a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Physics
Kinematics: Projectile Motion

Science and Mathematics
Education Research Group

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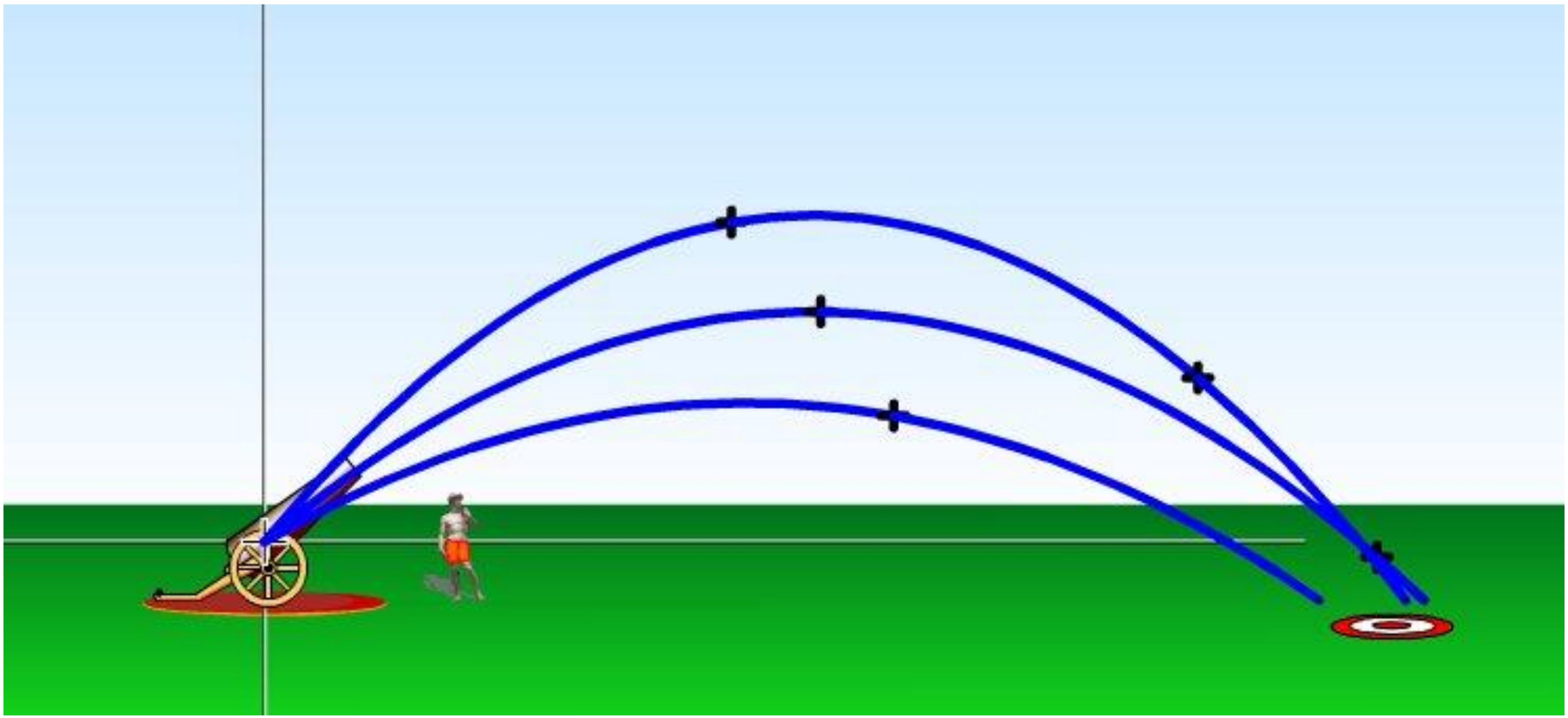
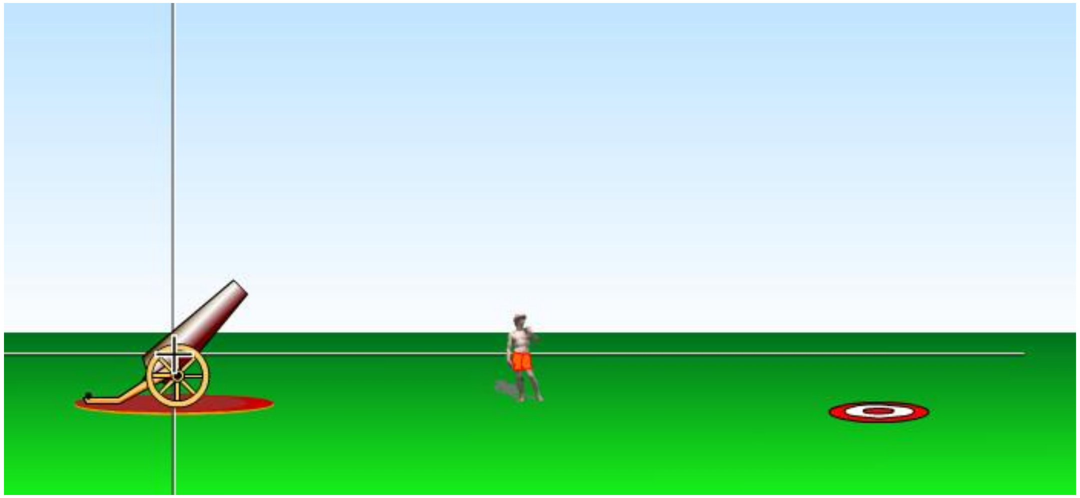
Cannonball Trajectories



