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#### FACULTY OF EDUCATION

Department of Curriculum and Pedagogy

# Mathematics Numbers: Absolute Value of Functions I

Science and Mathematics Education Research Group

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### Absolute Value of Functions I



### **Absolute Values I**

Which one of the following tables corresponds to |f(x)|, given the table of f(x) at the right?

(Red numbers indicate values that were changed)

X	f(x)			
-4	11			
-2	0			
0	-3			
2	6			
4	-8			

Α.			В	•		C.			D.		
	x	f(x)		X	f(x)		X	f(x)		x	f(x)
	4	11		-4	11		4	11		-4	-11
	2	0		-2	0		2	0		-2	0
	0	-3		0	3		0	3		0	3
	2	6		2	6		2	6		2	-6
	4	-8		4	8		4	8		4	8

#### Answer: B

**Justification:** The table of values of |f(x)| are exactly the same as f(x), except when f(x) is negative. In this case, the value of |f(x)| is made positive by multiplying f(x) by -1. This corresponds to table B, since every value under the |f(x)| column is positive.

Table A and C are incorrect because the x-values were changed to positive. The domain of |f(x)| is always the same as the domain of f(x).

Table D is incorrect because all values of f(x) where multiplied by -1, not just the negative values.

### **Absolute Values II**

The graph of f(x) is shown below. What is the graph of |f(x)|?





#### Answer: A

**Justification:** The graph of |f(x)| must lie entirely above or on the x-axis since  $|f(x)| \ge 0$ . Only graph A satisfies this. Notice that the part of f(x) that was negative has been reflected across the x-axis in order to become positive. This is the effect of multiplying f(x) by -1 when it is negative.



### **Solution Cont'd**

Notice the graphs of |f(x)| lie completely above or on the xaxis. It is not possible for |f(x)| to be less than 0 since negative values are turned positive.

In this case, the range of |f(x)| is  $|f(x)| \ge 0$ .



#### **Absolute Values III**

Which of the following graphs corresponds to the function: f(x) = |x|



#### Answer: B

**Justification:** The definition of the absolute value states that:



Notice how the absolute value function is composed of 2 lines, one for when  $x \ge 0$  and one for when x < 0.

Graph D shows the graph of f(x) = x. What is the equation for graph A? (Write as a piecewise function.)

#### **Absolute Values IV**

Which of the following piecewise functions corresponds to the absolute value graph shown below?



A.  $f(x) = \begin{cases} -2x+4, & x \ge 2\\ 2x+4, & x < 2 \end{cases}$ 

B. 
$$f(x) = \begin{cases} 2x+4, & x \ge 2\\ -2x+4, & x < 2 \end{cases}$$

C. 
$$f(x) = \begin{cases} 2x - 4, & x \ge 2\\ -2x + 4, & x < 2 \end{cases}$$

$$f(x) = \begin{cases} -2x+4, & x \ge 2\\ 2x-4, & x < 2 \end{cases}$$

E. 
$$f(x) = \begin{cases} 2x - 4, & x \ge 2\\ -\frac{1}{2}x + 4, & x < 2 \end{cases}$$

Answer: C  
Justification: 
$$f(x) = \begin{cases} 2x - 4, & x \ge 2 \\ -2x + 4, & x < 2 \end{cases}$$

The graph is best described by 2 different lines (one when x > 2 and one when x < 2). Find the equation of these lines individually and combine them into a single function in piecewise notation:

$$f(x) = \begin{cases} 2x - 4, & x \ge 2\\ -2x + 4, & x < 2 \end{cases}$$

Notice that both the lines pass through the point (2,0), so the x = 2 case can be applied to either line.

