a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Mathematics
Numbers: Factors

Science and Mathematics
Education Research Group

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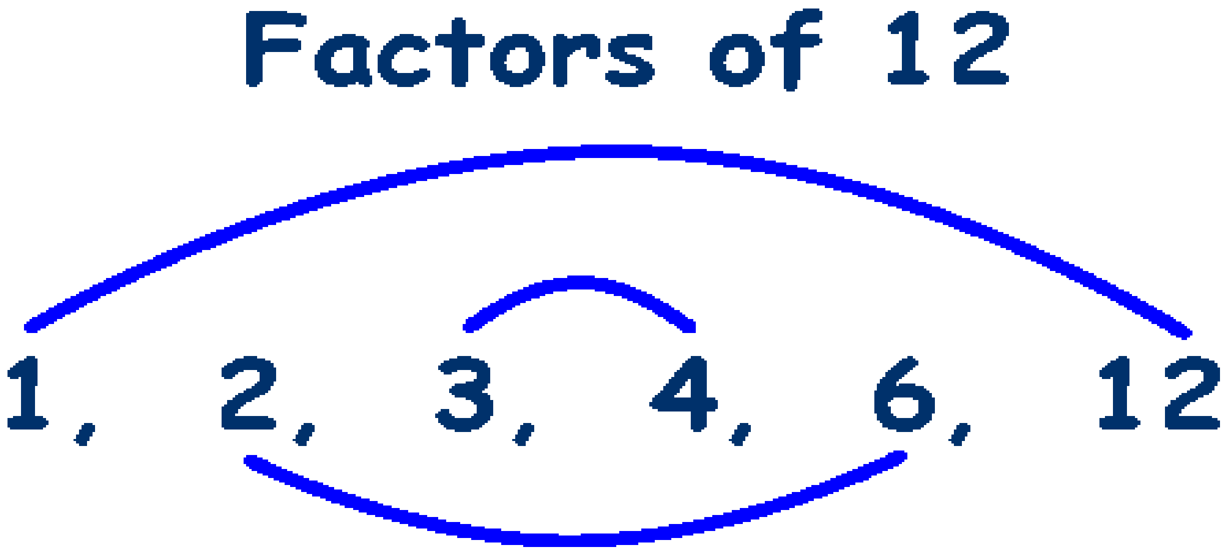
Factors



Retrieved from http://www.11plusforparents.co.uk/Maths/factors.html

Factors

Which number is not a factor of 18?

A. 1
B. 2
C. 3
D. 4
E. 6

Solution

Answer: D

Justification: Factors of a certain number are natural numbers (1, 2, 3 …) that multiply into that certain number.

For example, for factors of 8, we are looking for a pair of numbers that when we multiply them, the result (product) is 8.

$$\underbrace{8}_{\text{Product}} = \underbrace{1 \times 8}_{\text{Factors}}$$

Solution I Cont'd

Factors of 18 can be found by finding two numbers that multiply into 18. Therefore,

$$18 = 1 \times 18$$
$$18 = 2 \times 9$$
$$18 = 3 \times 6$$

Thus, factors of 18 are 1, 2, 3, 6, 9, and 18. Since 4 is not in this list, the correct answer for this question is D.

Solution II

Factors of 18 can be found using the following method:

1. List the first two factors of 1 and itself on the let and right end of a horizontal line.

$$18 = 1 \times 18$$

1 18

2. Now, move on to the next possible factor. For our case, we know that 18 is divisible by 2 because it is an even number. The number that multiplies with 2 to get 18 is 9.

$$18 = 1 \times 18$$

$$18 = 2 \times 9$$

1 2 9 18

Thus, from this step, the left side is increasing and the right is decreasing. At the end, we will have exhausted all the pairs of numbers that multiply into 18.

Solution II Cont'd

3. Continue with step 2. We know that 18 is divisible by 3, because if we add up the digits of 18, it gives 9, which is divisible by 3. The number that multiplies by 3 to get 18 is 6.

