a place of mind

# Mathematics <br> Mean, Median, Mode, Range 

## Science and Mathematics Education Research Group

Test Scores


## Best Practices

When solving mean, median, mode and range questions, it is often very helpful to rewrite the data from smallest to largest.

scores Median scores
$20,20,20,40,40,70,80$ The range can be found by subtracting the first data point from the last one:

Mode $=20$

## Best Practices II

It is often not necessary to calculate the exact mean (average) of a data set to solve a lot of these questions.
In many cases all you need to do is decide which data set has the larger or smaller mean by estimating:

$$
64,67,76,68,74
$$

Which data set has the larger mean?

Noticing patterns in a data set can sometimes make calculating the mean easier.
$75,75,80,80,85,85:$ Mean $=80$
Notice that 75 is 5 less than 80, and 85 is 5 greater than 80

## Test Scores I

Jeremy scored the following on his last seven math tests (out of 100):
$70,80,70,90,80,100,70$
What is the mean of Jeremy's test scores?

A. 280<br>B. 90<br>C. 80<br>D. 70<br>E. 30

## Solution

## Answer: C

Justification: The mean is the average of the scores.

Note: Scores have been arranged from smallest to largest below.

$$
\begin{aligned}
\text { Mean } & =\frac{70+70+70+80+80+90+100}{7} \\
& =\frac{560}{7} \\
& =80
\end{aligned}
$$

## Test Scores II

Jeremy scored the following on his last seven math tests (out of 100):
$70,80,70,90,80,100,70$
What is the median of Jeremy's test scores?
A. 100
B. 90
C. 80
D. 75
E. 70

## Solution

## Answer: C

Justification: The median is the score in the middle when arranged from least to greatest (or greatest to least).


Median
If there are 2 scores in the middle (due to an even number of scores), the median is the mean of those 2 scores.

## Test Scores III

Jeremy scored the following on his last seven math tests (out of 100):
$70,80,70,90,80,100,70$
What is the mode of Jeremy's test scores?
A. 100
B. 90
C. 80
D. 75
E. 70

## Solution

## Answer: E

Justification: The mode is the number that occurs most frequently in a set of data.


The test score 70 out of 100 appears 3 times while the other scores only appear once or twice.

## Test Scores IV

Jeremy scored the following on his last seven math tests (out of 100):
$70,80,70,90,80,100,70$
What is the range of Jeremy's test scores?
A. 100
B. 50
C. 30
D. 15
E. None of the above

## Solution

## Answer: C

Justification: The range is the difference between the largest and smallest test score.
$70,70,70,80,80,90,100$

$$
\text { Range }=100-70=30
$$

## Test Scores V

Jeremy scored the following on his last seven math tests (out of 100):

$$
75,81,92,75,86,90,75
$$

Jeremy decides to calculate the mean, median, mode and range of his scores, which are shown below. How high does Jeremy have to score on his next test in order to improve his average?

A. Greater than 17
B. Greater than 75
C. Greater than 81
D. Greater than 82
E. None of the above

## Solution

## Answer: D

Justification: In order for Jeremy to improve his average, he must score higher than his current average. A score lower than his current average will lower his average.

## Test Scores VI

Alex, Betty, Chris, David and Eliza all score 100 (out of 100) on their latest math test. Their previous 7 test scores are listed below. Whose average will increase the most?
A. Alex: 71, 64, 63, 77, 80, 79, 80
B. Betty: 90, 92, 96, 99, 99, 89, 97
C. Chris: 100, 100, 100, 100, 100, 100, 100
D. David: 83, 89, 80, 82, 89, 79, 79
E. Eliza: 63, 61, 66, 66, 70, 71, 65

## Solution

## Answer: E

Justification: Getting a perfect score on the most recent test will make the biggest impact on the person who had the lowest average to begin with. This is because a high score is being added to a low score, resulting in the biggest overall change. Eliza has the lowest average at 66 (remember: average is the same as mean) before the 100 test score is added.

Before:

$$
\begin{aligned}
\text { Mean } & =\frac{61+63+65+66+66+70+71}{7} \\
& =\frac{462}{7} \\
& =66
\end{aligned}
$$

After 100 is added:

$$
\begin{aligned}
\text { Mean } & =\frac{61+63+65+66+66+70+71+100}{8} \\
& =\frac{562}{8} \\
& =70.25
\end{aligned}
$$

## Solution

## Answer: E

Justification Cont'd: Consider another example of Alex's scores before and after the last test.

