a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Mathematics
Numbers: Percentages

Science and Mathematics
Education Research Group

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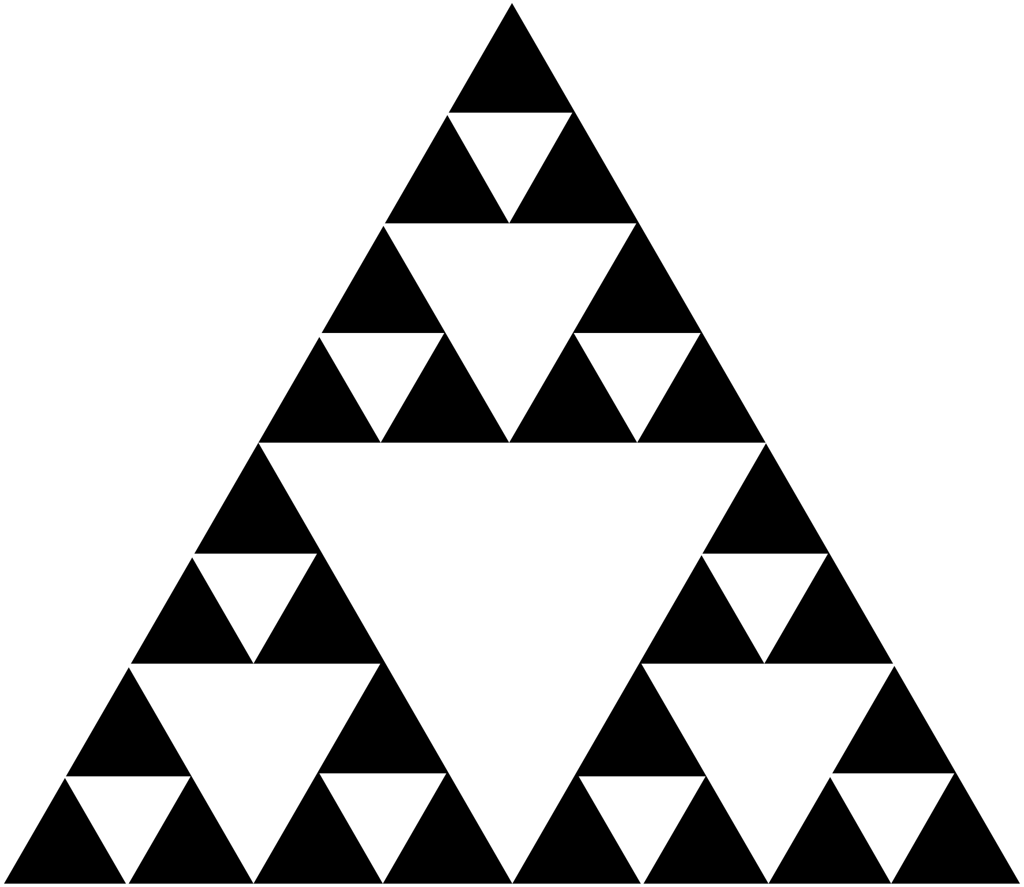
Sierpinski Triangle



Sierpinski Triangle I

An equilateral triangle is divided into 4 equal parts. The centre piece is then removed. What fraction of the original triangle remains?