$$18 = 1 \times 18$$

$$18 = 2 \times 9$$

$$18 = 3 \times 6$$

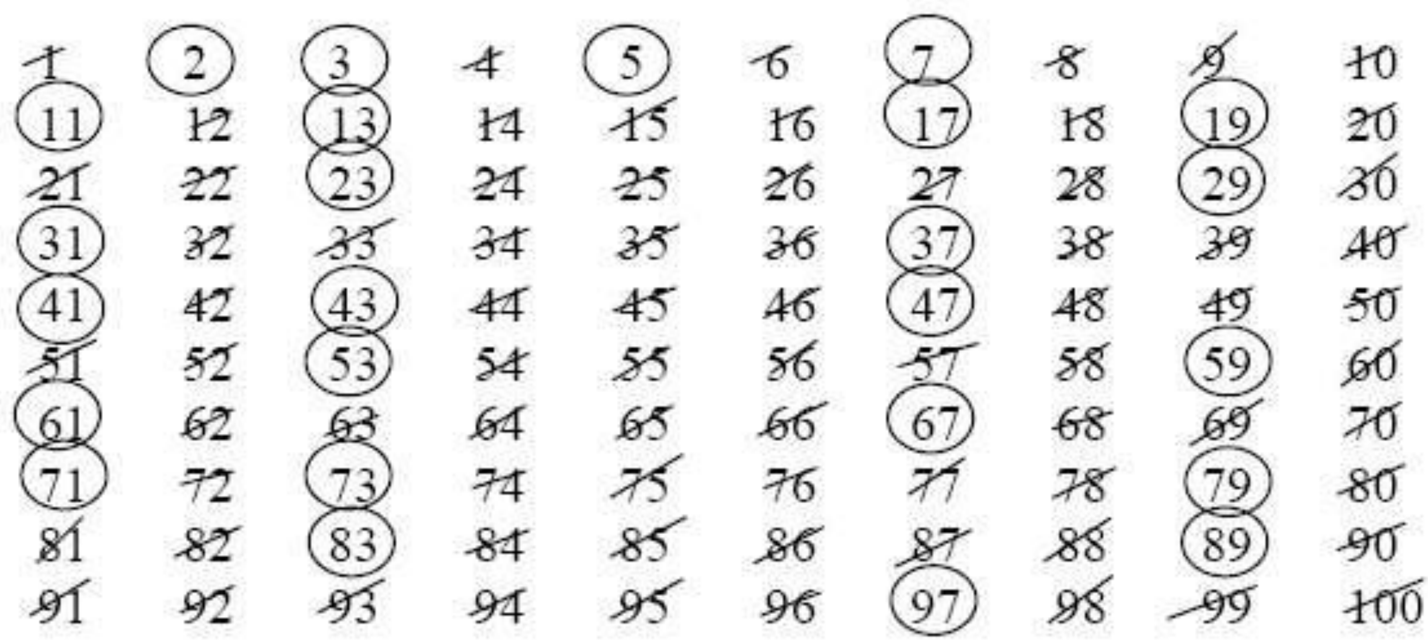
1 2 3 6 9 18

4. The next possibility is 4 or 5. Since 18 is not divisible by either of them, we are done.

Thus, factors of 18 are 1, 2, 3, 6, 9, and 18. Our answer is D.

Prime Numbers

Sieve of Eratosthenes



Retrieved from http://www.onlinemathlearning.com/image-files/sieve-prime.jpg

Why Prime Numbers?

Prime numbers are used for internet security of our financial-related medium. This includes credit card numbers and bank account numbers.

For example, a secure way that a bank communicate important information with like customer's credit card number is using Public Key Cryptography, which uses prime numbers.

When someone tries to see your credit card number, he will see a long series of numbers called cipher. The bank decrypts this cipher using two different and large prime numbers called Private Key, and decodes the secret message. Then, when the bank sends the information in a number that is a product of these Private Keys. This number can be used to generate credit card numbers for customers using Public Key, which the bank sends to customers.

Retrieved from http://www.itsnotmagicitsscience.com/science.asp?newsid=492

Prime Numbers

Which number is not a prime number?

A. 2
B. 9
C. 11
D. 13
E. 19

Solution

Answer: B

Justification: Prime numbers are numbers that have factors of 1 and itself.

