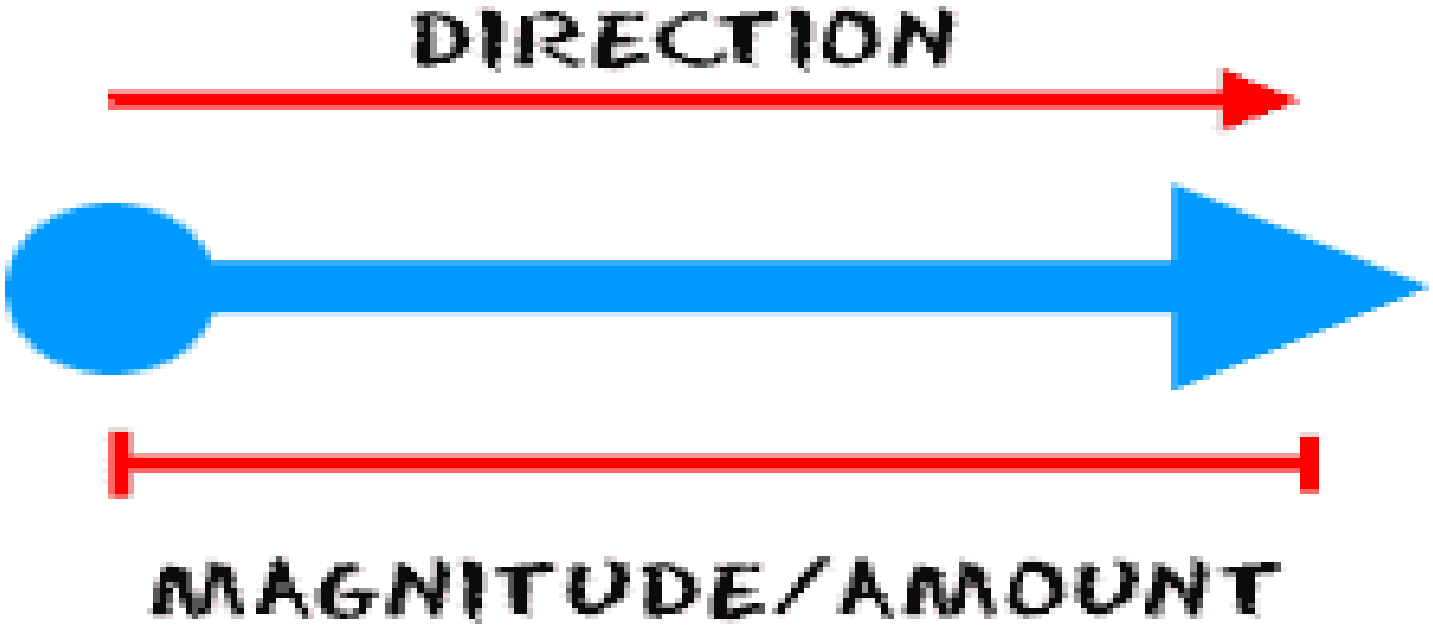
Vector Problems



Retrieved from http://www.physics4kids.com/files/art/motion_vector1_240x180.gif

Vector Problems

The following questions have been compiled from a collection of questions submitted on PeerWise (https://peerwise.cs.auckland.ac.nz/) by teacher candidates as part of the EDCP 357 physics methods courses at UBC.

Vector Problems I

Which of the following are vector quantities?

i. The velocity of a frisbee

ii. The width of a crater made by an asteroid

iii. The speed of a car on the highway

iv. The displacement of a billiard ball after it is struck by the cue ball

A. i only

B. i and ii

C. ii and iii

D. i and iv

E. ii and iv

Solution

Answer: D

Justification: A vector quantity indicates that the number has a **magnitude** (size) and **direction**. Using this knowledge to answer i through iv it can be determined that:

i the **velocity** of a frisbee is a vector quantity because the velocity indicates magnitude and direction

ii. the **width** of a crater made by an asteroid is not a vector quantity because the width does not indicate direction

iii. the **speed** of a car on the highway is not a vector quantity because the speed of a car does not indicate direction

iv. the **displacement** of a billiard ball after it is struck by the cue ball is a vector quantity because the displacement indicates magnitude and direction

Solution continued

Further explanation:

Velocity indicates **speed** and **direction**. For example, if a frisbee is thrown at 10 m/s to the northwest, its velocity is 10 m/s to the northwest. The speed of the frisbee is 10 m/s.

