a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Mathematics
Intersection of Lines

Science and Mathematics
Education Research Group

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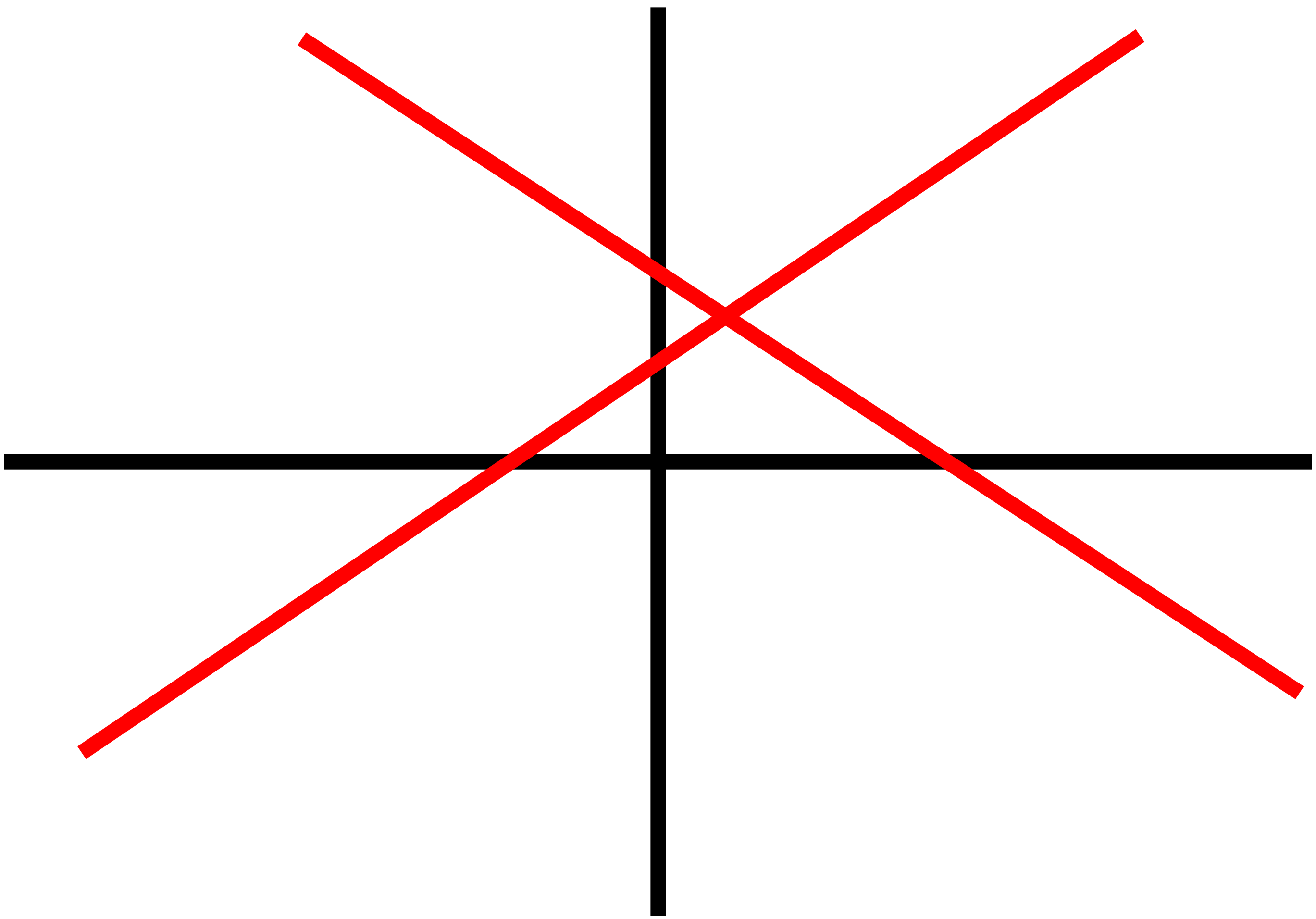
Intersection of Lines

