a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Physics
Nuclear Physics:
Binding Energy

Science and Mathematics
Education Research Group

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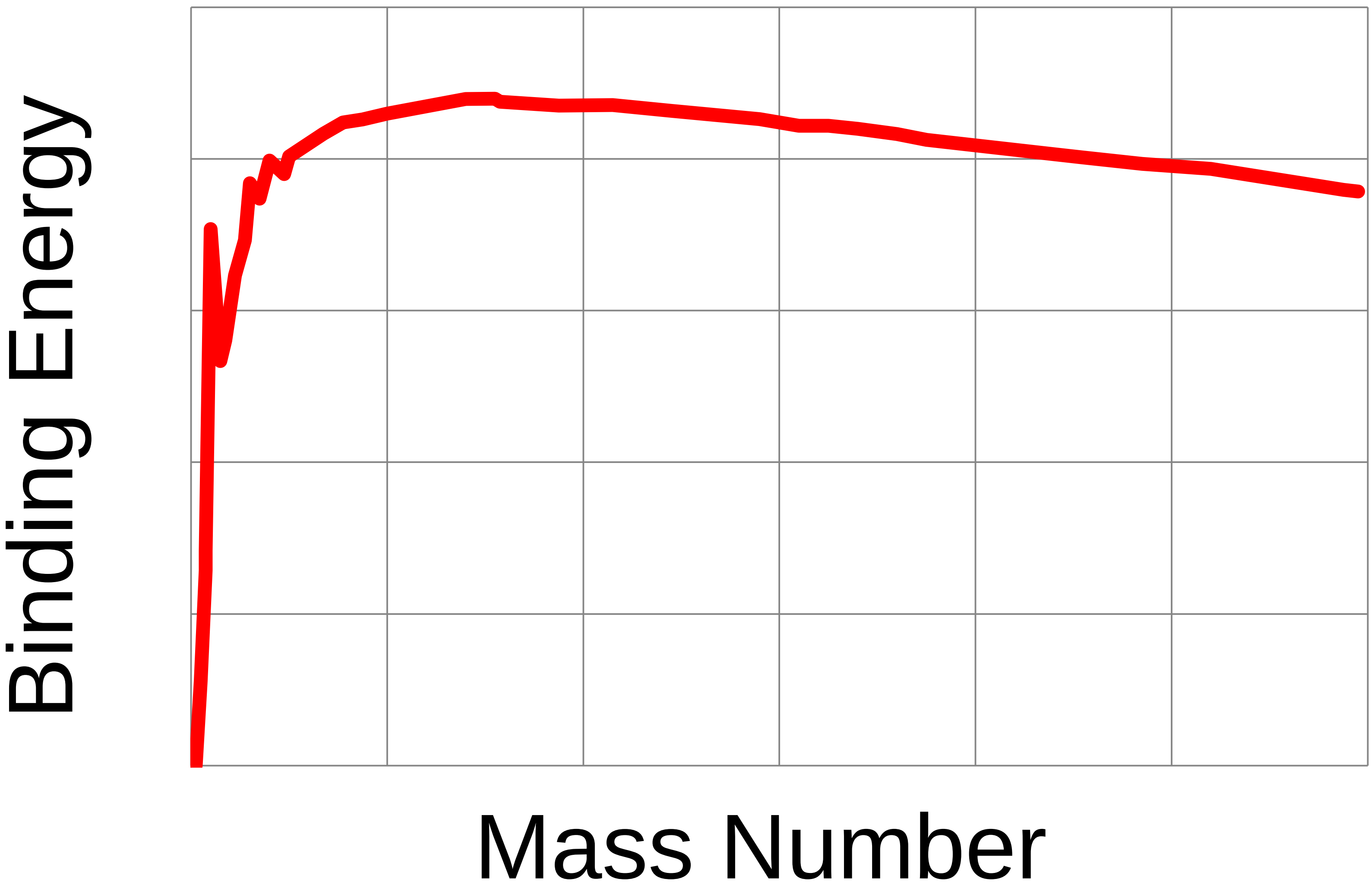
Nuclear Binding Energy



Units of Mass and Energy

When studying particles, we often deal with very small masses. For example, the mass of a proton is approximately $1.67 \times 10^{-27}\ \mathrm{kg}$.