Displacement indicates **distance** and **direction**. For example, if a billiard ball travels 0.5m at an angle of 30 degrees from the long axis of the table, its displacement is 0.5m at an angle of 30 degrees from the long axis of the table. The distance the ball traveled is 0.5m.

Note that terms such as length, width and height are usually referring to distances. Also note that the speed of an object can also be called the magnitude of the velocity.

Vector Problems II

Which of the following groups are vector quantities?

A. Velocity, Displacement, Energy
B. Momentum, Acceleration, Work
C. Weight, Momentum, Displacement
D. Power, Force, Acceleration

Solution

Answer: C

Justification: A vector quantity always has a **magnitude** (size) and **direction**. A scalar quantity only has magnitude. Let us look at each term separately:

Velocity has both a **speed** and **direction**. For example, if a car is travelling at 15 m/s to the south, its speed is 15 m/s and its direction is south.

Displacement indicates both **distance** and **direction**. For example, if a sack is dropped to the ground from 10 m above the ground, the distance it travelled was 10 m, and the direction was vertically down towards the ground.

Energy has **magnitude**, but has **no direction**. It is a scalar, not a vector. For example, a ball travelling north at 10 m/s and a ball travelling south at 10 m/s have the same amount of kinetic energy.

Solution continued

Momentum has both **magnitude** and **direction**. It is defined as the quantity of motion of a moving body. For example, if a car with a mass of 500 kg is travelling east at 10 m/s, the magnitude of its momentum is 5000 kg.m/s (mass and velocity multiplied) and its direction is east. The same car travelling in the opposite direction would have the same magnitude of momentum, but in the west direction.

Acceleration is the rate at which an object changes its velocity. Since velocity is a vector, and changing velocity may also include a change in direction, acceleration has both **magnitude** and **direction**. For example, acceleration due to gravity close to the Earth's surface is approximately 9.8 m/s^2. This means that the magnitude of the acceleration is 9.8m/s^2, and its direction is towards the center of the Earth.

Solution continued 2

Work results when a force acts upon an object to cause a displacement. The formula for work is: Work = Force × Displacement × Cosine(θ), where θ is the angle between the Force and Displacement vectors. Note that when θ = 90º the result will be zero (cos90º = 0). In other words when Force and Displacement are perpendicular, the force does no work on the body! Work is a scalar product with only **magnitude**, and **NO direction**. The SI unit of work is the joule (which is also used for Energy). Work can also be thought of as the **change in Energy**, and since Energy is a scalar so is Work.

Weight of an object is the force of gravity exerted on that object. It is equal to mass times the acceleration due to gravity. The direction of this acceleration is towards the center of the Earth (or other planet/moon). Therefor weight has both **magnitude** and **direction**, and is a vector.

Solution continued 3

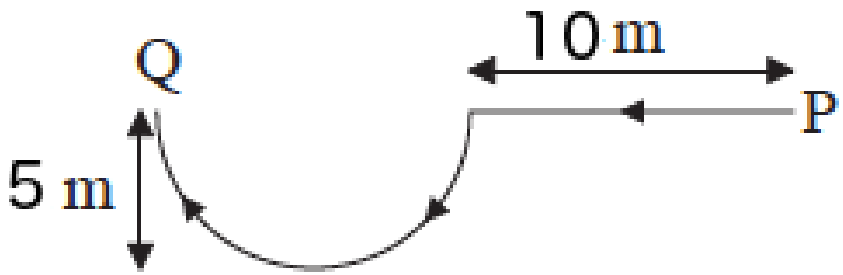
Power is the rate of doing Work. It is equivalent to an amount of Energy consumed per unit time (Joules per second). Since Work and Energy are scalars, we know that Power must also be a scalar. It only has **magnitude**, and no **direction**.