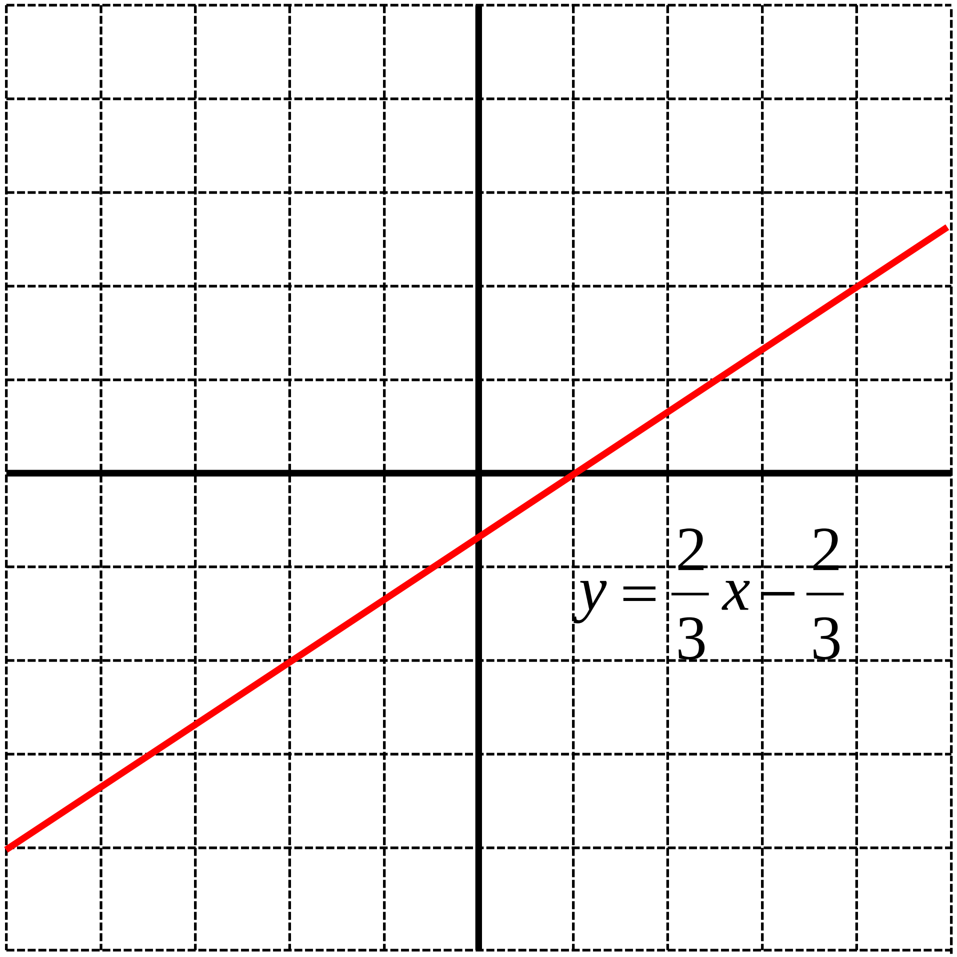


Intersection of Lines

Does the line shown in the graph pass through the point (9,5)?