This question set introduces a new unit of mass called the unified atomic mass unit, u. We will also see the relationship between the unified atomic mass unit and the electronvolt (eV), an unit of energy.

Note that many of these questions may require a calculator.

Binding Energy I

How many moles of molecules are there in $12\ \mathrm{g}$ of carbon-12?

6
C
Carbon
12.011

A. Approximately 1 mole

B. Exactly 1 mole

C. Approximately 12 moles

D. Exactly 12 moles

E. Approximately 6.02×10^{23} moles

Solution

Answer: B

Justification: A mole is defined as the number of atoms in $12\ \mathrm{g}$ of carbon-12, a carbon isotope that consists of 6 protons, 6 neutrons, and 6 electrons. From the definition of a mole, we can conclude that there is exactly 1 mole of carbon-12 molecules in a $12\ \mathrm{g}$ sample.

Recall that the value of 1 mol is:

$$1\,\mathrm{mol} = 6.022 \cdot 10^{23}$$

This means there are $6.022 \cdot 10^{23}$ atoms in $12\ \mathrm{g}$ of carbon-12.

Binding Energy II

The unified atomic mass unit (u) is defined to be one-twelfth the mass of an atom of carbon-12. What is the mass of $1\ \mathrm{u}$, in terms of kg?

6
C
Carbon
12.011

A. $1\,\mathrm{u} = 1.6605 \cdot 10^{-24}\ \mathrm{kg}$

B. $1\,\mathrm{u} = 1.9927 \cdot 10^{-24}\ \mathrm{kg}$

C. $1\,\mathrm{u} = 1.6605 \cdot 10^{-27}\ \mathrm{kg}$

D. $1\,\mathrm{u} = 1.6605 \cdot 10^{-30}\ \mathrm{kg}$

E. $1\,\mathrm{u} = 1.9927 \cdot 10^{-30}\ \mathrm{kg}$

Solution

Answer: C

Justification: Let the mass of 1 carbon atom be m_c. We can find the value of m_c by recalling that there is exactly 1 mole of carbon atoms in a 12 g sample:

$$1\,\text{mol} \times m_c = 12\,\text{g}$$

$$6.022 \cdot 10^{23} \times m_c = 12 \cdot 10^{-3}\ \text{kg}$$

$$m_c = 1.99269 \cdot 10^{-26}\ \text{kg}$$

By the definition of the unified atomic mass unit, it is one-twelfth the mass of 1 carbon atom:

$$1\,\text{u} = \frac{1}{12} m_c = 1.6605 \cdot 10^{-27}\ \text{kg}$$

Solution Continued

Answer: C

Justification: This question can be solved without a calculator by estimating the order of magnitude of the unified atomic mass unit:

Mass of a carbon atom:

$$1\,\text{mol} \times m_c = 12\,\text{g}$$

$$6 \cdot 10^{23} \times m_c = 12 \cdot 10^{-3}\ \text{kg}$$

$$m_c = 2 \cdot 10^{-26}\ \text{kg}$$

One-twelfth the mass of a carbon atom:

$$1\,\text{u} = \frac{1}{12} m_c \approx \frac{1}{10} m_c = 2 \cdot 10^{-27}\ \text{kg}$$

Binding Energy III

What is the mass of a proton, in terms of the atomic mass unit u?

Particle	Mass
Proton	$1.67262 \cdot 10^{-27}$ kg
Neutron	$1.67492 \cdot 10^{-27}$ kg
Electron	$9.10938 \cdot 10^{-31}$ kg

A. 0.1007 u

B. 1.0070 u

C. 1.6726 u

D. $5.4859 \cdot 10^{-4}$ u

E. Cannot convert kg into u

Solution

Answer: B

Justification: The mass of the proton is given in the table in kg. In order to convert this to u, use the conversion rate discovered in the previous question:

$$1\,\mathrm{u} = 1.6605 \cdot 10^{-27}\ \mathrm{kg}$$

This gives us:

$$1.67262 \cdot 10^{-27}\ \mathrm{kg} \times \frac{1\,\mathrm{u}}{1.6605 \cdot 10^{-27}\ \mathrm{kg}} = 1.007\,\mathrm{u}$$

(You should be able to see that this value is just slightly larger than 1 u without using a calculator)

Alternative Solution

Answer: B

Justification: You can also estimate that the answer is very close to 1 u using the definition of the unified atomic mass unit.

