a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Mathematics
Geometry - Diagonals

Science and Mathematics
Education Research Group

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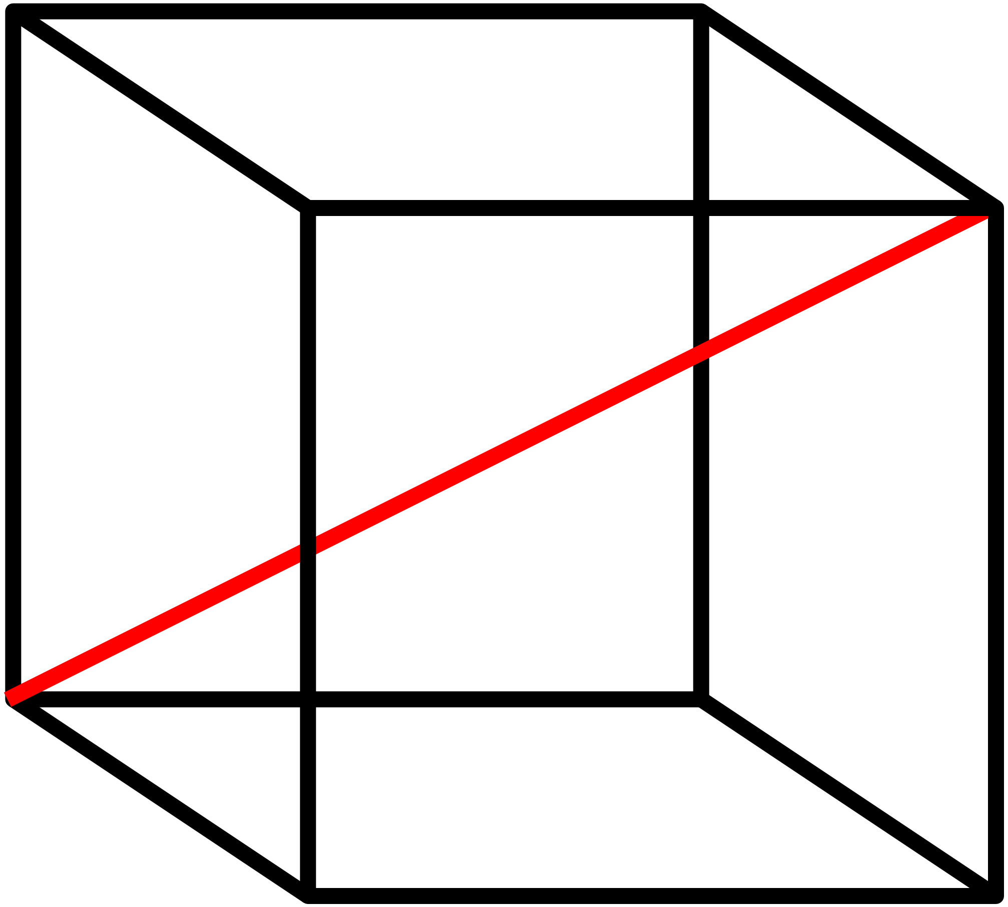
Diagonals



Diagonals I

A wireframe model of a cube is in front of you. Each side has length a. What is the length of the blue line?