Force has both **magnitude** and **direction**, and is a vector. It is defined as the mass of an object multiplied by its acceleration. Since acceleration is a vector with a direction, then Force also has direction.

Therefore the answer is **C**

Vector Problems III

Steve walks from Point P to Point Q. The curved part of his path is a semicircle. What is the magnitude of his displacement from point P to point Q?



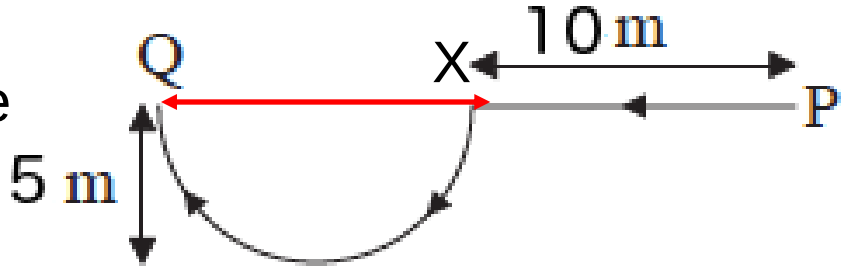
A. 5
B. 15
C. 20
D. 20.7
E. 25.7

Solution

Answer: C

Justification: Displacement is a vector quantity that refers to an object's overall change in position. It is the distance from the start to the end point(which in this case would be from P to Q), the path taken does not matter. Since we are asking here for the magnitude of the displacement, we are not required to specify the direction.

If we look at the diagram, we can see that the distance from P to X is 10 m. The distance from X to Q is the diameter of the semicircle. Since the radius of the circle is 5 m, we know that the diameter must be twice the size, therefore 10 m.



So the distance from P to Q = 10 + 10 = 20 m

So the answer is **C**

Solution continued

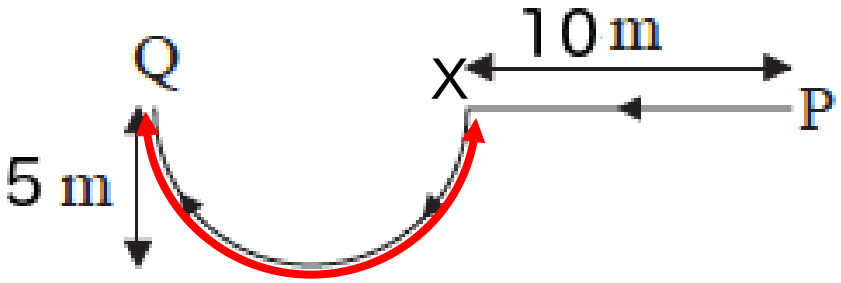
Extra information:

If we wanted to calculate the distance that Steve travelled, we would need to calculate the circumference of the semi-circle that he travelled along (distance XQ). We can use the equation for the circumference of a circle: Circumference = 2 × π × radius

In this case, since it is a semi-circle we only need to find out half the circumference. So we get:

XQ = π × radius = π × 5 = 15.7 m

To get the total distance we add up the distance PX (10 m) and XQ (15.7) to get 25.7 m