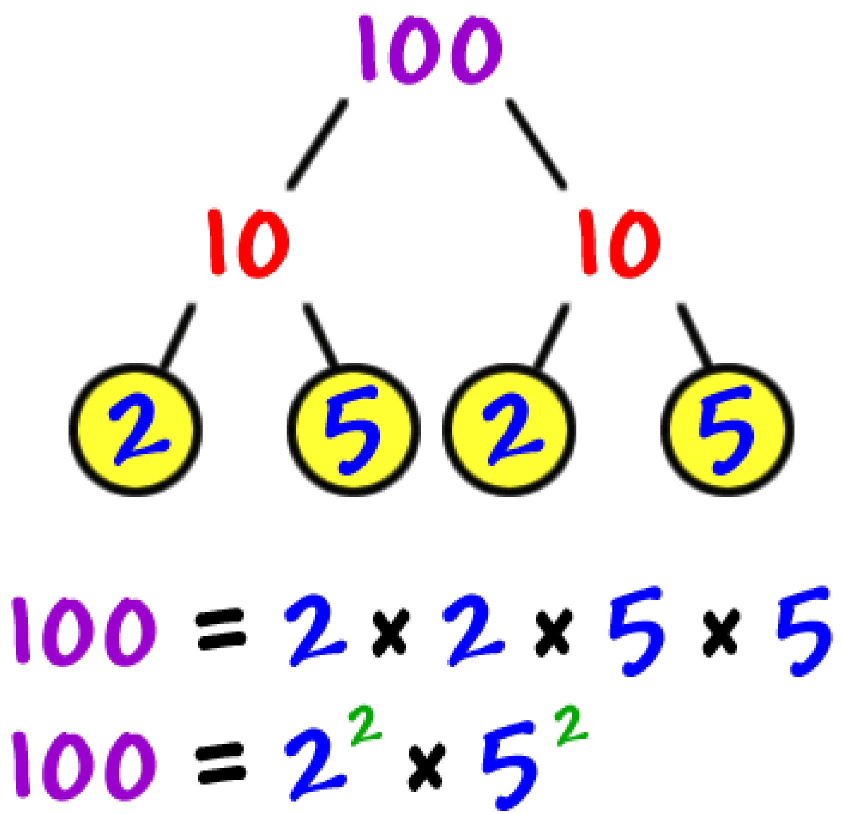
Taking a look at our choices:

$$8 = 1 \times 8$$
$$9 = 1 \times 9, 3 \times 3$$
$$11 = 1 \times 11$$
$$13 = 1 \times 13$$
$$19 = 1 \times 19$$

Remember!
Prime numbers have
Factors of 1 and itself only!

Since 9 has factors of 1, 3, and 9, it cannot be a prime number. Our answer is B.

Prime Factorization



Retrieved from http://coolmath.com/prealgebra/00-factors-primes/04-prime-factorizations-04.htm

Prime Factorization

What is the Prime Factorization of 196?

A. 4×49

B. $2^2 \times 7^2$

C. 14^2

D. $2^2 + 7^2$

E. $2 \times 7 \times 14$

Solution I

Answer: B

Justification: We can divide 196 by the smallest possible prime number. The first prime number we can try dividing is 2.

$$196 \div 2 = 96$$

$$96 \div 2 = 49$$

Since 49 cannot be divided by 2, 3, nor 5, the next prime number we can choose to divide is 7.

$$49 \div 7 = 7$$

Since we have multiplied two 2s and two 7s, we can write as prime factorization of $2^2 \times 7^2$. Thus, the answer is B.

Solution II

Answer: B

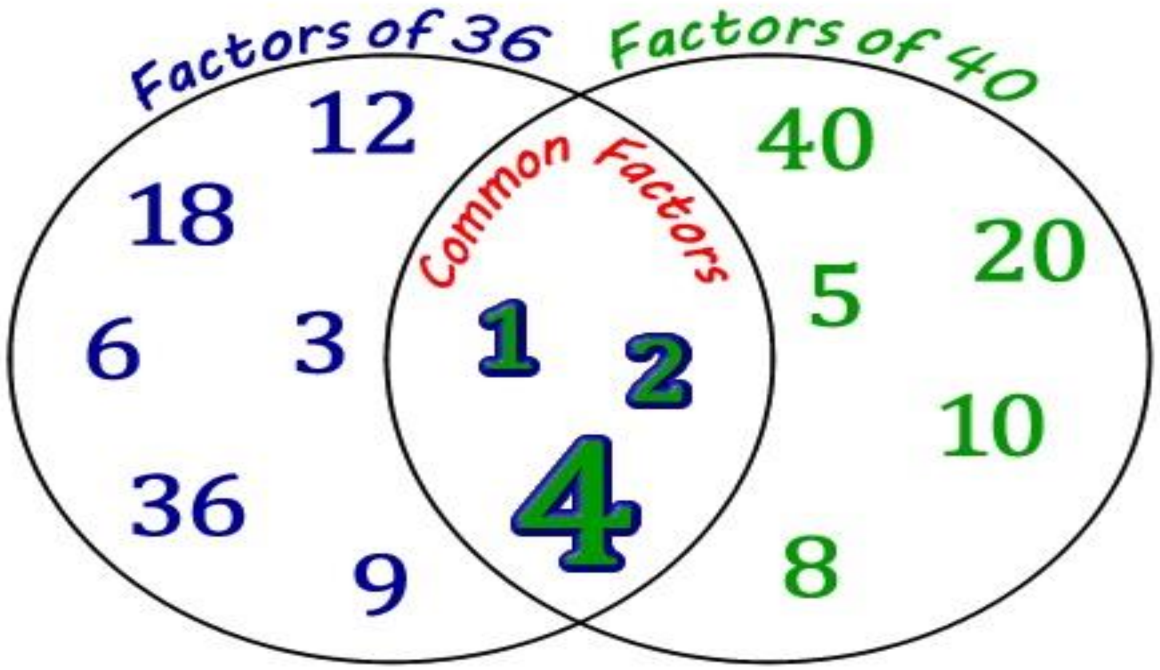
Justification: Similar to the previous solution, we can start dividing 196 by the smallest possible prime number, which is 2.