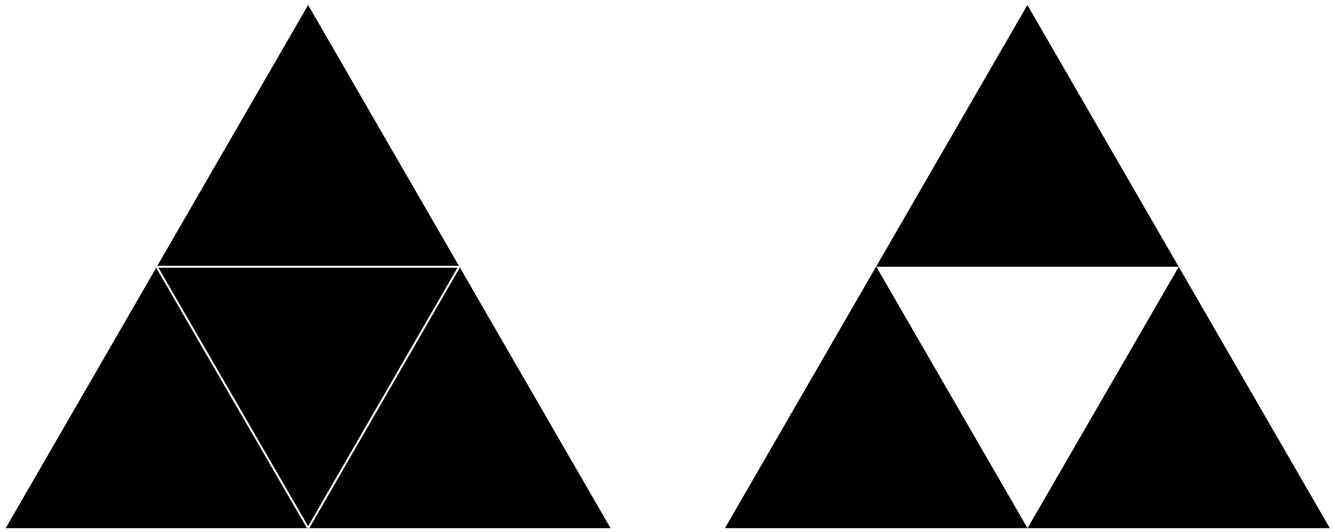
A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

E. $\frac{4}{3}$



Solution

Answer: D

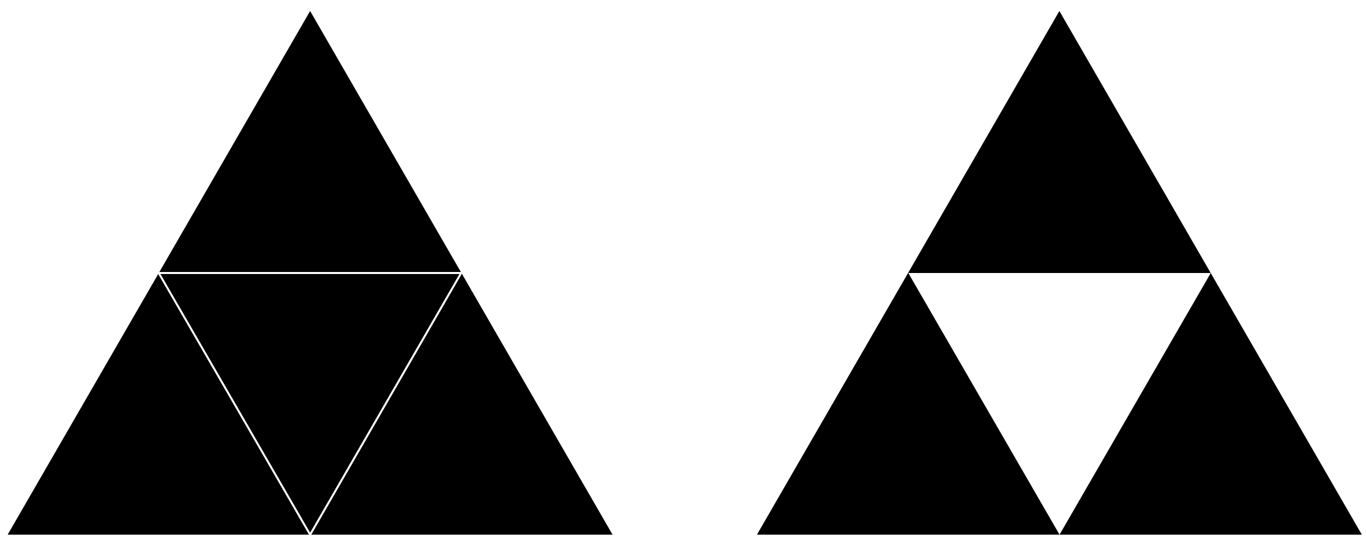
Justification: The original triangle was split into 4 equal parts, or 4 quarters. Since one part was removed, there are 3 left, constituting ¾ of the original triangle.