The unit u is defined to be one-twelfth the mass of an atom of carbon-12. The mass of carbon-12 is primarily made up of the mass of 6 protons and 6 neutrons (since electrons have very small mass compared to these particles). Since protons and neutrons have approximately the same mass, one twelfth the mass of carbon-12 is approximately the mass of a single proton or neutron.

Particle	Mass
Proton	$1.0073\ \mathrm{u}$
Neutron	$1.0087\ \mathrm{u}$
Electron	$0.0005486\ \mathrm{u}$

Binding Energy IV

Approximately how many times heavier (in terms of mass) is a proton compared to an electron?

Particle	Mass (kg)	Mass (u)
Proton	$1.67262 \cdot 10^{-27}$ kg	1.0073 u
Neutron	$1.67492 \cdot 10^{-27}$ kg	1.0087 u
Electron	$9.10938 \cdot 10^{-31}$ kg	0.0005486 u

A. 100
B. 200
C. 1000
D. 2000
E. 10000

Solution

Answer: D

Justification: It is easier to compare the relative mass of a proton compared to an electron using the units u since the mass of a proton is approximately $1\ \mathrm{u}$:

$$\frac{\text{mass of proton}}{\text{mass of electron}} = \frac{m_p}{m_e} \approx \frac{1\,\mathrm{u}}{0.0005\,\mathrm{u}} = 2000$$

Binding Energy V

The atomic mass of ^{62}Ni is 61.928 u. ^{62}Ni consists of 28 protons, 34 neutrons, and 28 electrons. If we add up the mass of 28 protons, 34 neutrons, and 28 electrons, how would this combined mass compare to the mass of a ^{62}Ni atom?

Particle	Mass
Proton	1.0073 u
Neutron	1.0087 u
Electron	0.0005486 u

A. A ^{62}Ni atom has more mass

B. The protons, neutrons, and electrons have more mass

C. They have exactly the same mass

Solution

Answer: B

Justification: If we add up the mass of 28 protons, 34 neutrons, and 28 electrons we will get:

$$28m_p + 34m_n + 28m_e = 62.516\,\mathrm{u}$$

It turns out that the mass of the nucleus of ^{62}Ni is less than the mass of 28 individual protons and 34 individual neutrons. The difference in the mass is called the mass defect/

In the case of ^{62}Ni, it is:

$$62.516\,\mathrm{u} - 61.928\,\mathrm{u} = 0.587\,\mathrm{u}$$

Binding Energy VI

The “missing mass” in the ^{62}Ni nucleus can be found in the form of energy. Mass can be transformed into energy by the formula

$E = mc^2$ where c is the speed of light.

Why might a ^{62}Ni nucleus contain more energy than 28 individual protons and 34 individual neutrons?

A. A Ni-62 nucleus is positively charged
B. Energy is required to hold the Ni-62 nucleus together
C. A Ni-62 nucleus has more kinetic energy than the individual nucleons
D. A Ni-62 nucleus is unstable while the individual nucleons are stable
E. None of the above

Solution

Answer: B

Justification: Strong forces are required to hold nucleons (protons and neutrons) together due to the electromagnetic forces pushing protons apart. Therefore a large amount of work (energy) is required to separate the nucleons from this force.

The binding energy of a nucleus is the work required to separate all the nucleons that make up the nucleus. If m_{defect} is the mass defect of Ni-62, then the binding energy of Ni-62 can be found by:

$$E_{binding} = m_{defect} c^2$$

The charge and stability of a nucleus do not say anything about the energy of the nucleus.

Binding Energy VII

We will use the formula $E = mc^2$ in order to convert mass defect into binding energy.

Suppose that the constant c (the speed of light) is given in the units meters per second. What must the units of m (mass) be if we want energy to be given in joules?

