a place of mind

# Mathematics <br> Finance: Compound Interest 

## Science and Mathematics Education Research Group

## Compound Interest



## Compound Interest I

A. $\$ 450$
B. $\$ 500$
C. $\$ 550$
D. $\$ 600$
E. $\$ 1,000,000$

For your birthday, you received $\$ 500$ from your friends and family. Being the smart individual that you are, you decided to bank your money in a high interest savings account, which has 10\% interest over the course of a year. If the interest is calculated once a year, how much money do you have in your bank by your next birthday (excluding the money you get on that birthday)?

## $\$ 500$

$+10 \%$

## Solution

## Answer: C

Justification: Because the interest is only calculated once a year, and it is $10 \%$ a year, by the end of the year you will have an additional $10 \% \times 500=\$ 50$, which means in total you will have $\$ 550$.

## Compound Interest II

A. $\$ 500$
B. $\$ 525$
C. $\$ 550$
D. $\$ 575$
E. \$600

Instead of giving you 10\% interest every year, the bank decides to give you $5 \%$ interest every half year. If you initially had $\$ 500$, how much money would you have after 6 months has gone by?

## Solution

## Answer: B

Justification: Since the bank gives you 5\% every half a year, and half a year has gone by, you get $100 \%+5 \%$ of your money, which is $\$ 525$.

## Compound Interest III

A. $\$ 500$
B. $\$ 525$
C. $\$ 534.75$
D. $\$ 551.25$
E. \$575

Consider the same situation as the last question, except this time a year has passed instead of 6 months. How much money do you have at the end of the year?

## \$525 <br> $+5 \%$

## Solution

## Answer: D

Justification: We know from the last question that you have $\$ 525$ at the end of 6 months. So after another 6 months have passed, you get 105\% of what you have after the first 6 months, which is $\$ 525$. By the end of the year you will have $105 \% \times 525=\$ 551.25$. The interest here has been compounded twice, so you will have $105 \% \times 105 \%$ of your original money. This is $10 \%$ interest per year compounded biannually, which means $5 \%$ is applied the first half year, and $5 \%$ is applied in the second year. This does not mean that you will have $10 \%$ at the end.

## Compound Interest IV

A. $\$ 500$
B. $\$ 525$
C. $\$ 534.75$
D. $\$ 551.25$
E. \$575

You store $\$ 500$ in a bank with $10 \%$ interest per year, compounded every 6 months (twice a year). How much money do you have by the end of the year?


## Solution

## Answer: D

Justification: This is the same question as question 3, except worded differently. The general equation is 500(100\%+10\%/2)(100\%+10\%/2).

## Compound Interest V

A. $P(1-r / 2)$
B. $P(1+r / 4)$
C. $P(1+r / 2)$
D. $\mathrm{P}(1+\mathrm{r})$
E. None of the above

You store P dollars in a bank with r\% interest per year, compounded twice a year. How much money do you have by the end of 6 months? Note that $100 \%=1$, so B could also be written as $P(100 \%+r / 4)$.

## Solution

## Answer: C

Justification: As half of the year has gone by, you get half of the interest per year added to your total, or $1+r / 2$. Multiply this by your original amount of money to get the amount of money you have after 6 months.

## Compound Interest VI

A. $P(1-r / 2)(1+r / 2)$
B. $P(1+r / 2)$
C. $P(1+r / 2)^{2}$
D. $P(1+r)(1+r / 2)$
E. None of the above

You store P dollars in a bank with $\mathrm{r} \%$ interest per year, compounded twice a year. How much money do you have by the end of the year? Note that $(1+r)^{2}=(1+r)(1+r)$.


## Solution

## Answer: C

Justification: When half of the year had passed you had $P(1+r / 2)$ money. After another 6 months, you would have $100 \%$ plus half of the interest rate (10\%) applied to what you had after the first 6 months. Therefore the answer is

$$
P(1+r / 2) \times(1+r / 2)=P(1+r / 2)^{2} .
$$

## Compound Interest VII

A. $P(1+r / n)^{n t}$
B. $P(1+r / 2)^{2 t}$
C. $P(1+r / n)^{n}$
D. $P(1+r)^{n}$
E. None of the above

You store P dollars in a bank with r\% interest per year, compounded $n$ times a year. How much money do you have by the end of $t$ years?

## Solution

## Answer: A

Justification: The answer to question 5 was $\mathrm{P}(1+\mathrm{r} / 2)^{2}$. That was for compounded twice a year. If we wanted to compound $n$ times a year, each time would have a r/n percent increase. Since it is $n$ times a year for $t$ years, there is $n t$ of such increases. Thus, our original savings are increased by $(1+r / n)^{n t}$, and $P(1+r / n)^{n t}$ is our final answer.