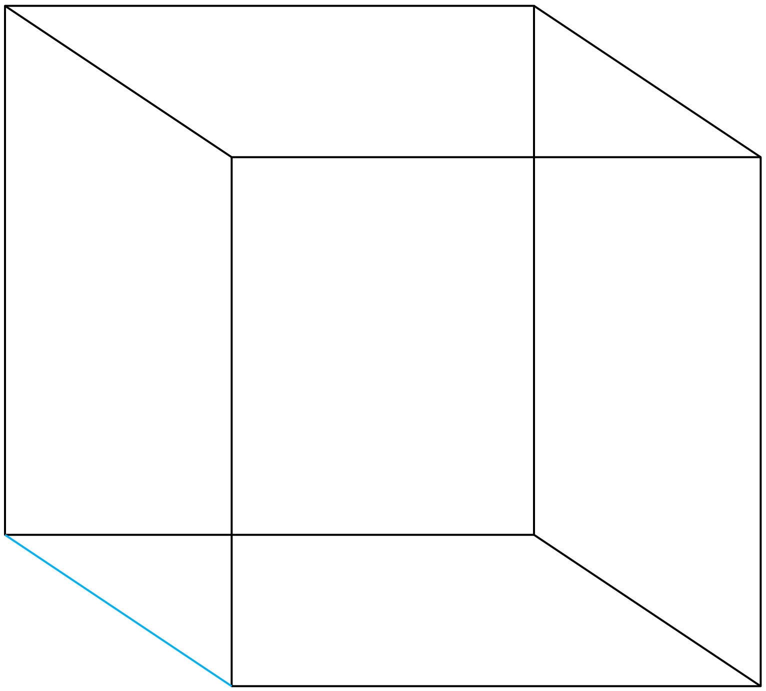
A. a

B. 2a

C. a/2

D. 12a

E. No idea



Solution

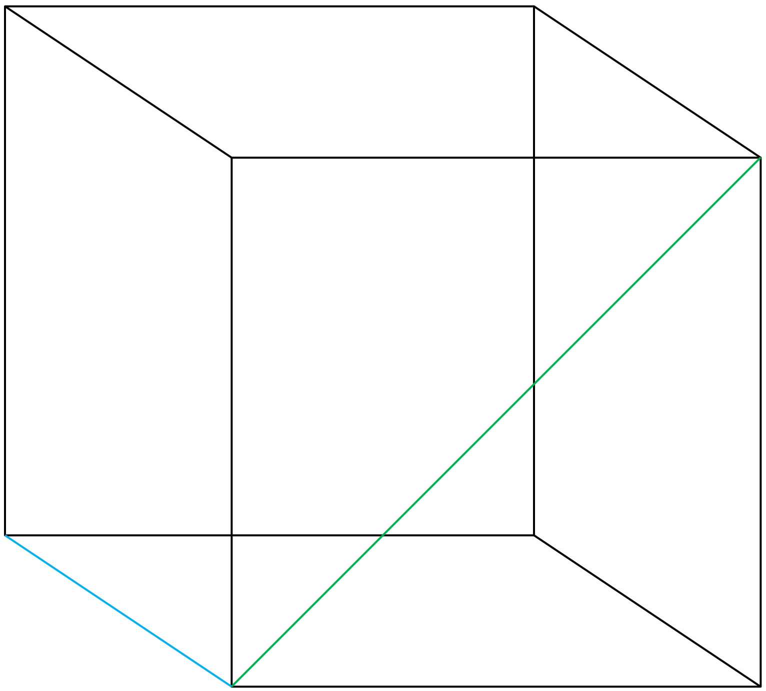
Answer: A

Justification: The side length is a. The blue line is a side. The answer is a.

Diagonals II

A wireframe model of a cube is in front of you. Each side has length a. What is the length of the green line?

A. $\sqrt{a}$
B. $2\sqrt{a}$
C. $\sqrt{2}a$
D. 2a
E. No idea



Solution

Answer: C

Justification: Using the Pythagorean theorem, where $a^2+b^2=c^2$, we see that $a^2+a^2=l^2$ where l is the length of the green line. Taking the square root of both sides we get $l=a\sqrt{2}$, which is our answer.

Remark: The following questions will become much easier when trigonometry is learned, but that is not the point of this problem set. Also, unless the radical has brackets following it, the radical is only applied to the first number, i.e. $\sqrt{2}a=(\sqrt{2})a\neq\sqrt{(2a)}$.

Diagonals III

A wireframe model of a cube is in front of you. Each side has length a. What is the length of the red line?