A. Yes

B. No



Solution

Answer: B

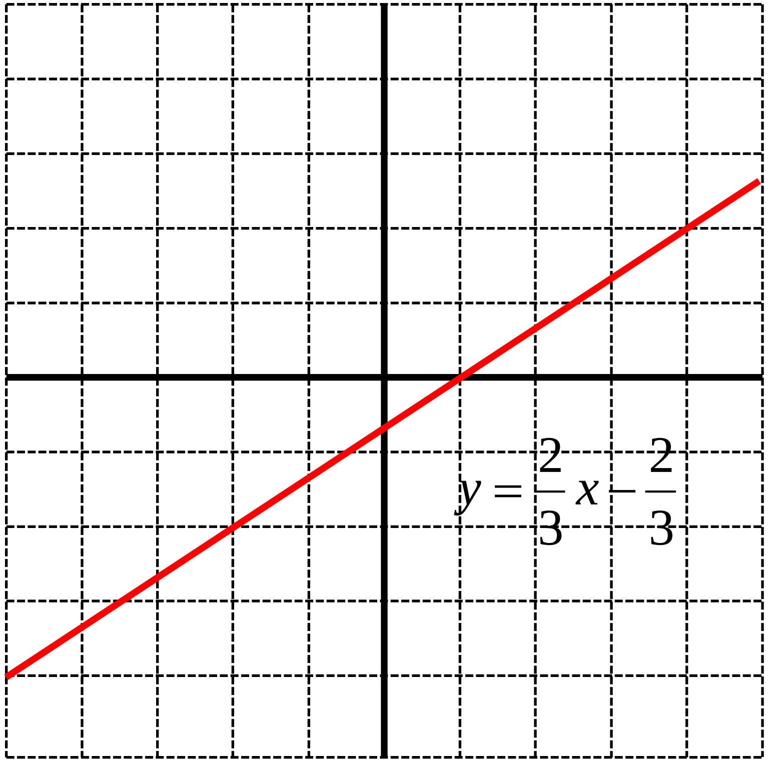
Justification: The equation of the line represents the y-value of a point, given an x-value. If the point (9,5) is inserted into the equation:

$$y = \frac{2}{3}x - \frac{2}{3}$$

$$5 = \frac{2}{3}(9) - \frac{2}{3}$$

$$5 \neq \frac{16}{3}$$

Since this is not true, the point does not lie on the line.



Intersection of Lines II

The lines $y_1 = m_1 x_1 + b_1$ and $y_2 = m_2 x_2 + b_2$ intersect at the point (x_i, y_i). Which one of the following statements must be true?

A. $y_i = m_1 x_i + b_1 \quad and \quad y_i = m_2 x_i + b_2$

B. $y_i = m_1 x_i + b_1 \quad or \quad y_i = m_2 x_i + b_2$

C. $m_1 = m_2$

D. $m_1 x_i + b_1 = m_2 x_i + b_2$

E. Both A and D

Solution

Answer: E (Both A and D)

Justification: Since (x_i,y_i) is the point of intersection of two lines, it is a point that lies on both lines. From the previous question, we know that when a point lies on a line, x_i and y_i can be plugged into the equation of the line. Therefore

A. $y_i = m_1 x_i + b_1 \quad and \quad y_i = m_2 x_i + b_2$

is true.

Since y_i is the same in both lines, they can be equated to give:

D. $m_1 x_i + b_1 = m_2 x_i + b_2$

Intersection of Lines III

At what point do the lines $y = \frac{2}{3}x - 1$ and $y = -2x + 3$ intersect?