Vector Problems IV

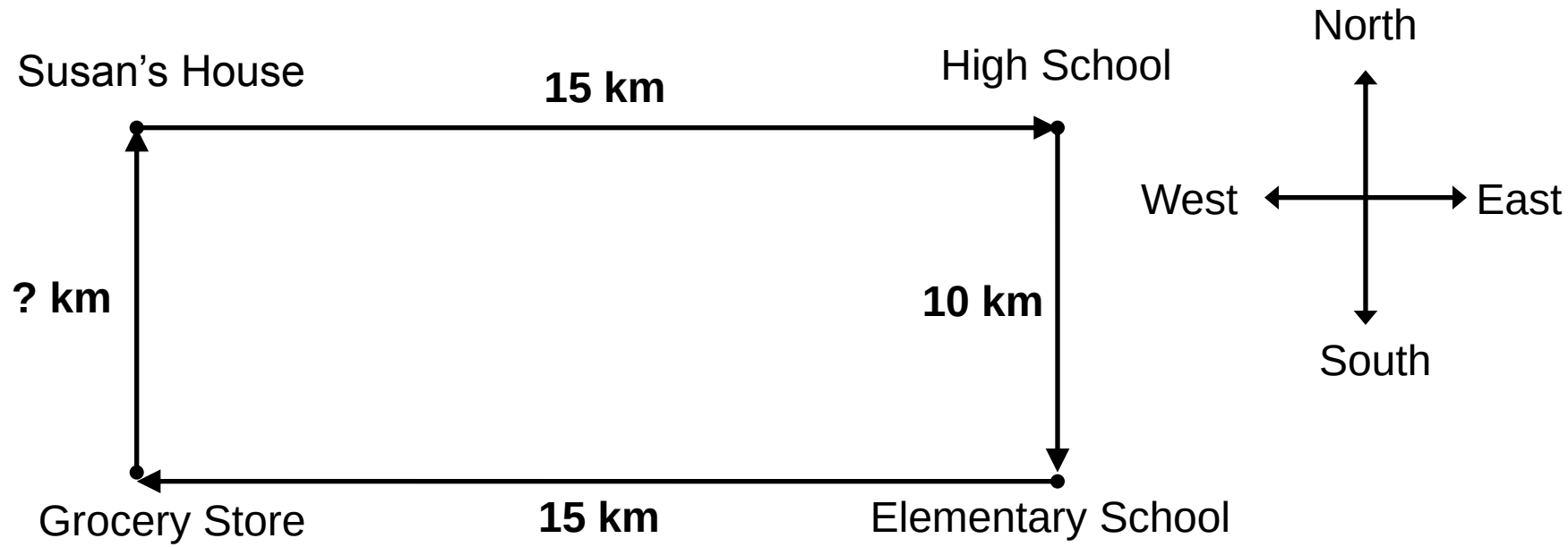
A Physics teacher, Susan, drove to her high school which is located 15km East from her house. After school, she drove to her children's elementary school which is 10 km South from her high school. Then, she drove to a grocery store, located 15km West from the elementary school. Finally, she drove back to home with her kids and several grocery bags. Choose a true statement from the following:

A. The total distance she traveled from her house to elementary school is 18 km.
B. The magnitude of the displacement vector from the high school to the grocery store is 25 km.
C. The magnitude of the displacement vector for the whole trip is 50 km.
D. The magnitude of the displacement vector from her house to the grocery store is 10 km.

Solution

Answer: D

Justification: In order to answer this question we need to draw the vectors that represent Susan's activities for the day:



We can see that Susan's path forms a perfect rectangle, and that the last trip she made home from the grocery store must be **10km**

Solution continued

We need to remember that **distance** is a scalar quantity that refers to how much ground an object has covered during its motion, and that **displacement** is a vector quantity that refers to how far out of place an object is (the object's overall change in position). Since we are only looking at the **magnitude** of the displacement vector in this question, the direction does not need to be stated.