A. Kilograms (kg)
B. Grams (g)
C. Unified atomic mass units (u)
D. Newtons (N)
E. Any unit of mass will work

Press for hint

Joules can be found by:

$$\mathrm{J} = \mathrm{N} \cdot \mathrm{m} = \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^2}$$

Solution

Answer: A

Justification: Whenever you are given a new formula, it is always a good idea to see how units combine to give the correct units. Recall that joules can be found by:

$$\mathrm{J} = \mathrm{N} \cdot \mathrm{m} = \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^2}$$

We get the units $\mathrm{m}^2/\mathrm{s}^2$ from the constant c^2. Therefore if mass is given in kg, our final answer from the formula will be in joules.

$$E = m \cdot c^2$$

$$\mathrm{J} = \mathrm{kg} \cdot \left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2$$

An example is given on the next slide

Solution Cont'd

We can now calculate the binding energy of ^{62}Ni using the formula:

$$E_{binding} = m_{defect} c^2$$

We discovered in question 5 that the mass defect of ^{62}Ni is 0.587 u. Before using the above formula, we must convert this mass back into kilograms using $1\ \text{u} = 1.6605 \cdot 10^{-27}$ kg. The binding energy of ^{62}Ni is therefore:

$$\begin{aligned} E_{binding} &= m_{defect} c^2 \\ &= (0.587\ \text{u})(2.9979 \cdot 10^8\ \text{m/s})^2 \\ &= (9.747 \cdot 10^{-28}\ \text{kg})(2.9979 \cdot 10^8\ \text{ms}^{-1})^2 \\ &= 8.755 \cdot 10^{-11}\ \text{J} \end{aligned}$$

Binding Energy VIII

Mass and energy are related by the formula $E = mc^2$, where

$c = 2.9979 \cdot 10^8$ m/s

Also recall that 1 u = $1.6605 \cdot 10^{-27}$ kg.

How much energy (in joules) is equivalent to 1 u of mass?

A. $4.98 \cdot 10^{-19}$ J

B. $1.49 \cdot 10^{-10}$ J

C. $3.00 \cdot 10^{8}$ J

D. $8.97 \cdot 10^{16}$ J

E. Cannot convert u into J

Solution

Answer: B

Justification: Notice how the mass-energy equivalence formula gives units of joules when mass is given in terms of kg:

$$E = mc^2$$

$$\mathrm{J} = \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^2}$$

We must therefore convert the unified atomic mass unit into kg before inserting it into the equation:

$$E = 1\,\mathrm{u} \times c^2$$

$$= 1.6605 \cdot 10^{-27}\ \mathrm{kg} \times (2.9979 \cdot 10^8)^2\ \mathrm{m}^2/\mathrm{s}^2$$

$$= 1.49 \cdot 10^{-10}\ \mathrm{J}$$

Binding Energy IX

Binding energies are most often given in terms of millions of electronvolts (MeV), where $1\ \text{MeV} = 10^6\ \text{eV}$. The conversion between joules and electronvolts is:

$$1\,\text{eV} = 1.602 \cdot 10^{-19}\ \text{J}$$

How much energy (in MeV) is equivalent to $1\ \text{u}$ of mass?

A. $2.391 \cdot 10^{-35}\ \text{MeV}$

B. $2.391 \cdot 10^{-23}\ \text{MeV}$

C. $9.315\,\text{MeV}$

D. $931.5\,\text{MeV}$

E. $9.315 \cdot 10^{14}\ \text{MeV}$

We found in the previous question that

$$1\,\text{u} = 1.4923 \cdot 10^{-10}\ \text{J}$$

Solution

Answer: D

Justification: Two unit conversions are required to convert from joules to MeV:

$$1\,\mathrm{u} = 1.4923 \cdot 10^{-10}\ \mathrm{J} \times \frac{1\,\mathrm{eV}}{1.602 \cdot 10^{-19}\ \mathrm{J}} \times \frac{1\,\mathrm{MeV}}{10^{6}\ \mathrm{eV}} = 931.5\,\mathrm{MeV}$$

We can now use the conversion $1\ \mathrm{u} = 931.5\ \mathrm{MeV}$ to quickly convert from mass defect to binding energy. For example, the binding energy of Ni-62 is:

$$E_{binding} = 0.587\,\mathrm{u} \times \frac{931.5\,\mathrm{MeV}}{1\,\mathrm{u}} = 547\,\mathrm{MeV}$$

Compare this value with the result obtained in the solution to question 7. We can get the same answer using a single unit conversion.