A. $\left(0, \frac{3}{2}\right)$

B. $\left(\frac{3}{2}, 0\right)$

C. $(3, 2)$

D. $(2, 3)$

E. $(0, 2)$

Solution

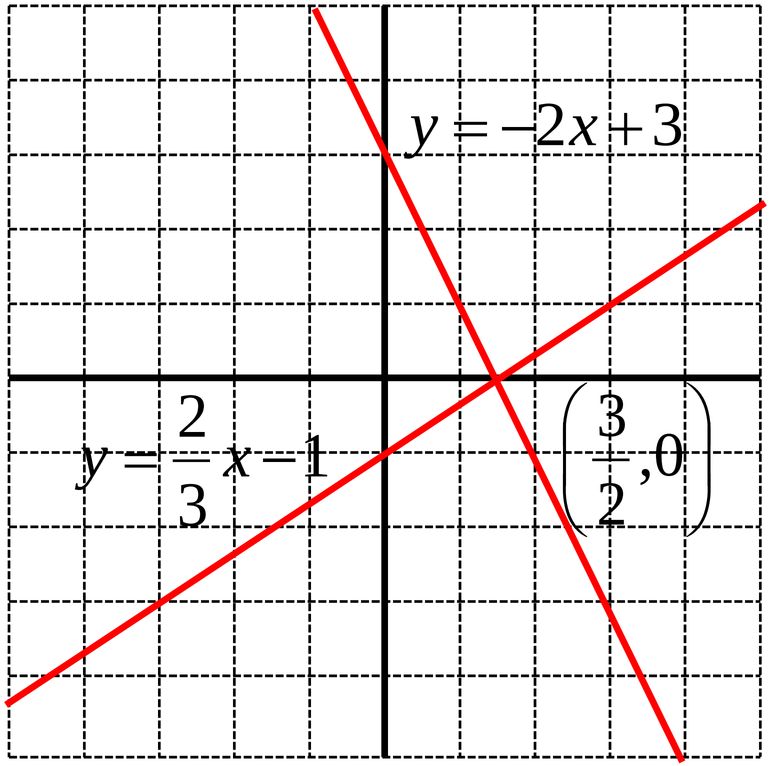
Answer: B

Justification: The x-value of the point of intersection can be found by equating the equations:

$$-2x_i + 3 = \frac{2}{3}x_i - 1$$

$$-6x_i + 9 = 2x_i - 3$$

$$x_i = \frac{12}{8} = \frac{3}{2}$$



Only B has this x-value. Double check that y_i=0 by inserting x_i to either line equation. The point of intersection is $\left(\frac{3}{2}, 0\right)$.

$$y_i = \frac{2}{3}\left(\frac{3}{2}\right) - 1 \qquad y_i = -2x\left(\frac{3}{2}\right) + 3$$

$$y_i = 0 \qquad y_i = 0$$

Intersection of Lines IV

Which of the following lines intersects the red line at the largest x-coordinate?