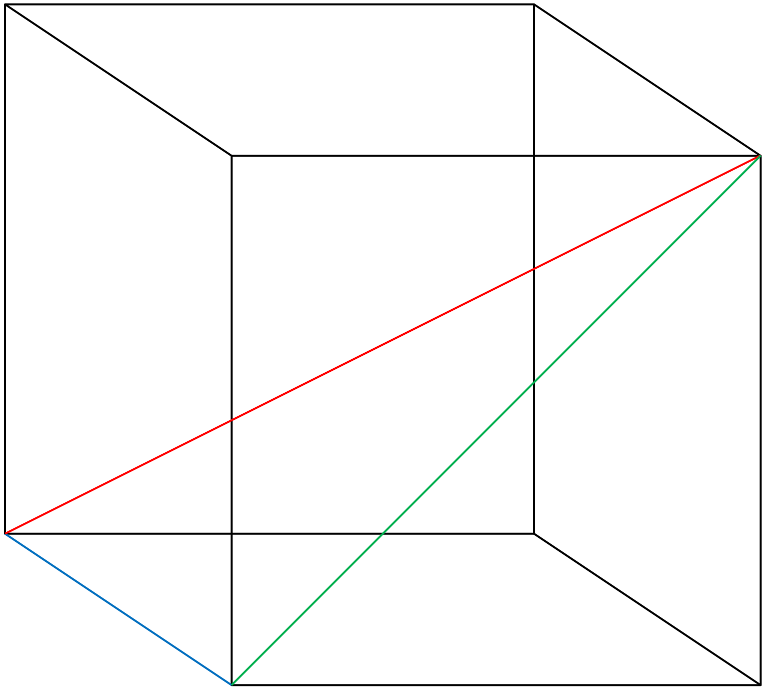
A. √a

B. 3√a

C. √3a

D. 2a

E. No idea



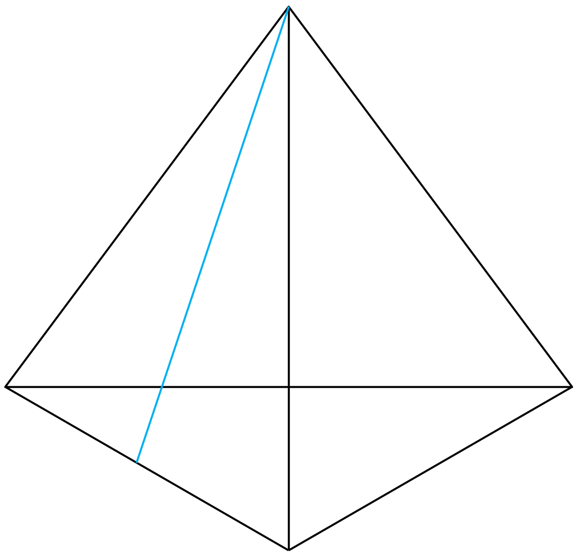
Solution

Answer: C

Justification: Again, we use the Pythagorean theorem. We can see the right triangle with the green line and blue line as sides. Since we know the green line has a length of $\sqrt{2}a$, $l^2=a^2+(\sqrt{2}a)^2$, where l is the length of the red line. Therefore, $l=\sqrt{3}a$, which is the answer.

Diagonals IV

A wireframe model of a regular tetrahedron is in front of you. Each side has length a. What is the length of the blue line that bisects the equilateral triangle?