Think about having four quarters in your pocket. When you take away one quarter, you are left with the other three, and you have ¾ of a dollar left.

Sierpinski Triangle II

A. 25%
B. 30%
C. 60%
D. 67%
E. 75%

An equilateral triangle is divided into 4 equal parts. The upside down triangle in the center is then removed. What percentage of the original triangle remains?



Solution

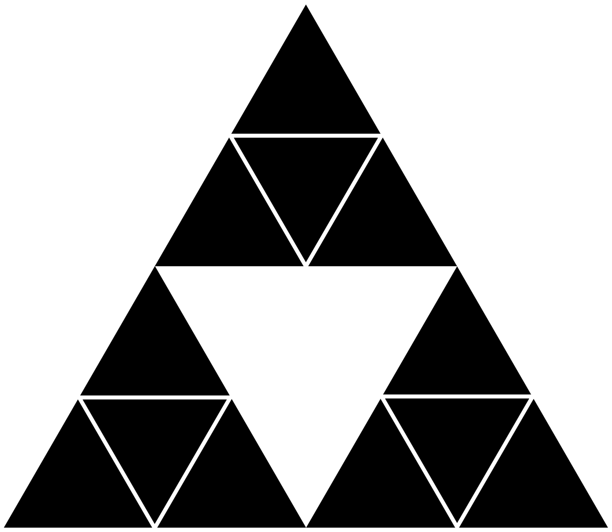
Answer: E

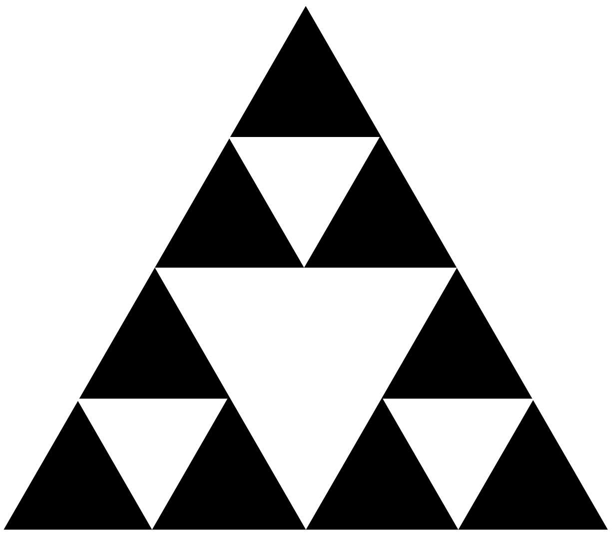
Justification: From question 1 we know that ¾ of the original triangle remains. The definition of a percent is a ratio to 100, so we want to find the number that is ¾ of 100. Multiplying 100 by ¾, we get 75, which is correct as

$$75/100 = 3/4 = 75\%$$

Sierpinski Triangle III

The equilateral triangle from the last question is divided even further by removing the centre triangles of each of the remaining black triangles. What fraction of the original whole triangle (without any holes) remains?





A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{9}{16}$

D. $\frac{11}{16}$

E. $\frac{3}{4}$

Solution

Answer: C

Justification: In this question, each of the three smaller black triangles had the center removed. We know that each center is ¼ of the triangle, so ¼ of each black triangle was removed. This leaves ¾ of the total area of each black triangle.

As we saw in question 2, the three black triangles make up 75%, or ¾, of the original triangle.

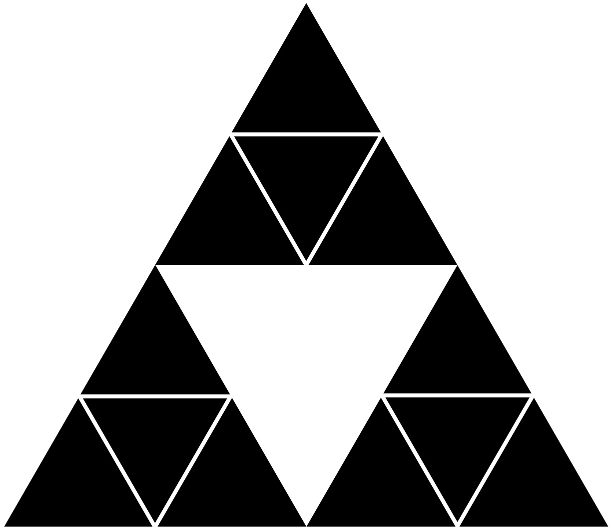
So the black area remaining is ¾ of ¾ of the original triangle.

