# Math <br> Shape and Space: Perimeter 

## Science and Mathematics Education Research Group

## Investigating Perimeters



## Perimeter I

What is the perimeter of the square below?

A. 30 m
B. 45 m
C. 60 m
D. 225 m
E. Not enough information

## Solution

## Answer: C

Justification: A square has 4 sides with equal length. Adding up all 4 sides give:

$$
P=15 m+15 m+15 m+15 m=60 m
$$

The perimeter can also be calculated using multiplication since there are 4 sides are the same:

$$
P=15 \mathrm{~m} \times 4=60 \mathrm{~m}
$$

Answer $D$ is the area of the square: $A=15 \mathrm{~m} \times 15 \mathrm{~m}$

## Perimeter II

What is the perimeter of the figure below?

A. 45 m
B. 52 m
C. 53 m
D. 60 m
E. Not enough information

## Solution

## Answer: D

Justification: Even though some of the sides do not have their lengths given, they can be found as follows:


## Alternate Solution

## Answer: D

Justification: The highlighted sides can be moved along the perimeter of a 15 m by 15 m square.


## Perimeter III

What is the perimeter of the figure below?

A. 55 m
B. 65 m
C. 75 m
D. 80 m
E. Not enough information

## Solution

## Answer: D

Justification: We do not know the individual lengths of the sides highlighted red, but we do know their sum must be 15 m .


## Perimeter IV

What is the perimeter of the figure below?

A. 42 m
B. 51 m
C. 54 m
D. 57 m
E. Not enough information

15 m

## Solution

## Answer: C

Justification: The sum of the red sides must be 15 cm , and the sum of the green sides must be 12 cm .


$$
\begin{aligned}
& P=15 m+6 m+6 m+15 m+12 m=54 m \\
& \text { OR } \\
& P=(15 m \times 2)+(12 m \times 2)=54 m \\
& 6 m+6 m=12 m \\
& \text { The perimeter is the same as a } 15 \mathrm{~m} \\
& \text { by } 12 \mathrm{~m} \text { rectangle. }
\end{aligned}
$$

## Perimeter V

The figure below is a 15 m by 15 m square with 3 rectangles taken away from the corners. What is the perimeter of the figure?

A. Less than 60 m
B. Exactly 60 m
C. Greater than 60 m
D. Not enough information

## Solution

## Answer: B

Justification: The inner rectangle sides can be moved to the outline of the square as shown. The perimeter then becomes the perimeter of the original square.


$$
P=15 m+15 m+15 m+15 m=60 m
$$

OR

$$
P=15 \mathrm{~m} \times 4=60 \mathrm{~m}
$$

## Perimeter VI

Which of the following has the greatest perimeter?


E. They all have the same perimeter

## Solution

## Answer: E

Justification: All of the highlighted sides can be moved to form the 5 m by 9 m rectangle.


## Perimeter VII

Four squares with a perimeter of 20 m each are arranged as shown to form a larger square. What is the perimeter of the larger square?

A. 20 m
B. 40 m
C. 60 m
D. 80 m
E. Not enough information

## Solution

## Answer: B

Justification: The small squares with $P=20 \mathrm{~m}$ must have side length $5 m$ since $5 m+5 m+5 m+5 m=20 m$.

|  | 5 m | 5 m | 5 m P ${ }^{\text {m }} \mathrm{m} \times 8=40 \mathrm{~m}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| 5 m | $\mathrm{P}=20 \mathrm{~m}$ | $\mathrm{P}=20 \mathrm{~m}$ |  |  |
| 5 m | $\mathrm{P}=20 \mathrm{~m}$ | $\mathrm{P}=20 \mathrm{~m}$ | 5 m |  |
|  | 5 m | 5 m |  |  |

## Alternative Solution

## Answer: B

Justification: The total perimeter of 4 separate squares is 80 m . When joined together, the highlighted sides will be glued together.


Instead of summing the exterior sides, the interior sides can be subtracted from the total perimeter.

$$
P=80 \mathrm{~m}-5 \mathrm{~m} \times 8=40 \mathrm{~m}
$$

## Perimeter VIII

Four squares with a perimeter of 20 m are arranged in two different ways as shown. Which has the greater perimeter?
A.

B.

C. Both have the same perimeter
D. Not enough information

## Solution

## Answer: A

Justification: Even though both shapes are made up of the same blocks, the arrangement on the left has 2 more revealed sides.


## Alternative Solution

## Answer: A

Justification: The arrangement with the fewest interior sides will have the largest perimeter. Interior sides do not add to perimeter.


## Perimeter IX

Can four squares with a perimeter of 20 m be arranged to give a perimeter greater than 50 m ? Squares can only be glued together such that at least 1 side is completely touching the side of a different square

A. Yes
B. No

## Solution

## Answer: B

Justification: The 4 blocks can only be arranged as follows:


$$
P=5 \mathrm{~m} \times 10=50 \mathrm{~m}
$$



$$
P=5 \mathrm{~m} \times 10=50 \mathrm{~m}
$$

$$
P=5 \mathrm{~m} \times 10=50 \mathrm{~m}
$$



## Perimeter X (Hard)

You are now given 100 squares with a perimeter of 20 m to arrange like before. What is the maximum perimeter you can have?


## 100

A. Less than 1000 m
B. Exactly 1000 m
C. Greater than 1000 m

## Solution

## Answer: C

Justification: The first 2 blocks must be arranged like so:


In order to get the largest perimeter possible, the next square should only cover 1 side, but add 3 more exterior sides.


The first two squares give a perimeter of 30 m . There are 98 remaining squares that will each add 10 m to the final shape.

$$
P=30 m+(10 \mathrm{~m} \times 98)=1010 \mathrm{~m}
$$

## Alternative Solution

## Answer: C

Justification: Notice the following pattern:
2 blocks: 2 interior sides (1 from each block)
3 blocks: $(3-1)(2)=4$ interior sides
100 blocks: $(100-1)(2)=198$ interior sides
Each time a new block is added, the minimum number of interior sides added is 2 sides since each block must be glued to another block.

The total perimeter of 100 separate blocks is $P=20 \mathrm{~m} \times 100=2000$ m . Subtracting the interior sides from the total perimeter gives:

$$
P=2000 m-(198 m \times 5)=1010 m
$$

## Perimeter XI (Hard)

You are now given 100 squares with a perimeter of 20 m to arrange like before. What is the minimum perimeter you can have?


## 100

A. Less than 200 m
B. Exactly 200 m
C. Greater than 200 m

## Solution

## Answer: B

Justification: The perimeter can be minimized by arranging the squares to form a larger square. In this arrangement, only the squares on the outside contribute to the perimeter of the shape.


$$
P=50 \mathrm{~m} \times 4=200 \mathrm{~m}
$$