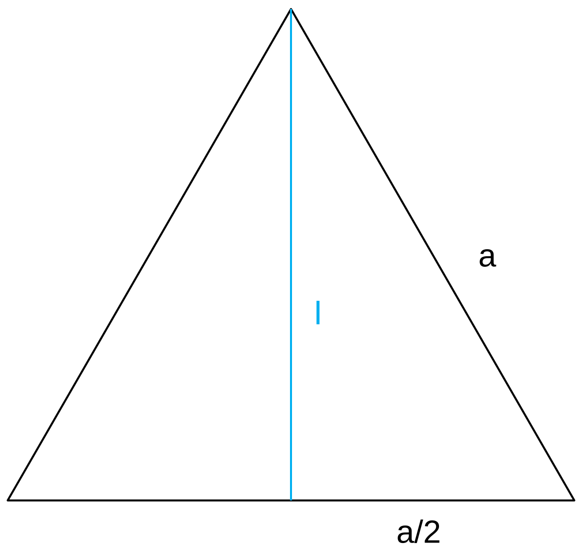


A. $2\sqrt{3}a/3$
B. $a/2$
C. $\sqrt{3}a/2$
D. $\sqrt{3}a$
E. $\sqrt{a}$

Solution

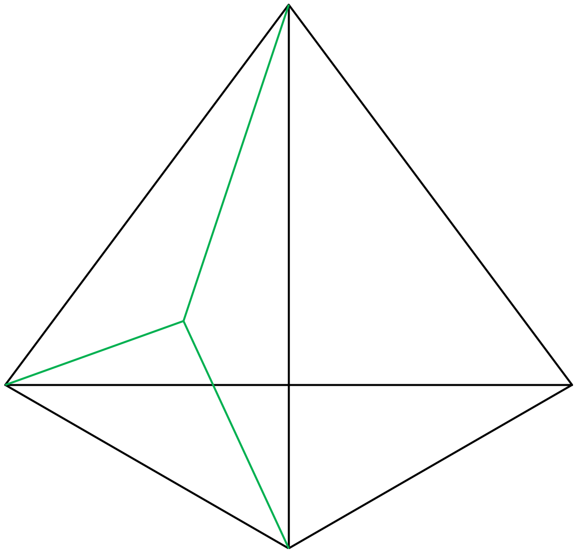
Answer: C

Justification: Look at the image below. The blue line constitutes a triangle with a/2 as the base and a as the hypotenuse. Using Pythagoras' theorem, we find that $l^2+(a/2)^2=a^2$, $l^2=3/4a^2$, and $l=\sqrt{3}a/2$.



Diagonals V

A wireframe model of a regular tetrahedron is in front of you. Each side has length a. What is the length of the red line if all four lines in the tetrahedron are equal and connected to the center?

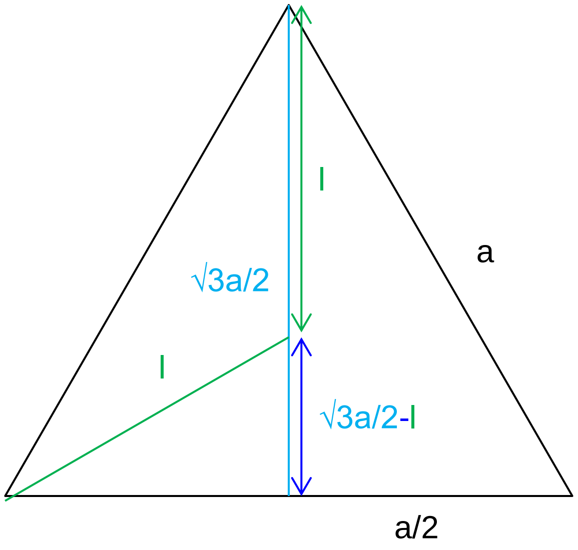


A. $\sqrt{3}a/3$
B. $\sqrt{3}a$
C. $2\sqrt{3}a/3$
D. $\sqrt{3}a/2$
E. $\sqrt{3}a/4$

Solution

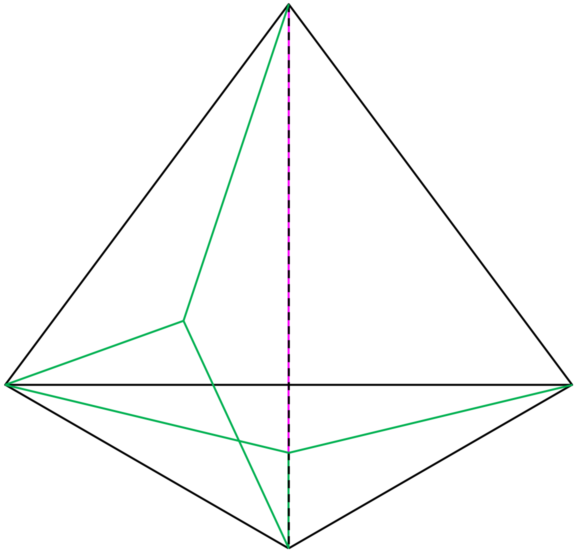
Answer: A

Justification: From the right triangle in the image below, we can see that $(\sqrt{3}a/2-l)^2+(a/2)^2=l^2$. Expanding we get $3a^2/4-\sqrt{3}al+l^2+a^2/4=l^2$. Simplifying, this is $\sqrt{3}al=a^2$. l is therefore $a/\sqrt{3}=\sqrt{3}a/3$.