#### **Alternative Solution**

**Answer:** C 
$$f(x) = \begin{cases} 2x - 4, & x \ge 2\\ -2x + 4, & x < 2 \end{cases}$$

**Justification:** First find the equation of the line when x < 2. The slope is -2 and the y-intercept is 4, so

 $f(x) = -2x + 4 \quad when \quad x < 2$ 

Notice that when  $x \ge 2$ , f(x) = -2x+4 is negative. Since the graph shown is an absolute value graph, we multiply f(x) = -2x+4 by -1 when it is negative. Therefore

$$f(x) = \begin{cases} 2x - 4, & x \ge 2\\ -2x + 4, & x < 2 \end{cases}$$

# **Absolute Values V**

What is the equation of the graph shown to the right, written with absolute values?

- A. f(x) = |2x 4|
- $\mathbf{B.} \quad f(x) = \left|-2x + 4\right|$
- $\mathbf{C.} \quad f(x) = \left| 2x + 4 \right|$
- $\mathbf{D.} \quad f(x) = \left|-2x 4\right|$
- E. More than one of the above are correct



**Answer:** E (both A and B are correct) **Justification:** 



 $f(x) = |2x-4| = |-2x+4| = \begin{cases} 2x-4, & x \ge 2\\ -2x+4, & x < 2 \end{cases}$ 

Answers A and B are the same because the expressions inside the absolute values only differ by a factor of -1.

$$\left|f(x)\right| = \left|-f(x)\right|$$

When f(x) = |2x-4|, the graph is reflected across the x-axis when x < 2. When f(x) = |-2x+4|, the graph is reflected across the x-axis when x > 2.

#### **Absolute Values VI**

Which of the following graphs corresponds to the equation:



#### Answer: D

$$f(x) = \left| -\frac{1}{2}x + 2 \right| - 2$$

**Justification:** Since 2 units are subtracted from the absolute value, the entire function is shifted 2 units down. Instead of negative values being reflected in the x-axis, they will be reflected across the line y = -2. This rules out answers A and B.

In order to decide between C and D, try finding an ordered pair. If we plug in x = 0 into the function, we find that the graph should pass through (0,0):

 $f(0) = \left| -\frac{1}{2}(0) + 2 \right| - 2 = 0 \quad \text{Only C passes though (0,0)}$ 

(Another strategy to solve this problem is to first graph

$$f(x) = \left| -\frac{1}{2}x + 2 \right|$$
, then shift it down by 2 units.)

#### **Absolute Values VII**

The graph of  $f(x) = |x^2 - 1|$  is shown below.

How many solutions are there to the equation  $|x^2 - 1| = 1$ ?



#### Answer: B

**Justification:** The function  $f(x) = |x^2 - 1|$  crosses the line y = 1 three times. This means that there are 3 values of x where  $f(x) = |x^2 - 1| = 1$ 



The equation should have 3 solutions.

$$|x^2 - 1| = 1$$

Answer continues on the next slide

### **Solution Cont'd**

#### Answer: B

**Justification:** The exact 3 solutions can be found by working with the equation. Divide the absolute value into 2 cases:

$$|x^{2} - 1| = 1$$

$$x^{2} - 1 = 1 \quad or \quad x^{2} - 1 = -1$$

$$x^{2} = 2 \quad x^{2} = 0$$

$$x = \pm \sqrt{2} \quad x = 0$$

The 3 solutions are  $x = -\sqrt{2}, 0, \sqrt{2}$ .

#### **Absolute Values VIII**

The graph of  $f(x) = |x^2 - 4x - 6|$  is shown below. For what values of q does the equation  $|x^2 - 4x - 6| = q$  have exactly 4 unique solutions?



- A. 0 < q < 10
- B.  $0 \le q \le 10$
- C.  $0 < q \le 10$
- D.  $0 \le q < 10$

E. *q* ≥10

#### Answer: A

**Justification:** If the equation  $|x^2 - 4x - 6| = q$  has 4 solutions, the line y = q must cross  $f(x) = |x^2 - 4x - 6|$  four times.



This happens when q is between: 0 < q < 10

When q = 0, the equation will only have 2 solutions.

When q = 10, the equation will only have 3 solutions.

Notice that it is impossible for this equation to only have 1 solution.