Introduction

The following questions can be modelled using the PhET simulation: http://phet.colorado.edu/en/simulation/projectile-motion

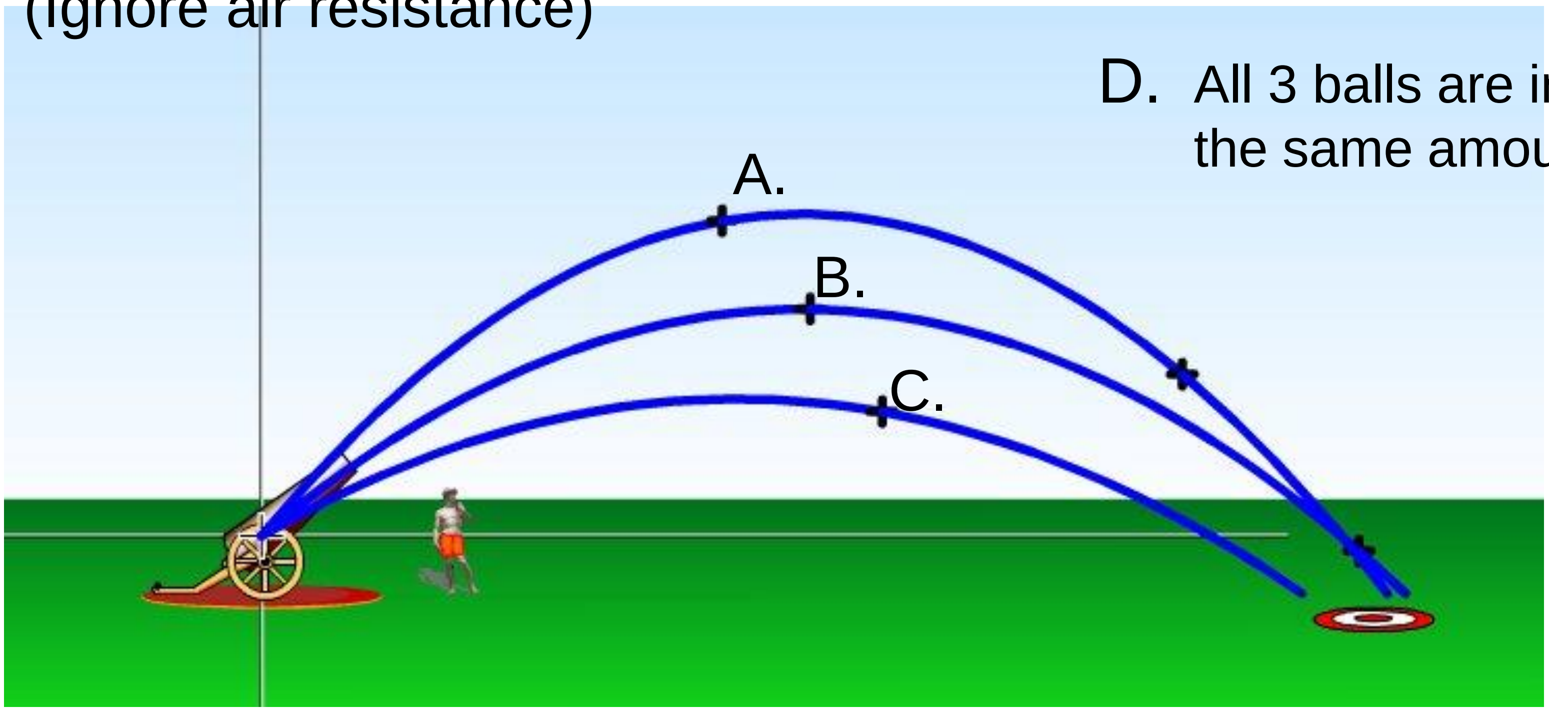
Try to solve these problems first and then test your answers.