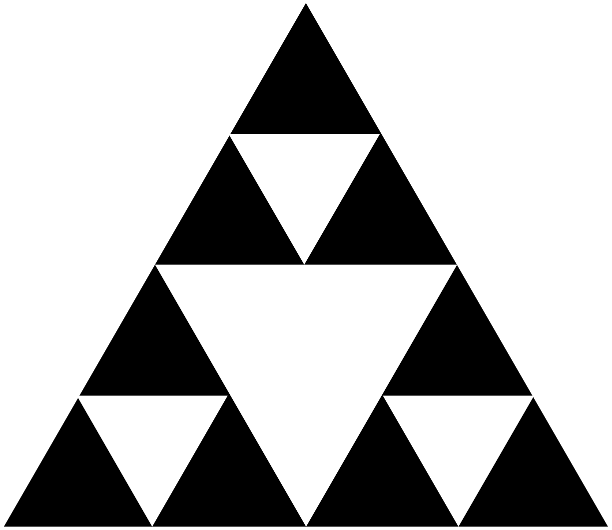
$$\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

Sierpinski Triangle IV

What percentage of the original whole triangle (without any holes) remains?

A. 24.25%
B. 30.25%
C. 56.25%
D. 67.75%
E. 75.25%





Solution

Answer: C

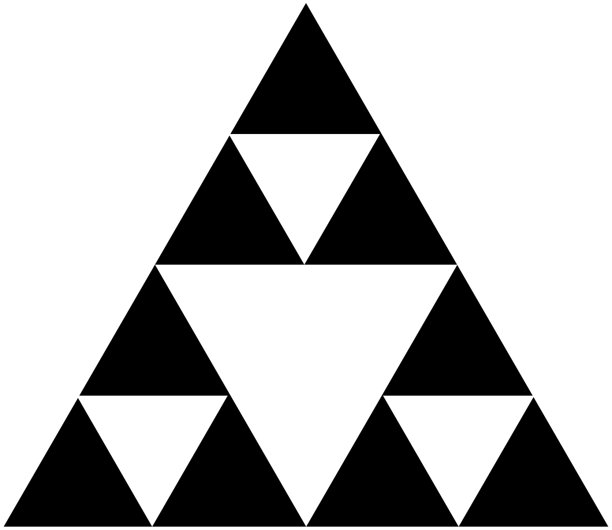
Justification: We know from question 3 that 9/16 of the original triangle remains. To turn a fraction into a percentage, we must multiply the fraction by 100%.

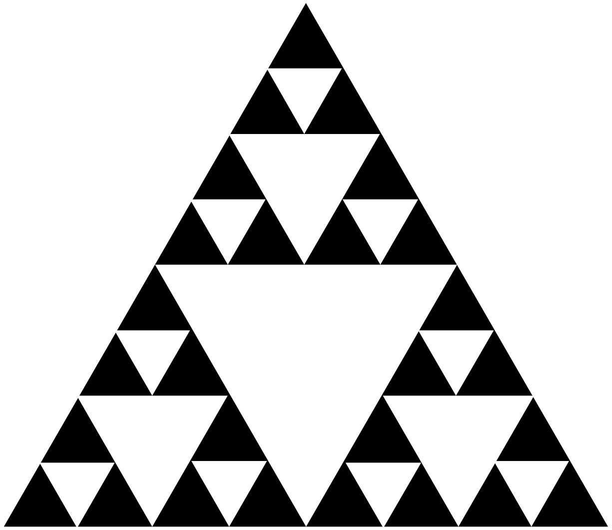
$$100\% \times \frac{9}{16} = \frac{900\%}{16} = 56.25\%$$

Sierpinski Triangle V

The equilateral triangle from the last question is divided even further by removing the center triangles from the remaining black triangles. What fraction of the original whole triangle (without any holes) remains?