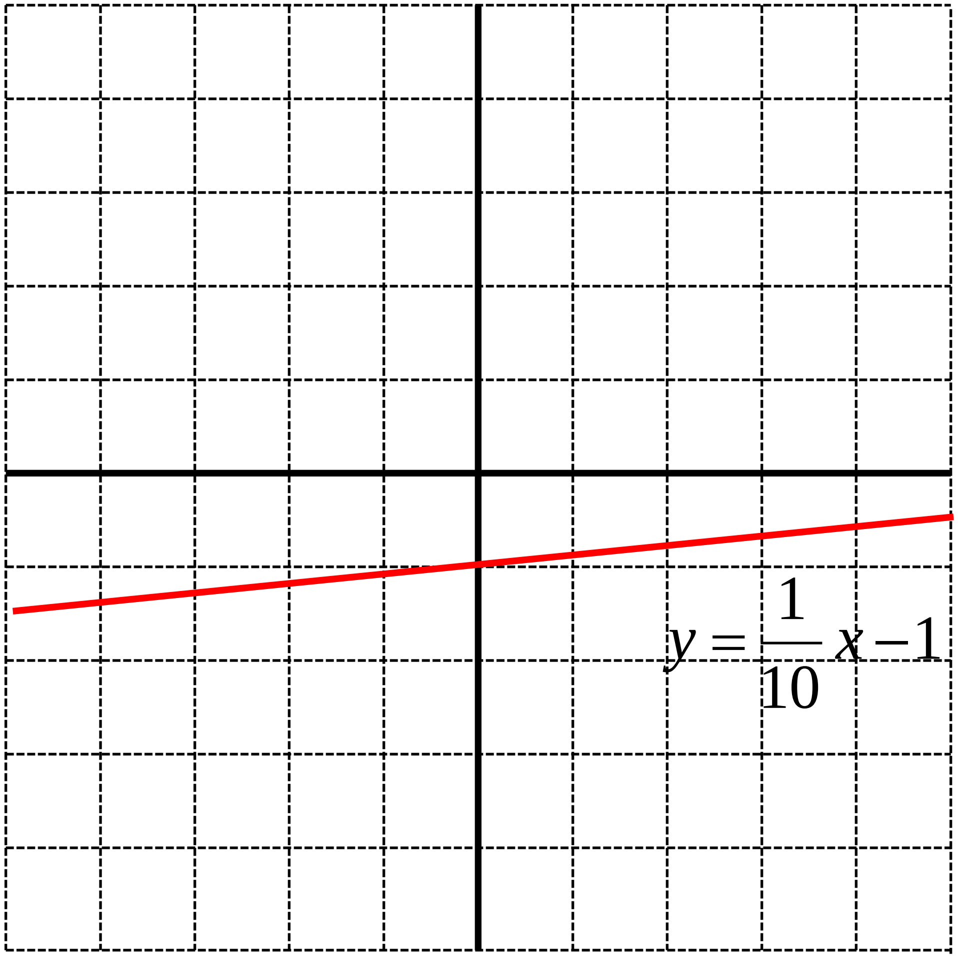
A. $y = \frac{1}{10}x + 1$

B. $y = -\frac{1}{10}x + 1$

C. $y = -10x + 1$

D. $y = x + 1$

E. $y = -x + 1$

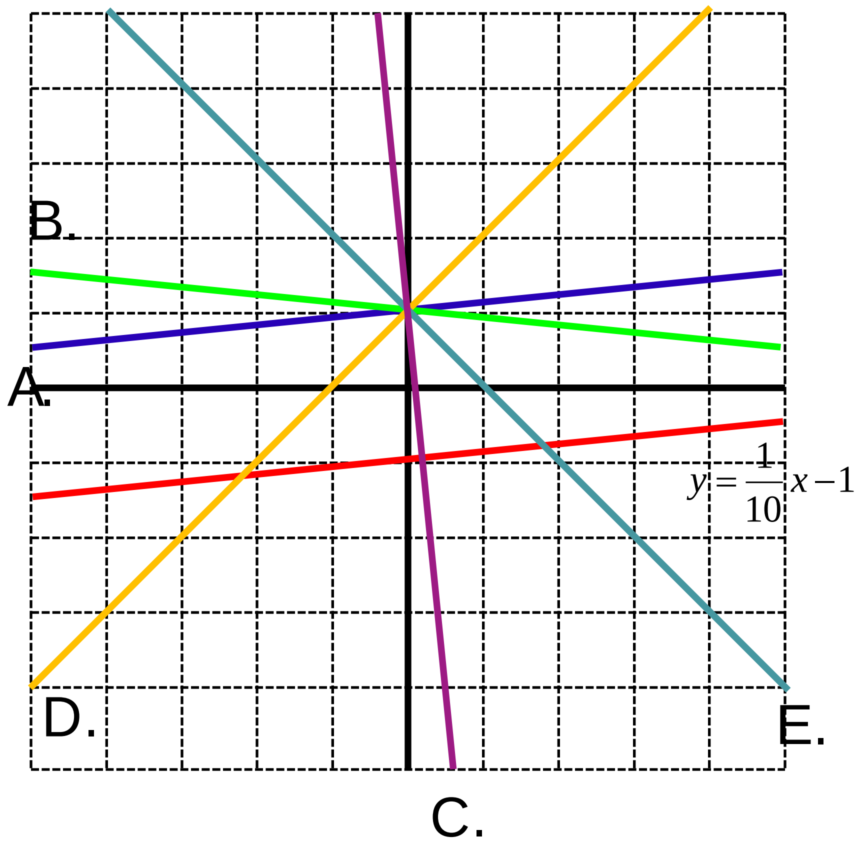


Solution

Answer: B

Justification: A good way to solve this problem is by drawing quick sketches of each line. Notice that all the lines intersect the y-axis at (0,1).

Starting at the point (0,1), only a negative slope will intersect the red line at a positive x-coordinate. This rules out answers A and D. Compared to C and E, line B has a much larger run than rise. The graph shows line B (green) intersects the red line at the largest x-value.



Intersection of Lines V

Jimmy is running towards Kerry at 7 m/s while she is running away from him at 4 m/s. Kerry begins 30 meters away. How long does it take for Jimmy to catch up to Kerry?