Cannonball Trajectories I

The barrel of a cannon is 1.2 m above the ground and can fire cannonballs at 15 m/s. Three balls are launched at different angles above the horizontal. The trajectories of the 3 balls are shown. Which ball was in the air for the longest amount of time?

(Ignore air resistance)



D. All 3 balls are in the air for the same amount of time

Solution

Answer: A

Justification: The time of flight of a projectile depends entirely on the height of the trajectory. **WHY?** The time of flight is the time it takes to reach its maximum height plus the time it takes to fall from there to the ground.

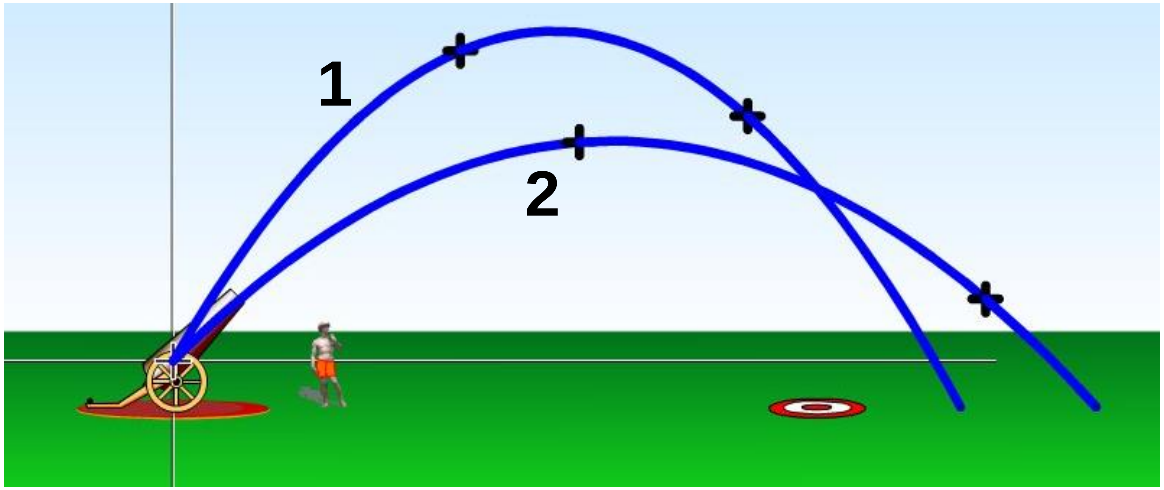
Since ball A has the highest trajectory, it will have the longest flight time.

Simulation Note: Ball A was launched at 50°, Ball B at 40° and Ball C at 30°.