Binding Energy X

Suppose we look up the mass of a ^{4}He nucleus in a table of values. We also know A (the mass number of helium) and Z (the atomic number of helium).

How can we calculate the mass defect of ^{4}He?

A. $m_{defect} = (A-Z)m_p + Zm_n - m_{He}$

B. $m_{defect} = m_{He} - m_p - (Z-A)m_n$

C. $m_{defect} = Am_p + (Z-A)m_n - m_{He}$

D. $m_{defect} = m_{He} - m_p - (A-Z)m_n$

E. $m_{defect} = Zm_p + (A-Z)m_n - m_{He}$

Solution

Answer: E

Justification: Mass defect is the difference in mass between the individual nucleons of helium and the mass of a helium nucleus. Recall that:

A – The number of protons and neutrons in a nucleus

Z – The number of protons in a nucleus

There are Z protons, and $(A-Z)$ neutrons.

Mass defect can then be found by:

$$m_{defect} = \underbrace{Zm_p + (A-Z)m_n}_{\text{Mass of individual nucleons}} - \underbrace{m_{He}}_{\text{Mass of helium nucleus}}$$

Binding Energy XI

Looking up the mass of helium in a table of values, we find that the mass of a ^4He nucleus is 4.0015 u. From the previous question, we know we can calculate the mass defect of He by:

$$m_{defect} = Zm_p + (A-Z)m_n - m_{He}$$

What is the binding energy per nucleon of ^4He?

A. 1.78 eV
B. 0.0305 MeV
C. 1.78 MeV
D. 7.10 MeV
E. 28.4 MeV

Useful information:

$$m_{He} = 4.0015\,\text{u}$$

$$m_p = 1.0073\,\text{u}$$

$$m_n = 1.0087\,\text{u}$$

$$m_e = 5.486 \cdot 10^{-4}\,\text{u}$$

$$1\,\text{u} = 931.5\,\text{MeV}$$

Solution

Answer: D

Justification: We find that the mass defect of helium is:

$$m_{defect} = 2m_p + 2m_n - m_{He} = 0.0305\,\mathrm{u}$$

We convert this to binding energy by using the following conversion:

$$E_{binding} = 0.0305\,\mathrm{u} \times \frac{931.5\,\mathrm{MeV}}{1\,\mathrm{u}} = 28.41\,\mathrm{MeV}$$

To find the binding energy <u>per nucleon</u>, we divide the binding energy by the number of nucleons in He.

$$\text{Binding energy per nucleon} = \frac{E_{binding}}{\#\text{ of nucleons}} = \frac{28.41\,\mathrm{MeV}}{4} = 7.10\,\mathrm{MeV}$$

Binding Energy XII

The first ionization energy of helium is the energy required to remove 1 electron from a helium atom. How does this compare to the binding energy per nucleon of helium?