We can use the previous diagram to calculate each of the following:

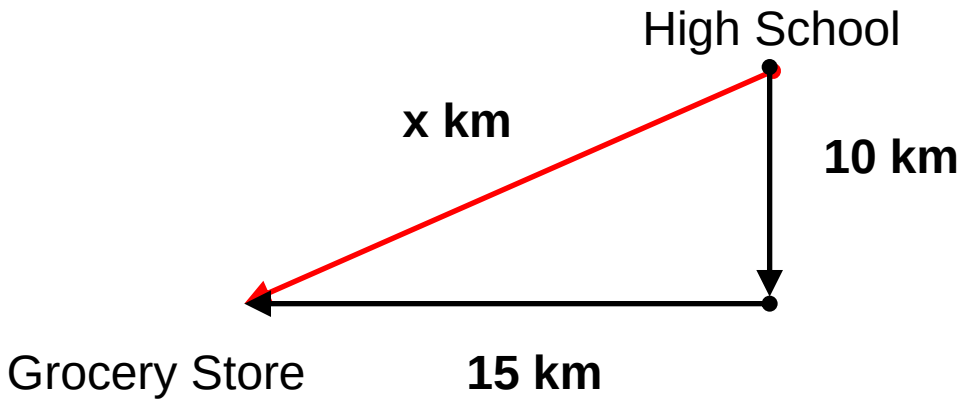
A. The total distance Susan travelled from her house to the elementary school is equal to the distance from Susan's house to the high school plus the distance from the high school to the elementary school:

Distance = 15 km + 10 km = 25km So **A** is incorrect

Solution continued 2

B. In order to calculate the magnitude of the displacement vector (x) from the high school to the grocery store we need to use Pythagoras' Theorem:

$$x = \sqrt{10^2 + 15^2}$$
$$x = \sqrt{100 + 225}$$
$$x = \sqrt{325}$$
$$x = 18\ km$$



Therefore **B** is incorrect

C. The **distance** of the whole trip is 15 + 10 + 15 + 10 = 50 km. However, since Susan ended up in the same place as she started, the magnitude of her displacement is **zero**.

Therefore **C** is incorrect

Solution continued 3

D. The magnitude of the displacement vector from Susan's house to the grocery store can be calculated if we remember that displacement only takes into account Susan's starting position and end position (and not what route she took to get from start to end). If Susan started at her house, and ended at the grocery store, then her displacement was only 10 km.

Therefore **D** is the correct answer

Note: The **distance** Susan covered from her house to the grocery store was 15 + 10 + 15 = 40 km

Vector Problems V

Steve is driving in his car to take care of some errands. The first errand has him driving to a location 2 km East and 6 km North of his starting location. Once he completes that errand, he drives to the second one which is 4 km East and 2 km South of the first errand. What is the magnitude of the vector that describes how far the car has traveled from its starting point, rounded to the nearest km?

A. 6 km
B. 7 km
C. 8 km
D. 10 km
E. 14 km

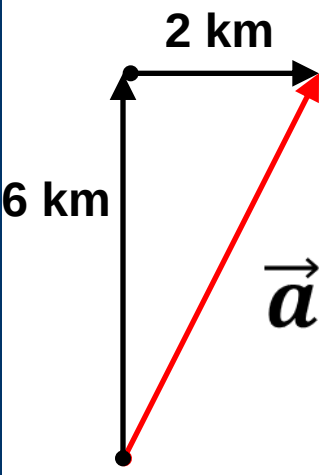
Solution

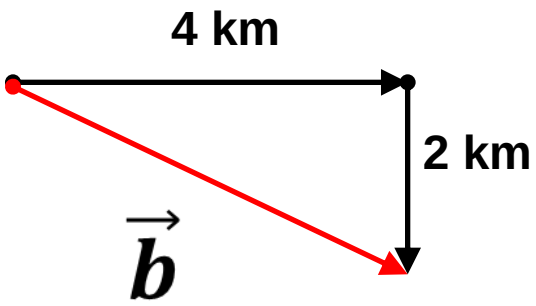
Answer: B

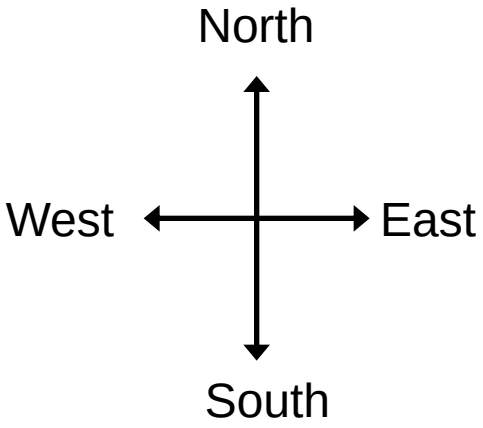
Justification: First, we describe the two sections of the journey as two vectors, **a** and **b**. If we define West-East as the x-coordinates (East is positive) and North-South as the y-coordinates (North is positive):

a = (2 , 6)

b = (4 , – 2)