Cannonball Trajectories II

The 2 balls shown were launched at the same speed. The ball following trajectory 1 is in the air for the longest amount of time. Why does the ball following trajectory 2 travel farther horizontally?

(Ignore air resistance)



A. Ball 2 accelerates downwards towards the ground slower
B. Ball 2 has a larger horizontal velocity than Ball 1
C. Ball 2 has less mass than Ball 1
D. It is not possible for Ball 2 to travel further than Ball 1

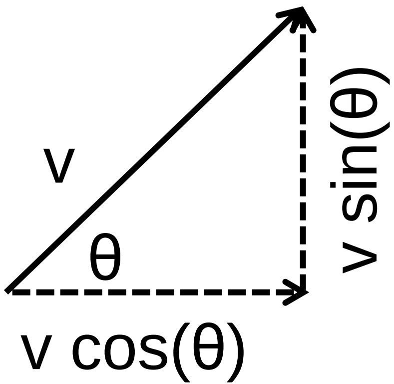
Solution

Answer: B

Justification: Acceleration due to gravity is constant, and affects both balls equally. A change in mass will not effect the downward acceleration of the balls.

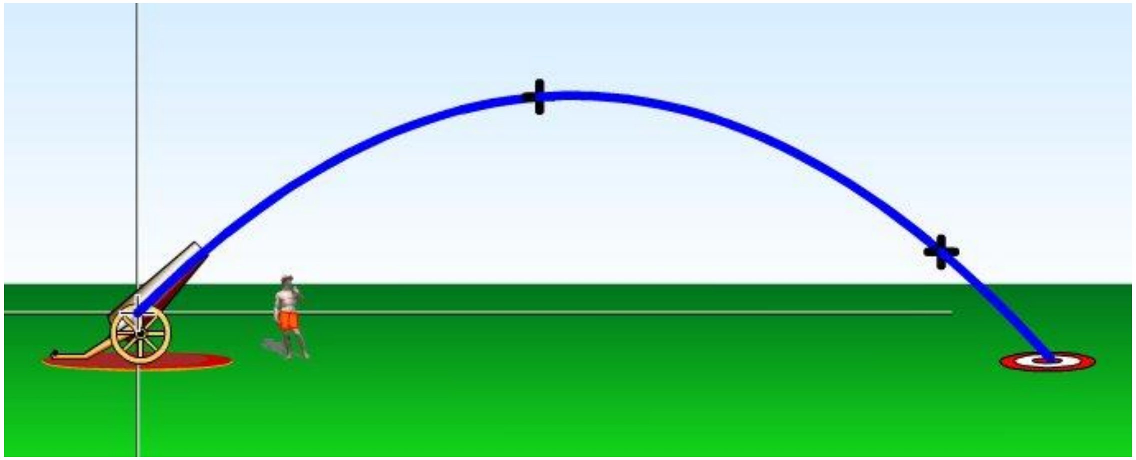
The horizontal and vertical components of velocity depend on the angle at which the ball is fired. Even though Ball 2 is in the air for a shorter amount of time, it has a larger horizontal velocity than Ball 1. On the other hand, Ball 1 has a larger vertical velocity than Ball 2. Since Ball 2 has a larger horizontal velocity, which is constant, it will travel further horizontally.

Notice that the horizontal component is largest when $\theta = 0°$ (firing straight forward), and smallest when $\theta = 90°$ (firing straight up).



Cannonball Trajectories III

The diagram below shows the trajectory of a ball launched from a cannon. What happens when a ball with greater mass is fired at the same velocity and height? (Ignore air resistance)