Diagonals VI

A wireframe model of a regular tetrahedron is in front of you. Each side has length a. What is the length of the purple line that goes from the top vertex to the center of the base?

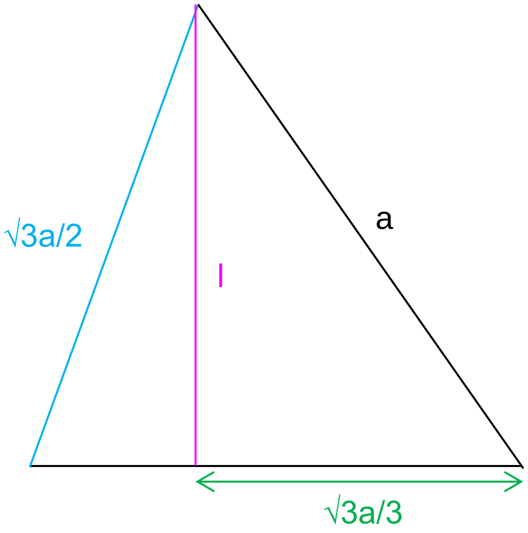


A. $\sqrt{3}a$
B. $\sqrt{3}a/2$
C. $\sqrt{6}a/2$
D. $\sqrt{6}a/3$
E. $\sqrt{3}a/\sqrt{3}$

Solution

Answer: D

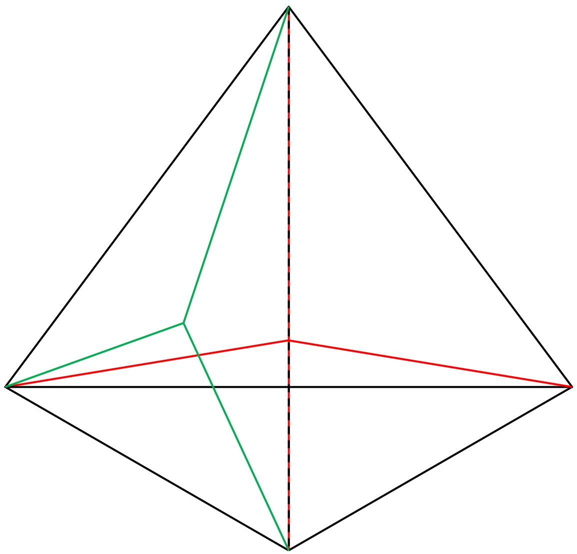
Justification: Splitting the tetrahedron in half and looking at the cross section, we see that the green trisectional lines and the edge of the tetrahedron make a right triangle with the purple line. From this we know that $a^2-(\sqrt{3}a/3)^2=l^2$, where $l^2=2a^2/3$, $l=a\sqrt{(2/3)}=\sqrt{6}a/3$.



Diagonals VII

A wireframe model of a regular tetrahedron is in front of you. Each side has length a. What is the length of the red line if all four lines in the tetrahedron are equal and connected to the center?

A. √3a
B. √3a/2
C. √6a/2
D. √6a/3
E. √3a/√3



Solution

Answer: C

Justification: Splitting the tetrahedron in half and looking at the cross sectional view, we can see that $(\sqrt{6}a/3-l)^2+(\sqrt{3}a/3)^2=l^2$. Expanding we get $2a^2/3-\sqrt{6}al/3+l^2+a^2/3=l^2$. Simplifying, this is $\sqrt{6}al/3=a^2$. l is therefore $3a/\sqrt{6}=\sqrt{6}a/2$.