Before:
Mean $=\frac{63+64+71+77+79+80+80}{7}$
$=\frac{514}{7}$
$=73.4$

## After 100 is added:

$$
\begin{aligned}
\text { Mean } & =\frac{63+64+71+77+79+80+80+100}{8} \\
& =\frac{614}{8} \\
& =76.75
\end{aligned}
$$

Notice when you calculate Chris's average that it will not change when a score of 100 is added because his average before the test was 100 as well.

## Test Scores VII

Alex, Betty, Chris, and David all score 85 (out of 100) on their latest math test. Their previous test scores are listed below. Students that are absent on the test days have their scores "omitted." Omitted scores do not affect the grade of the students. Whose average will increase the most?
A. Alex: 80, omit, omit, 80, omit
B. Betty: $80,80,80$, omit, omit
C. Chris: 80, omit, 80, 80, 80
D. David: $80,80,80,80,80$
E. The average of all 4 students will increase the same amount

## Solution

## Answer: A

Justification: All four students have the same average (80) before the last test. However, notice the difference in the calculation between Alex and David averages before the final test:

$$
\begin{aligned}
& \text { Alex: Mean }=\frac{80+80}{2}=80 \\
& \text { David: Mean }=\frac{80+80+80+80+80}{5}=80
\end{aligned}
$$

Alex and David's averages after the final test:

$$
\begin{aligned}
& \text { Alex: Mean }=\frac{80+80+85}{3}=81.7 \\
& \text { David: Mean }=\frac{80 \cdot 5+85}{6}=80.8
\end{aligned}
$$

The more terms in a data set, the harder it is to change the mean.

## Test Scores VIII

Jeremy has the following test scores:

$$
96,97,98,98,98,99,99,100
$$

On his latest test, Jeremy decided not to study because he was doing so well and ended up with 64/100. How will the mean, median, mode, and range change?
A. The mean, median, mode, and range will change.
B. Only the mean, median, and range will change.
C. Only the mean, mode, and range will change.
D. Only the mean and range will change.

E . The mean, median, mode, and range will all stay the same.

## Solution

## Answer: D

Justification: The mean will change because a lower score is being added to the set, making the mean lower.

The median is 98 before the low score is added (the two middle numbers out of the 8 test scores are 98 and 98). After the low score is added, 98 is still the median: $64,96,97,98,98,98,99$, 99, 100

The most frequent score is 98 , which occurs 3 times. This is still the most frequent score after 64 is added. Therefore, the mode does not change.

The range will change because a new lowest score is added. The original range is $100-96=4$. The new range is $100-64=36$.

## Test Scores IX

Consider the following set of 10 test scores:

$$
1,2,3,4,5,6,7,8,9,10
$$

What is the median of the test scores?

> A. 6
> B. 5.5
> C. 5
> D. 4.5
> E. 4

## Solution

Answer: B
Justification: There is an even number of test scores, so the median will be the mean of the 2 middle numbers.


The two numbers in the middle are 5 and 6. Taking the average of these two numbers give:

$$
\frac{5+6}{2}=5.5
$$

## Test Scores X

Consider the following set of 10 test scores:

$$
1,2,3,4,5,6,7,8,9,10
$$

What is the mean of the 10 test scores?

> A. 6
> B. 5.5
> C. 5
> D. 4.5
> E. 4

## Solution

## Answer: B

Justification: Grouping sets of data that are in a sequence (each number is 1 larger than the previous) different ways can make it easier to calculate the mean. For example:

$$
\begin{aligned}
& 1+2+3+4+5+6+7+8+9+10 \\
= & (1+10)+(2+9)+(3+8)+(4+7)+(5+6) \\
= & 11 \times 5 \quad \text { Group the first and last numbers } \\
= & 55 \\
& 1+2+3+4+5+6+7+8+9+10 \\
= & (1+9)+(2+8)+(3+7)+(4+6)+5+10 \\
= & 10 \times 4+15 \quad \text { Group the numbers that sum to } 10 \\
= & 55
\end{aligned}
$$

## Test Scores XI (Bonus)

Consider a set of 99 test scores. The test scores go from 1 to 99 such that the first score is $1 / 100$, the second is $2 / 100$, and so on until the last score is 99/100.

What is the median of the 99 test scores?
A. 51
B. 50.5
C. 50
D. 49.5
E. 49


## Solution

Answer: C
Justification: There are an odd number of test scores (99 in total) so there will be a number in the middle of the set. Notice that $99 / 2=49.5$, so there will be 49 test scores on either side of the median score:


## Test Scores XII (Bonus)

Consider a set of 99 test scores. The test scores go from 1 to 99 such that the first score is $1 / 100$, the second is $2 / 100$, and so on until the last score is $99 / 100$.

What is the mean of the 99 test scores?