A. The ball will travel a larger distance

B. The ball will follow the same trajectory

C. The ball will travel a smaller distance

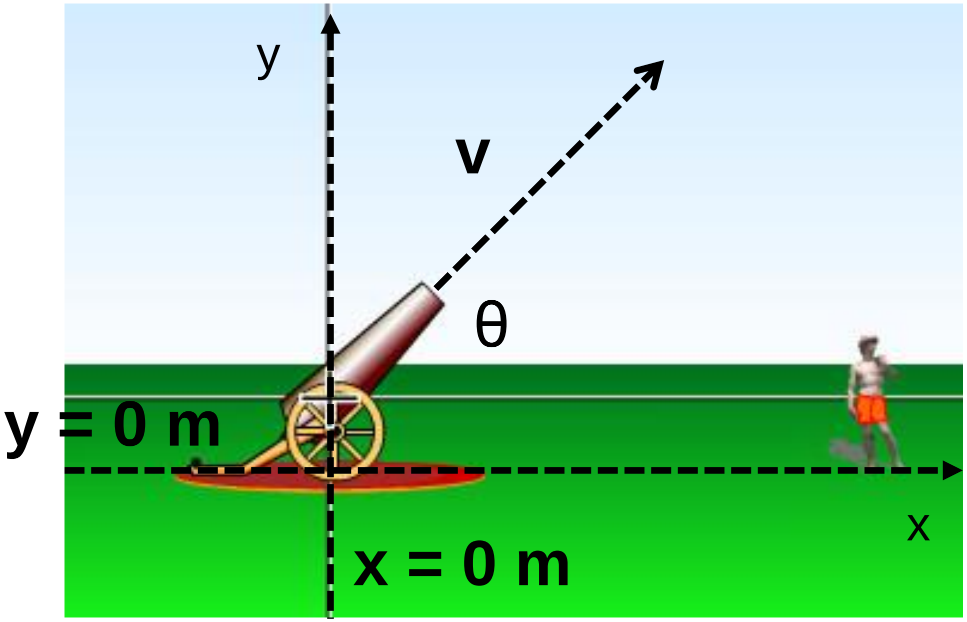
Solution

Answer: B

Justification: Without air resistance, the mass of the ball does not affect its trajectory. Acceleration due to gravity is the same for all objects, regardless of size or mass. Because the velocity remains unchanged, horizontal and vertical components of velocity will be unchanged and the ball will follow the same trajectory.

Cannonball Trajectories IV

For simplicity, imagine the cannon is buried into the ground so that the ball is launched from y = 0 m and lands at y = 0 m. (The cannon is no longer 1.2 m above the ground.) Which equation can be used to determine the time the ball spends in the air? (Ignore air resistance)



A. $-v = v - gt$

B. $-v\sin\theta = v\sin\theta - gt$

C. $0 = v - gt$

D. $0 = v\sin\theta - gt$

E. $0 = v\sin\theta - \frac{1}{2}gt^2$

Solution

Answer: B

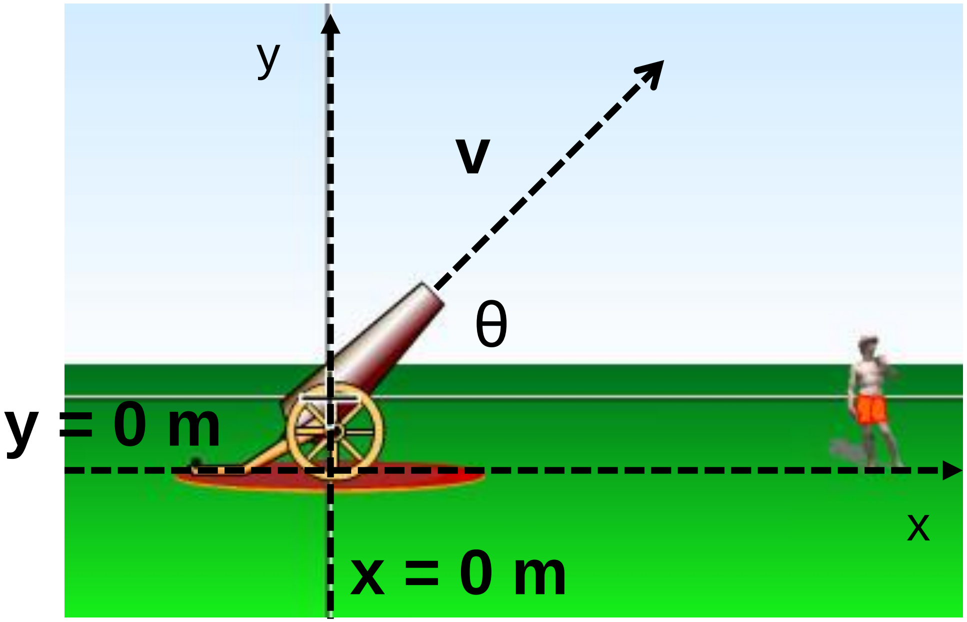
Justification: What we know (see x-y coordinate axes) : **a** = -g, y_0, y_f, **d**, **v**, and θ. The vertical component of velocity is needed to determine the y position of the ball, so v sin(θ) must be used instead of v. The final velocity of the ball is the velocity the instant before it hits the ground, which would be –v sin(θ) instead of 0. Since the cannon fires at 0 m, the displacement will be 0 m.

