a place of mind

# Physics <br> Electrostatics: Coulomb's Law 

## Science and Mathematics Education Research Group

## Coulomb's Law

$$
F_{e}=k \frac{\left|q_{1}\right|\left|q_{2}\right|}{r^{2}}
$$



## Coulomb's Law I

Consider two opposite charges, $\mathrm{q}_{1}$ and $\mathrm{q}_{2}$ a distance r apart. According to Coulomb's law, the magnitude of the force between the two charges is:

$$
F_{e}=k \frac{\left|\dot{q}_{1}\right|\left|q_{2}\right|}{r^{2}}
$$

If $\mathrm{q}_{1}$ is doubled, the magnitude of the force will:
A. Decrease by a factor of 4
B. Decrease by a factor of 2
C. Remain the same
D. Increase by a factor of 2
E. Increase by a factor of 4


## Solution

## Answer: D

Justification: The electric force one charge exerts on another is directly proportional to the product of the two charges, and thus is also proportional to each individual charge. If the magnitude of one charge is doubled then the magnitude of the force is also doubled.

The force is not proportional to the square of the charge, so the magnitude of the force will not change by a factor of four.

$$
\begin{aligned}
& F_{1}=k \frac{\left|q_{1}\right|\left|q_{2}\right|}{r^{2}} \\
& F_{2}=k \frac{\left|2 q_{1} \| q_{2}\right|}{r^{2}}=2 k \frac{\left|q_{1}\right|\left|q_{2}\right|}{r^{2}}=2 F_{1}
\end{aligned}
$$

## Coulomb's Law II

Consider two opposite charges, $\mathrm{q}_{1}$ and $\mathrm{q}_{2}$ a distance r apart. According to Coulomb's law, the magnitude of the force between the two charges is:

$$
F_{e}=k \frac{\left|q_{1}\right|\left|q_{2}\right|}{r^{2}}
$$

If $r$ is doubled, the magnitude of the force will:
A. Decrease by a factor of 4
B. Decrease by a factor of 2
C. Remain the same
D. Increase by a factor of 2
E. Increase by a factor of 4


## Solution

Answer: A
Justification: The electric force is proportional to the inverse square of the distance between the charges. If the distance between the charges double, the magnitude of the force decreases by a factor of 4. As the distance increases, the square increases. Dividing by a larger number gives a smaller answer. Similarly, if the distance decreases, the square decreases. Dividing by a smaller answer gives a larger answer.

$$
\begin{aligned}
& F_{1}=k \frac{\left|q_{1}\right|\left|q_{2}\right|}{r^{2}} \\
& F_{2}=k \frac{\left|q_{1}\right|\left|q_{2}\right|}{(2 r)^{2}}=k \frac{\left|q_{1}\right|\left|q_{2}\right|}{4 r^{2}}=\frac{1}{4} F_{1}
\end{aligned}
$$

## Coulomb's Law III

Consider two opposite charges, $q_{1}$ and $q_{2}$ a distance $r$ apart. According to Coulomb's law, the magnitude of the force between the two charges is:

$$
F_{e}=k \frac{\left|q_{1}\right|\left|q_{2}\right|}{r^{2}}
$$

If $q_{1}, q_{2}$, and $r$ are all doubled, the magnitude of the force will:
A. Decrease by a factor of 4
B. Decrease by a factor of 2
C. Remain the same
D. Increase by a factor of 2
E. Increase by a factor of 4


## Solution

## Answer: C

Justification: Doubling one charge increases the force by a factor of 2 , so doubling both charges increases the force by a factor of 4 . Doubling distance decreases the force by a factor of 4. The increase caused by the doubled charge is negated by the decrease caused by the doubled distance.

$$
\begin{aligned}
& F_{1}=k \frac{\left|q_{1}\right|\left|q_{2}\right|}{r^{2}} \\
& F_{2}=k \frac{\left|2 q_{1} \| 2 q_{2}\right|}{(2 r)^{2}}=4 k \frac{\left|q_{1}\right|\left|q_{2}\right|}{4 r^{2}}=F_{1}
\end{aligned}
$$

## Coulomb's Law IV

Consider two uniformly charged spheres a small distance apart. Sphere 1 has a $+3 q$ charge while sphere 2 has a $-q$ charge.

Which of the following diagrams correctly shows the magnitude and direction of the electrostatic forces?
A.

C. 1


B. ${ }_{\text {(3q) }}^{1}$

D.



## Solution

Answer: C
Justification: Opposite charges attract, like charges repel. This discounts D, where the charges push each other apart.

Newton's Third Law states that for every action there must be an equal and opposite reaction. This means that the force that charge 1 exerts on charge 2 has the same magnitude as the force that charge 2 exerts on charge 1 , but acts in the opposite direction.

Alternatively, the equation for Coulomb's Law does not change when we consider the force of charge 1 on charge 2 or vice versa.


## Coulomb's Law V

Three charges with equal magnitudes are arranged horizontally and spaced evenly apart as shown:


Which of the following free-body diagrams best represents the electric forces acting on $Q_{3}$ ?
A.

$F_{2 \text { on } 3}$
B.

$F_{2 \text { on } 3}$
C.

D.


## Solution

## Answer: E

Justification: Both $Q_{1}$ and $Q_{2}$ are positive, while $Q_{3}$ is negative. Opposite charges attract, so $Q_{3}$ is attracted to the other two charges and experiences a net force to the left.

Both $Q_{1}$ and $Q_{2}$ have the same charge, $q$, so the magnitude of the forces they each exert on $Q_{3}$ will depend on the distance of each charge from $Q_{3}$. The distance between $Q_{1}$ and $Q_{3}$ is twice the distance between $Q_{2}$ and $Q_{3}$. The force one charge exerts on another is inversely proportional to the square of the distance between the two charges. $Q_{1}$ is twice as far as $Q_{2}$ from $Q_{3}$, and therefore, $F_{1 \text { on } 3}$ must be four times smaller than $F_{2 \text { on } 3 \text {. }}$.


## Coulomb's Law VI

Three charges with equal magnitudes are arranged horizontally and spaced evenly apart as shown:


Which of the following free-body diagrams best represents the electric forces acting on $Q_{1}$ ?


## Solution

Answer: A
Justification: Charges $Q_{2}$ and $Q_{1}$ have the same sign, so they repel each other. $Q_{2}$ is pushing Q1 away. $Q_{1}$ and $Q_{3}$ are of opposite sign, and will attract each other. $Q_{3}$ is pulling $Q_{1}$ closer.

The distance between $Q_{1}$ and $Q_{3}$ is twice the distance between $Q_{2}$ and $Q_{3}$. The force is proportional to the inverse of the distance, so $F_{3 \text { on } 1}$ must be smaller than $F_{2 \text { on } 1}$ by a factor of 4 .
A.