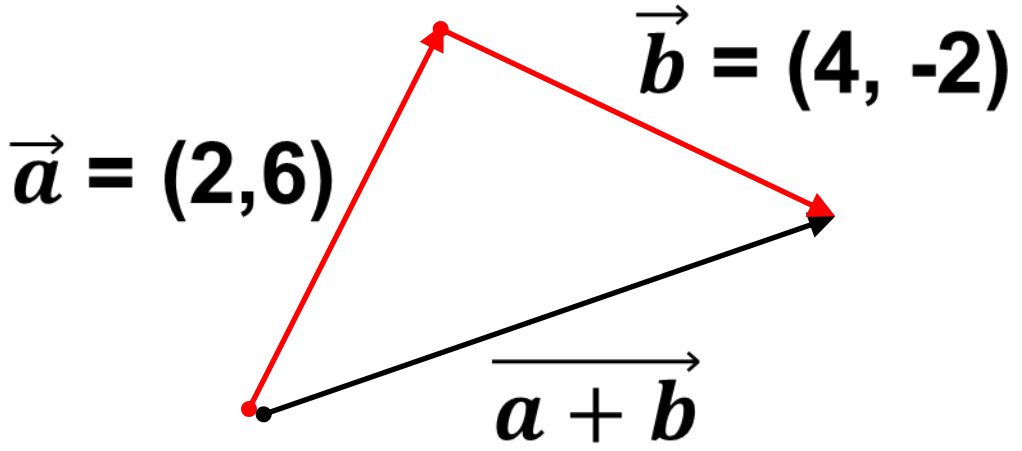




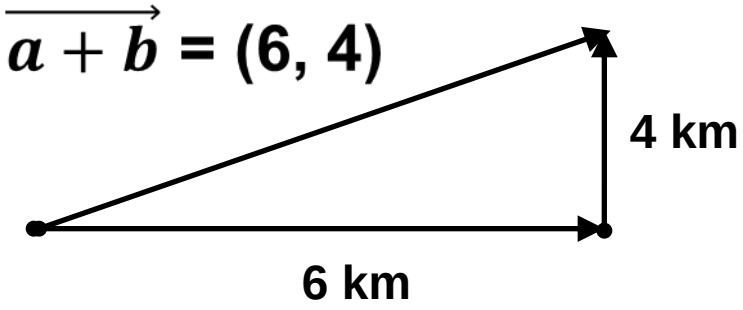
Solution continued

Then you need to add vectors **a** and **b** together to find the vector **a+b**.

a+b = (2 , 6) + (4 , – 2)
a+b = (2 + 4 , 6 – 2)
a+b = (6 , 4)



Using Pythagoras' Theorem, we can find the magnitude of the vector **a+b**:



$$\left|\overrightarrow{a+b}\right| = \sqrt{6^2 + 4^2}$$
$$\left|\overrightarrow{a+b}\right| = \sqrt{36 + 16}$$
$$\left|\overrightarrow{a+b}\right| = \sqrt{52}$$
$$\left|\overrightarrow{a+b}\right| = 7.2\ km$$

Solution continued 2

Rounded to the nearest kilometer, the magnitude of **a+b** is 7 km.

Therefore the correct answer is **B**.

Note:

If you didn't have a calculator to perform a square root, you could determine the correct answer with the knowledge that 52 is between 49 and 64, the squares of 7 and 8, respectively. However, 52 is also much closer to 49 than it is to 64, so the rounded answer would be 7 km.