A. 5/64
B. 9/32
C. 27/64
D. 9/16
E. 3/4





Solution

Answer: C

Justification: From similarities to question 3, we know that each black triangle has ¼ of its area removed when the center piece is removed.

The total area of each successive shape is ¾ of the shape preceding it.

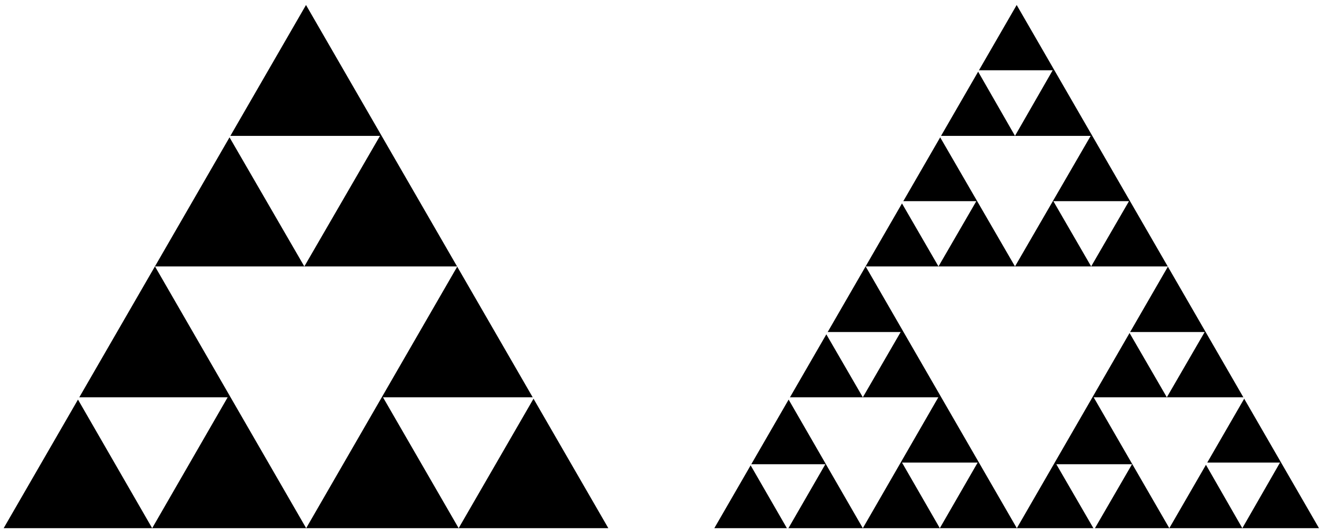
The last shape had an area of 9/16, therefore

$$\frac{3}{4} \text{ of } \frac{9}{16} \text{ is equal to } \frac{3}{4} \times \frac{9}{16} = \frac{27}{64}$$

Sierpinski Triangle VI

What percentage of the original whole triangle (without any holes) remains?

A. 22.34%
B. 42.19%
C. 56.25%
D. 67.75%
E. 75%



Solution

Answer: B

Justification: In question 5 we found that $^{27}\!/_{64}$ of the triangle remains.