$$\begin{array}{r|c}
2 & 196 \\ \hline
2 & 98 \\ \hline
7 & 49 \\ \hline
 & 7
\end{array}$$

Once we have a prime number at the very end of this operation, we can stop. From this method, we know how many prime factors 196 has – it has two prime factors: 2 and 7.

Thus, we can write $196 = 2^2 \times 7^2$. The answer is B.

Common Factors



Retrieved from http://edtech2.boisestate.edu/brianroska/506/finalproject/gcf.html

Common Factors

What are the common factors of 12 and 18?

A. 1

B. 1, 2

C. 1, 2, 3

D. 1, 2, 3, 6

E. 1, 2, 3, 6, 12

Solution

Answer: D
Justification: First, list all the factors of 12 and 18. Then, we can find the common factors of these two numbers.

Using solution II from our factor question,

Factors of 12 = 1 2 3 4 6 12
Factors of 18 = 1 2 3 6 9 18

Remember!
They have to be
Common (same)!

Circling all of the common numbers, our common factors of 12 and 18 are 1, 2, 3, and 6. Our answer is D.

Greatest Common Factor (GCF)

24 = 1, 2, 3, 4, 6, 8, 12, 24

36 = 1, 2, 3, 4, 6, 9, 12, 18, 36

Retrieved from http://www.mathemania.com/gcf.php

Greatest Common Factor (GCF)

What is the greatest common factor of 48 and 60?

A. 6
B. 12
C. 24
D. 48
E. 60

Solution I

Answer: B

Justification: First, list all the factors of 48 and 60. Then, we can find the greatest common factor of these two numbers.

Factors of 48 = 1 2 3 4 6 8 12 16 24 48
Factors of 60 = 1 2 3 4 5 6 10 12 15 20 30 60

Circling all of the common numbers, our common factors of 48 and 60 are 1, 2, 3, 4, 6, and 12. Out of these factors, 12 is the greatest. Our answer is B.

Solution II

Answer: B

Justification: For 48 and 60, we can start dividing them by the smallest possible prime number. In our case, we start with 2.

2	48	60
2	24	30
3	12	15
	4	5

Once we can no longer divide by same prime number, we can stop. In order to find the GCF, we have to multiply the numbers we have been dividing both 48 and 60 (the numbers in blue circle). By multiplying 2 x 2 x 3, we get 12. Thus, our GCF is 12. The answer is B.

Greatest Common Factor II

Suppose we have 3 dozens of eggs and 63 loaves of bread. We want to distribute these items evenly to as many people as possible. How many people can receive these items?

A. 9
B. 12
C. 21
D. 36
E. 63

Solution I

Answer: A
Justification: This question has to do with the GCF because we want to find the greatest number of people that we can divide both 36 and 63 without having a remainder.

First, list all the factors of 36 and 63. Then, we can find the greatest common factor of these two numbers.

Factors of 36 = 1 2 3 4 6 9 12 18 36
Factors of 63 = 1 3 7 9 21 63

Highlighting all of the common numbers, our common factors of 36 and 63 are 1, 3, and 9. Out of these factors, 9 is the greatest. Our answer is A.

Solution II

Answer: A

Justification: Knowing that this question is finding the GCF between 36 and 63, we can start dividing them by the smallest possible prime number. In our case, we start with 3.

$$\begin{array}{c|cc} 3 & 36 & 63 \\ \hline 3 & 12 & 21 \\ \hline & 4 & 7 \end{array}$$

Once we can no longer divide by same prime number, we can stop. In order to find the GCF, we have to multiply the numbers we have been dividing both 36 and 63 (the numbers in blue circle). By multiplying 3 x 3, we get 9. Thus, our GCF is 9. The answer is A.