$$v_f = v_i + at \qquad d = v_i t + \frac{1}{2}at^2 \qquad v_f^2 = v_i^2 + 2ad \qquad d = \frac{v_i + v_f}{2}t$$

Using what we know from the problem, we can eliminate the third and fourth equations of motion. Since we know initial and final velocities, we can use equation 1, eliminating the need to use a quadratic. Substituting v_i and v_f into equation 1 gives $-v\sin\theta = v\sin\theta - gt$

Cannonball Trajectories V

For simplicity, imagine the cannon is buried in the ground so that the ball is launched from y = 0 and lands at y = 0. (The cannon is no longer 1.2 m above the ground.) Which equation can be used to determine the x position of the ball at any time t? (Ignore air resistance)



A. $x = vt$

B. $x = tv\cos\theta$

C. $x = vt - \frac{1}{2}gt^2$

D. $x = tv\cos\theta - \frac{1}{2}gt^2$

E. $x = t^2 v\cos\theta - \frac{1}{2}gt$

Solution

Answer: B

Justification: The horizontal component of velocity is needed to determine the x position of the ball, so v*cos(θ) must be used instead of v_x. The horizontal velocity of the ball is constant, since there is no horizontal acceleration (the acceleration due to gravity acts in a vertical direction). We will use the equation

$$v = \frac{x - x_0}{t} = \frac{x}{t}$$

To solve for x, multiply the horizontal velocity by the time it is in the air to determine how far the ball travels.

$$x = t\left(v\cos\theta\right)$$

Cannonball Trajectories VI

The horizontal position of the ball when it lands can be found by substituting the flight time t into x = tvcosθ.

$$x = tv\cos\theta, \quad t = \frac{2v\sin\theta}{g} \quad (\mathit{from} \quad \mathit{question} \quad 4)$$

$$x_{\max} = \frac{2v^2\sin\theta\cos\theta}{g}, \quad \sin(2\theta) = 2\sin\theta\cos\theta$$

$$x_{\max} = \frac{v^2\sin(2\theta)}{g}$$

At what angle θ should the ball be launched so that it travels the *farthest away* from the base of the cannon?