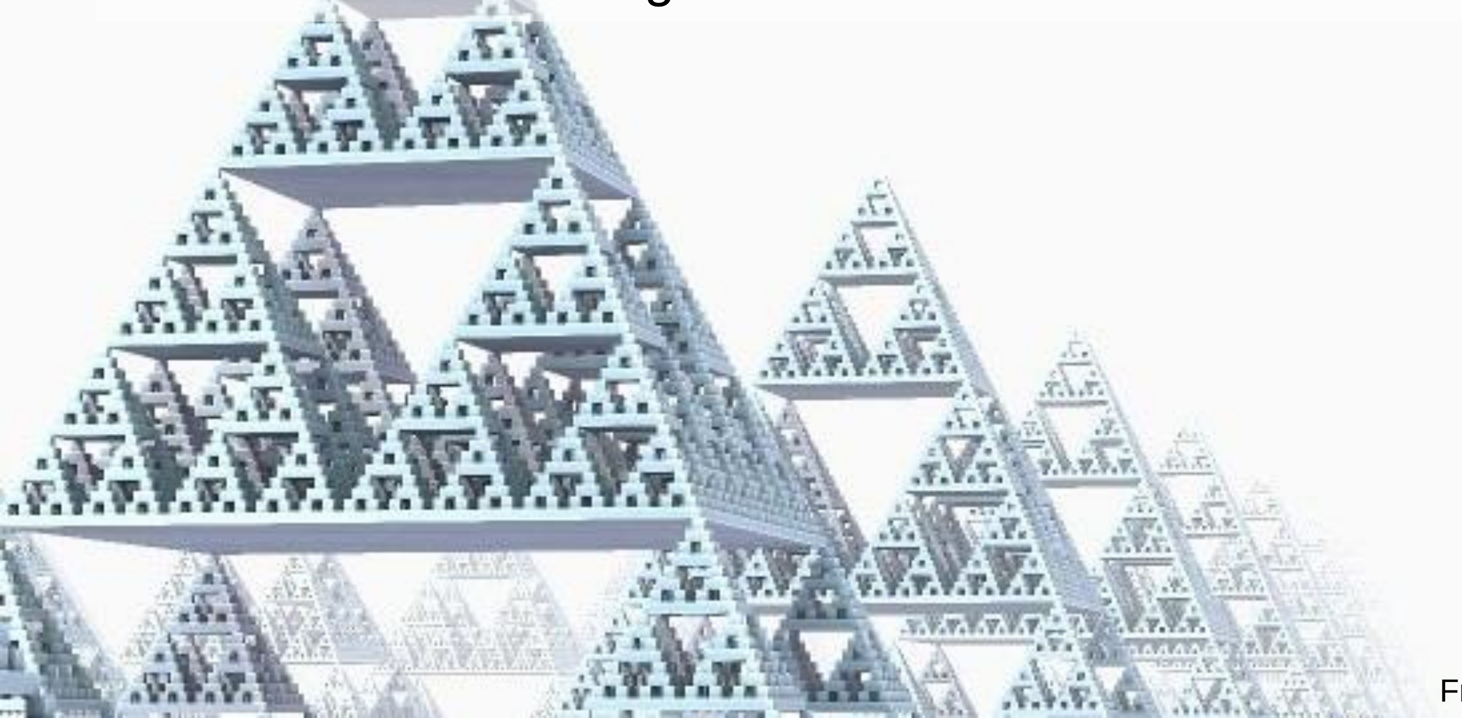
Multiplying that by 100, we get

$$100\% \times \frac{27}{64} = \frac{2700\%}{64} = 42.19\%$$

42.19% of the original triangle remains in this shape.

The Sierpinski Triangle and Other Fractals

The Sierpinski Triangle is part of a group of mathematical constructs called fractals. All fractals share the same property of having parts that look like the whole. Like many other fractals, it can be produced via a variety of methods. One of the methods we have just seen in this problem set, and the other methods include mathematical constructions such as Pascal's Triangle and the Chaos Game.



Fractal art by Krzysztof Marczak

The Sierpinski Triangle and Other Fractals

There are fractals of all shapes and sizes. Some look like a gothic cathedral, some look like a futuristic metal factory, and some look vaguely organic. The Mandelbrot Set, a famous fractal, has the property that it is a “map” or combination of another fractal, the Julia Set. One of the adaptations of the Mandelbrot Set into the third dimension, the Mandelbox (the one being used as a background for this slide), also shares the ability to contain other fractals, such as the Koch Snowflake, the Apollonian Gasket, the Maskit fractal, and even the Sierpinski Triangle.

The Sierpinski Triangle and Other Fractals

One might think that fractals are only mathematical constructs and serve no applications or connections to real life whatsoever. However, fractals are everywhere in nature. Things such as trees, mountains, coastlines, clouds, cracks, and even rivers all show fractal geometry. Some scientists also use fractals to model complex chaotic systems such as turbulence and weather


Looks like a rock right?
(it’s a computer generated fractal)