a place of mind

# Physics <br> 1-D Kinematics: Relative Velocity 

## Science and Mathematics Education Research Group

## Relative Velocities



## Relative Velocity I

Car A is moving at $\mathbf{v}_{\mathrm{A}}=60 \mathrm{~km} / \mathrm{h}$ to the right with respect to the ground. Car $B$ is moving at $\mathbf{v}_{B}=80 \mathrm{~km} / \mathrm{h}$ to the left with respect to the ground. What is the velocity of Car A with respect to Car B (the velocity of Car A as measured by the passenger in Car B)?

A. $20 \mathrm{~km} / \mathrm{h}$ left
B. $20 \mathrm{~km} / \mathrm{h}$ right
C. $60 \mathrm{~km} / \mathrm{h}$ left
D. $140 \mathrm{~km} / \mathrm{h}$ left
E. $140 \mathrm{~km} / \mathrm{h}$ right

## Solution

Answer: E, 140km/h [right]
Justification: The velocity of Car A with respect to $\operatorname{Car} B\left(\mathbf{v}_{\mathrm{AB}}\right)$ is given by: $\mathbf{v}_{\mathrm{AB}}=\mathbf{v}_{\mathrm{A}}-\mathbf{v}_{\mathrm{B}}$.

$$
\begin{aligned}
& \left.\mathbf{v}_{\mathrm{AB}}=60 \mathrm{~km} / \mathrm{h}[\text { right }]-80 \mathrm{~km} / \mathrm{h}[\mathrm{left}]\right)= \\
& =60 \mathrm{~km} / \mathrm{h}-(-80 \mathrm{~km} / \mathrm{h})=140 \mathrm{~km} / \mathrm{h}[\text { right }]=140 \mathrm{~km} / \mathrm{h}
\end{aligned}
$$

According to the person driving Car B (a frame of reference moving at the velocity of $80 \mathrm{~km} / \mathrm{h}$ [right]), Car B is stationary while Car A is moving at $140 \mathrm{~km} / \mathrm{h}$ [right]. Because the cars are travelling in opposite directions the magnitude of the velocity of one car relative to another will be the sum of the magnitudes of the velocity of each car. If the cars were moving in the same direction, the magnitude of the relative velocity would have equal the difference between the two magnitudes.

## Relative Velocity II

Car $A$ is moving at $\mathbf{v}_{\mathrm{A}}=60 \mathrm{~km} / \mathrm{h}$ to the right with respect to the ground. Car $B$ is moving at $\mathbf{v}_{\mathrm{B}}=80 \mathrm{~km} / \mathrm{h}$ to the left with respect to the ground. What is the velocity of Car B with respect to Car A (the velocity of Car B as measured by a passenger in Car A)?

A. $20 \mathrm{~km} / \mathrm{h}$ left
B. $20 \mathrm{~km} / \mathrm{h}$ right
C. $60 \mathrm{~km} / \mathrm{h}$ left
D. $140 \mathrm{~km} / \mathrm{h}$ left
E. $140 \mathrm{~km} / \mathrm{h}$ right

## Solution

## Answer: D

Justification: The velocity of Car B with respect to $\operatorname{Car} A\left(\mathbf{v}_{B A}\right)$ is given by:

$$
\begin{aligned}
& \mathbf{v}_{\mathrm{BA}}=\mathbf{v}_{\mathrm{B}}-\mathbf{v}_{\mathrm{A}}, \mathbf{v}_{\mathrm{BA}}=(80 \mathrm{~km} / \mathrm{h}[\mathrm{left}])-(60 \mathrm{~km} / \mathrm{h}[\mathrm{right}])= \\
& =-80 \mathrm{~km} / \mathrm{h}-60 \mathrm{~km} / \mathrm{h}=-140 \mathrm{~km} / \mathrm{h}=140 \mathrm{~km} / \mathrm{h} \text { [left] }
\end{aligned}
$$

Notice that Car A and Car B both measure the other car travelling at the same speed. However, the directions of relative velocities will be opposite:
$\mathbf{v}_{\mathrm{BA}}=\mathrm{v}_{\mathrm{B}}-\mathrm{v}_{\mathrm{A}} ; \mathrm{v}_{\mathrm{AB}}=\mathrm{v}_{\mathrm{A}}-\mathrm{V}_{\mathrm{B}}$
Therefore: $v_{B A}=-v_{B A}$

## Relative Velocity III

Car $A$ is moving at $\mathbf{v}_{\mathrm{A}}=60 \mathrm{~km} / \mathrm{h}$ to the right with respect to the ground. Car $B$ is moving at $\mathbf{v}_{\mathrm{B}}=80 \mathrm{~km} / \mathrm{h}$ to the right with respect to the ground. What is the velocity of Car A with respect to Car B (the velocity of Car A measured by Car B)?

A. $20 \mathrm{~km} / \mathrm{h}$ left
B. $20 \mathrm{~km} / \mathrm{h}$ right
C. $60 \mathrm{~km} / \mathrm{h}$ right
D. $140 \mathrm{~km} / \mathrm{h}$ left
E. $140 \mathrm{~km} / \mathrm{h}$ right

## Solution

## Answer: A

Justification: The velocity of Car A with respect to $\operatorname{Car} B\left(\mathbf{v}_{\mathrm{AB}}\right)$ is given by:

$$
\begin{aligned}
\mathbf{v}_{\mathrm{AB}} & =\mathbf{v}_{\mathrm{A}}-\mathbf{v}_{\mathrm{B}} \\
\mathbf{v}_{\mathrm{AB}} & =(60 \mathrm{~km} / \mathrm{h} \text { [right] })-(80 \mathrm{~km} / \mathrm{h} \text { [right] })=-20 \mathrm{~km} / \mathrm{h} \text { [right] } \\
& =20 \mathrm{~km} / \mathrm{h}[\text { left }]
\end{aligned}
$$

Even though both cars are moving to the right, Car A looks like it is moving $20 \mathrm{~km} / \mathrm{h}$ to the left while Car B is stationary. The cars appear to be getting farther away from each other at $20 \mathrm{~km} / \mathrm{h}$.