A. 0°

B. 30°

C. 45°

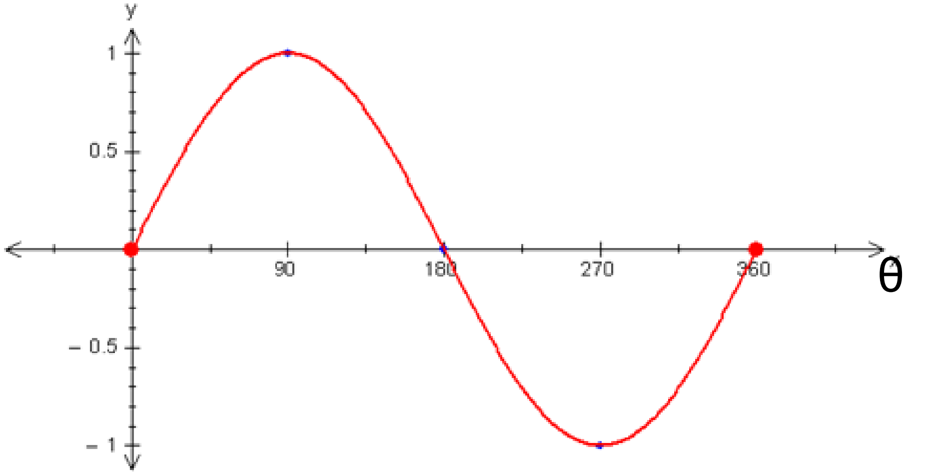
D. 60°

E. 90°

Solution

Answer: C

Justification: $$x = \frac{v^2 \sin(2\theta)}{g}$$



In order to get the largest possible x, the value for sin(2θ) must be as large as possible. The largest value of sine is sin(90°) = 1. Therefore, if we let θ = 45° then 2θ = 90°.

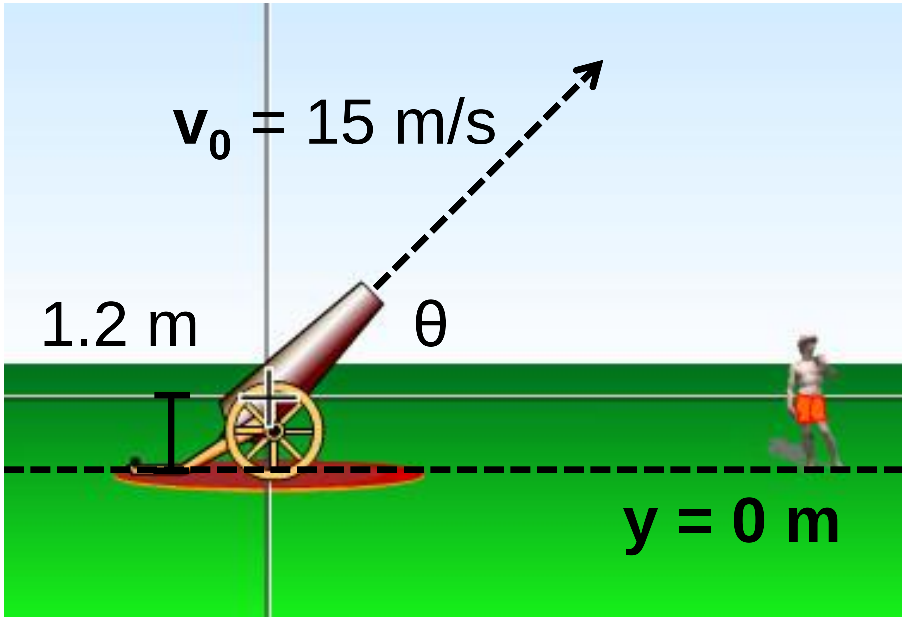
$$x = \frac{v^2 \sin(90°)}{g} = \frac{v^2}{g}$$

Check: It is a good idea to check the final solution to make sure the units come out as expected.

$$x = \frac{(\mathrm{m/s})^2}{\mathrm{m/s^2}} = \frac{\mathrm{m^2/s^2}}{\mathrm{m/s^2}} = \mathrm{m}$$

Cannonball Trajectories VII

Now suppose the cannonball is launched 1.2 m above the ground (The ball starts at y = 1.2 m and lands at y = 0 m.) At what angle should a ball be launched so that it travels the farthest away from the base of the cannon? (Ignore air resistance)



A. Less than 45°
B. Exactly at 45°
C. Greater than 45°

Solution

Answer: A

Justification: If the cannon were fired from ground level, then launching a ball at 45° will make the ball go the farthest. However, the cannon is 1.2 m above the ground. The added height adds a constant amount of time to the ball's flight. Therefore, a slightly larger horizontal velocity will make the ball move farther. As previously discussed, smaller angles will give a larger horizontal velocity. Therefore, an angle slightly smaller than 45 ° will make the ball go farther than a larger angle.

Simulation Note: In this case where the ball is launched from 1.2 m above the ground at 15 m/s, the best angle to launch the ball is 43.6°. Compared to a launch angle of 45°, the ball travels 0.027 m farther.