A. 3 s
B. 5 s
C. 10 s
D. 15 s
E. 30 s

Solution

Answer: C

Justification: Setting up 2 line equations is essential to solve this problem. Let the origin be the starting point of Jimmy so that we work with positive numbers. A point represents the position of the runners at different times. Jimmy starts at (0, 0) while Kerry starts at (0, 30). Since Jimmy moves 7 m every second, his position over time can be expressed as: Jimmy $x_J = 7t$

(where x is in meters and t is in seconds)

Kerry's position over time can be expressed as: Kerry $x_K = 4t + 30$

since she stars 30 m away

When these 2 lines intersect, Jimmy would have caught up with Kerry because they will be at the same position at the same time:

$$7t_i = 4t_i + 30$$

$$3t_i = 30$$

$$t_i = 10 \text{ seconds}$$

Intersection of Lines VI

How far does Jimmy travel before he catches up with Kerry?

A. 30 m
B. 35 m
C. 40 m
D. 70 m
E. 100 m

Solution

Answer: D

Justification: Remember from the previous question that it takes 10 seconds for Jimmy to catch up. Since he moves at 7 m/s, he should travel 70 m. The point of intersection is (10, 70).

Notice that Kerry, who runs at 4 m/s, only travels 40 m in 10 seconds. However, she started 30 meters ahead of Jimmy her position is also at 70 m after 10 seconds.

Jimmy:

$$x_i = 7t_i$$

$$x_i = 7(10)$$

$$x_i = 70\text{ m}$$

Kerry:

$$x_i = 4t_i + 30$$

$$x_i = 4(10) + 30$$

$$x_i = 70\text{ m}$$

Intersection of Lines VII

The population growth of country X is shown to the right. Which of the following countries will catch up in population the earliest? All the populations are modelled as linear relations.

Country X

Year	Pop.
1984	36000
1988	40000
1992	44000
1996	48000

A.

Year	Pop.
1984	29000
1988	33000
1992	37000
1996	41000

B.

Year	Pop.
1984	33000
1988	36000
1992	39000
1996	42000

C.

Year	Pop.
1984	20000
1988	26000
1992	32000
1996	38000

D.

Year	Pop.
1984	27000
1988	32000
1992	37000
1996	42000

Solution

Answer: D

Justification: Countries A and B will not catch up in population with Country X. This is because they have a smaller initial population and also increase at an equal (Country A) or slower (Country B) rate.

Country C increases in population by 6000 every 4 years, while Country X increases by 4000 every 4 years. Country D has a larger initial population, but increases only by 5000 every 4 years. To make calculations easier, let the variable t represent a change of 4 years.

C:

$$36000 + 4000t = 20000 + 6000t$$
$$2000t = 16000$$
$$t = 8$$

D:

$$36000 + 4000t = 27000 + 5000t$$
$$1000t = 9000$$
$$t = 9$$

Country C will catch up in 2016, while country D will catch up in 2020.

What are the limitations of using a linear relation to model population growth?

Intersection of Lines VIII

Consider two lines with an opposite slope:

$$y_1 = mx_1 + b \qquad y_2 = -mx_2 + c$$

At what y-coordinate do these two lines intersect?

A. $y_i = \dfrac{b+c}{2}$

B. $y_i = \dfrac{b-c}{2}$

C. $y_i = \dfrac{b+c}{2m}$

D. $y_i = \dfrac{b-c}{2m}$

E. $y_i = \dfrac{b+c}{m}$

Solution

Answer: A

Justification: Starting at the y-intercept, the two lines will approach each other at the same rate since they have opposite slopes. Intuitively we may infer that the two lines will intersect at the midpoint of the y-intercepts.

Working with the equations shows that the lines will intersect at the midpoint of *b* and *c*.

$$mx_i + b = -mx_i + c$$
$$2mx_i = c - b$$
$$x_i = \frac{c-b}{2m}$$

$$y_i = m\left(\frac{c-b}{2m}\right) + b$$
$$y_i = \frac{c-b}{2} + b$$
$$y_i = \frac{c+b}{2}$$