Notice, if we assume that a positive direction of velocity is to the right, then $20 \mathrm{~km} / \mathrm{h}$ [left] is equivalent to $-20 \mathrm{~km} / \mathrm{h}$

## Relative Velocity IV

Car $A$ is moving at $\mathbf{v}_{\mathrm{A}}=60 \mathrm{~km} / \mathrm{h}$ to the right with respect to the ground. Car $B$ is moving at $\mathbf{v}_{\mathrm{B}}=80 \mathrm{~km} / \mathrm{h}$ to the right with respect to the ground. What is the velocity of Car B with respect to Car A (the velocity of Car B measured by a passenger in Car A)?

A. $20 \mathrm{~km} / \mathrm{h} \mathrm{left}$
B. $20 \mathrm{~km} / \mathrm{h}$ right
C. $80 \mathrm{~km} / \mathrm{h}$ right
D. $140 \mathrm{~km} / \mathrm{h}$ left
E. $140 \mathrm{~km} / \mathrm{h}$ right

## Solution

## Answer: B

Justification: The velocity of Car B with respect to $\operatorname{Car} A\left(\mathbf{v}_{B A}\right)$ is given by:

$$
\begin{aligned}
& \mathbf{v}_{\mathrm{BA}}=\mathbf{v}_{\mathrm{B}}-\mathbf{v}_{\mathrm{A}} \\
& \mathbf{v}_{\mathrm{BA}}=(80 \mathrm{~km} / \mathrm{h}[\text { right }])-(60 \mathrm{~km} / \mathrm{h}[\text { right }])= \\
& (80 \mathrm{~km} / \mathrm{h}-60 \mathrm{~km} / \mathrm{h})=20 \mathrm{~km} / \mathrm{h}[\text { right }]
\end{aligned}
$$

Again, the cars are getting farther apart at $20 \mathrm{~km} / \mathrm{h}$.

## Relative Velocity V

Car $A$ is moving at $\mathbf{v}_{\mathrm{A}}=50 \mathrm{~km} / \mathrm{h}$ to the right with respect to the ground. Car B is moving at $\mathbf{v}_{\mathrm{B}}=50 \mathrm{~km} / \mathrm{h}$ to the right with respect to the ground. What is the velocity of Car B with respect to Car A (the velocity of Car B measured by a passenger in Car A)?

A. $0 \mathrm{~km} / \mathrm{h}$
B. $50 \mathrm{~km} / \mathrm{h}$ left
C. $50 \mathrm{~km} / \mathrm{h}$ right
D. $100 \mathrm{~km} / \mathrm{h}$ left
E. $100 \mathrm{~km} / \mathrm{h}$ right

## Solution

## Answer: A

Justification: The velocity of Car B with respect to $\operatorname{Car} A\left(\mathbf{v}_{B A}\right)$ is given by:

$$
\begin{aligned}
& \mathbf{v}_{\mathrm{BA}}=\mathbf{v}_{\mathrm{B}}-\mathbf{v}_{\mathrm{A}} \\
& \mathbf{v}_{\mathrm{BA}}=(50 \mathrm{~km} / \mathrm{h}[\text { right }])-(50 \mathrm{~km} / \mathrm{h}[\text { right }])=0 \mathrm{~km} / \mathrm{h}
\end{aligned}
$$

According to the person driving Car A, Car B appears to be stationary because they both have the same velocity: they are not moving relatively to each other. Likewise, the driver of Car B sees Car A stationary.

## Relative Velocity VI

Jeremy is running on a 100 m track at $4 \mathrm{~m} / \mathrm{s}$. An observer moving at $3 \mathrm{~m} / \mathrm{s}$ in the same direction as Jeremy sees Jeremy run at $1 \mathrm{~m} / \mathrm{s}$. According to the observer moving at $3 \mathrm{~m} / \mathrm{s}$, how long does it take for Jeremy to finish the race?

A. 14 seconds
B. 25 seconds
C. 33 seconds
D. 58 seconds
E. 100 seconds

## Solution

## Answer: B

Justification: The observer only sees Jeremy moving at $1 \mathrm{~m} / \mathrm{s}$. However, the moving observer also sees the finish line moving towards Jeremy at $3 \mathrm{~m} / \mathrm{s}$. In the frame of reference of the moving observer, Jeremy and the finish line are approaching each other at $4 \mathrm{~m} / \mathrm{s}$. Therefore, it will take Jeremy 25 s to finish the race. This is the same result as would have been observed by a stationary observer:

$$
\mathrm{t}=\frac{\Delta \mathrm{x}}{\mathrm{v}}=\frac{100 \mathrm{~m}}{4 \mathrm{~m} / \mathrm{s}}=25 \mathrm{~s}
$$

Regardless of who is observing, the time for Jeremy to run the 100 m track at a velocity of $4 \mathrm{~m} / \mathrm{s}$ will always be 25 s .