A. 51<br>B. 50.5<br>C. 50<br>D. 49.5<br>E. 49



## Solution

## Answer: C

Justification: Since the data is a sequential set of numbers (each number is 1 larger than the previous), we can pair the numbers that are an equal difference from the median:

$$
\left[\begin{array}{l}
1=50-49 \\
48=50-2 \\
{\left[\begin{array}{r}
49=50-1 \\
50=50 \\
51=50+1 \\
52=50+2 \\
99=50+49
\end{array}\right.}
\end{array}\right.
$$

$$
\begin{aligned}
& \frac{49+50+51}{3}=50 \\
& \frac{48+49+50+51+52}{5}=50 \\
& \frac{1+2+3+\ldots 50 \ldots+97+98+99}{99}=50
\end{aligned}
$$

Since 51 is 1 more than 50 , and 49 is 1 less than 50 , the mean of 49,50 and 51 is 50 . The same pattern can applied for 48 and 52, 47 and 53 , and so on until 1 and 99 . The mean is therefore 50.

## Alternative Solution

## Answer: C

Justification: We can alternatively pair the first and last numbers of the set, which always sum to 100 : $1,2,3,4, \ldots 48,49,50,51,52 \ldots .96,97,98,99$

49 pairs 100
of 10 $\left\{\begin{array}{ccc}1+99=100 \\ 2+98 & =100 \\ 3+97 & =100 & \text { Mean }=\frac{1+2+3+\ldots .97+98+99}{99} \\ \vdots & & =\frac{(49 \times 100)+50}{99} \\ 47+53=100 & \text { (49 pairs of 100, plus } \\ 48+52=100 \\ 49+51=100 & =\frac{4950}{99} & \\ \text { the remaining 50) }\end{array}\right.$

## Test Scores XIII (Bonus)

Now consider a set of 100 test scores. The test scores go from 1 to 100 such that the first score is $1 / 100$, the second is $2 / 100$, and so on until the last score is $100 / 100$. What is the median of the 100 test scores?
A. 51
B. 50.5
C. 50
D. 49.5
E. 49

## HINT:

The median of the numbers between 1 to 99 was 50 .

Press for hint


## Solution

## Answer: B

Justification: There are an even number of test scores (100 in total) so the median will be the mean of the two middle numbers. The numbers 1 to 100 can be divided into two equal sets, 1 to 50 and 51 to 100.


## 50 test scores

50 test scores

The two middle numbers are 50 and 51 . The median is therefore:

$$
\frac{50+51}{2}=\frac{101}{2}=50.5
$$

## Test Scores XIV (Bonus)

Now consider a set of 100 test scores. The test scores go from 1 to 100 such that the first score is $1 / 100$, the second is $2 / 100$, and so on until the last score is $100 / 100$.

What is the mean of the 100 test scores?
A. 51
B. 50.5
C. 50
D. 49.5
E. 49

## HINT:

Recall that the mean of the numbers from 1 to 99 is 50 .

## Solution

## Answer: B

Justification: From the previous question, we know the mean of 1 to 99 is 50 . This means that:

$$
\begin{aligned}
& \frac{1+2+3+\ldots+97+98+99}{99}=50 \\
& 1+2+3+\ldots+97+98+99=50 \times 99 \\
& 1+2+3+\ldots+97+98+99=4950
\end{aligned}
$$

(From the definition of the mean)

$$
1+2+3+\ldots+97+98+99=50 \times 99 \quad \text { (Multiply both sides of the equation by 99) }
$$

The sum of 1 to 100 is the sum of 1 to 99 plus 100:

$$
1+2+3+\ldots+98+99+100=4950+100=5050
$$

Since there are 100 terms from 1 to 100 , the mean of 1 to 100 is:

$$
\text { Mean }=\frac{1+2+3+\ldots+98+99+100}{100}=\frac{5050}{100}=50.5
$$

## Alternative Solution

## Answer: B

Justification: The question can also be solved similar to question 10. Pairing the first and last numbers of the data set always sum to 101:

50 pairs | of 101 |
| :--- |\(\left\{\begin{array}{cl}50+51=101 \& If all the numbers from 1 to 100 are <br>

49+52=101 \& grouped as shown, there will be 50 pairs <br>
48+53=101 \& of numbers that sum to 101 . <br>
\vdots \& The sum of the numbers from 1 to 100 is: <br>
1+100=101 \& (1+100)+(2+99)+··· .+(50+51)=101 \times 50=5050\end{array}\right.\)

Mean $=\frac{1+2+3+\ldots+98+99+100}{100}=\frac{5050}{100}=50.5$