A. The ionization energy of helium is <u>much larger</u> than its binding energy per nucleon

B. The ionization energy of helium is <u>approximately the same</u> as its binding energy per nucleon

C. The ionization energy of helium is <u>much smaller</u> than its binding energy per nucleon

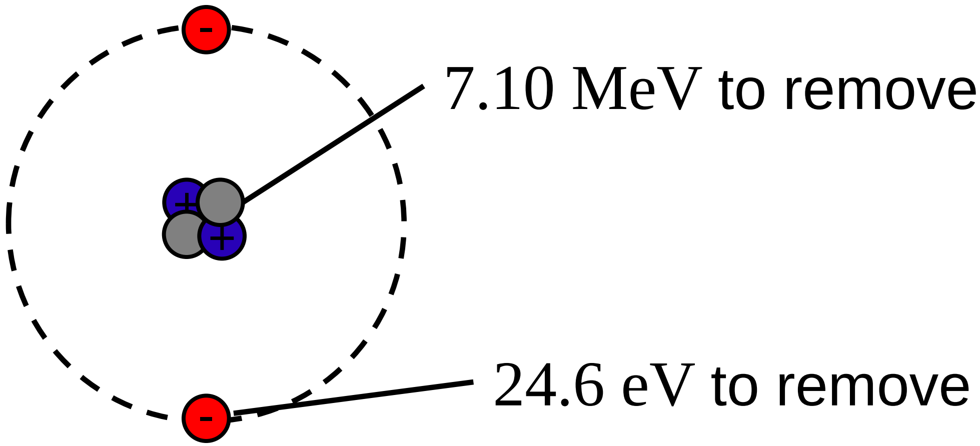
Solution

Answer: C

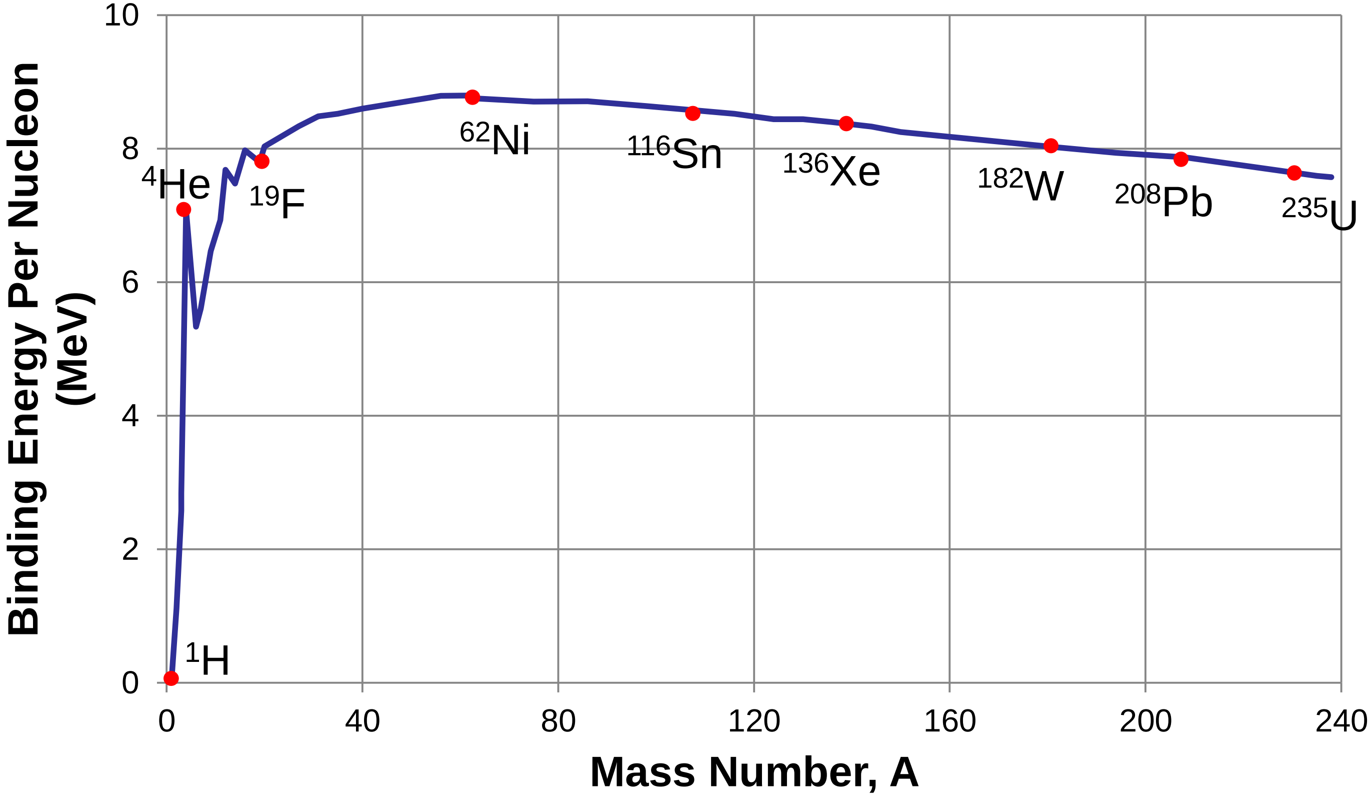
Justification: The ionization energy of helium is $24.6\ \mathrm{eV}$. This is 10^6 times smaller than its binding energy per nucleon ($7.10\ \mathrm{MeV}$).

It takes much more energy to remove nucleons than it does to remove electrons from an atom. The nucleus of an atom is held together tightly by the strong nuclear force. This is much stronger than electromagnetic forces holding electrons.

Bohr Model of Helium:



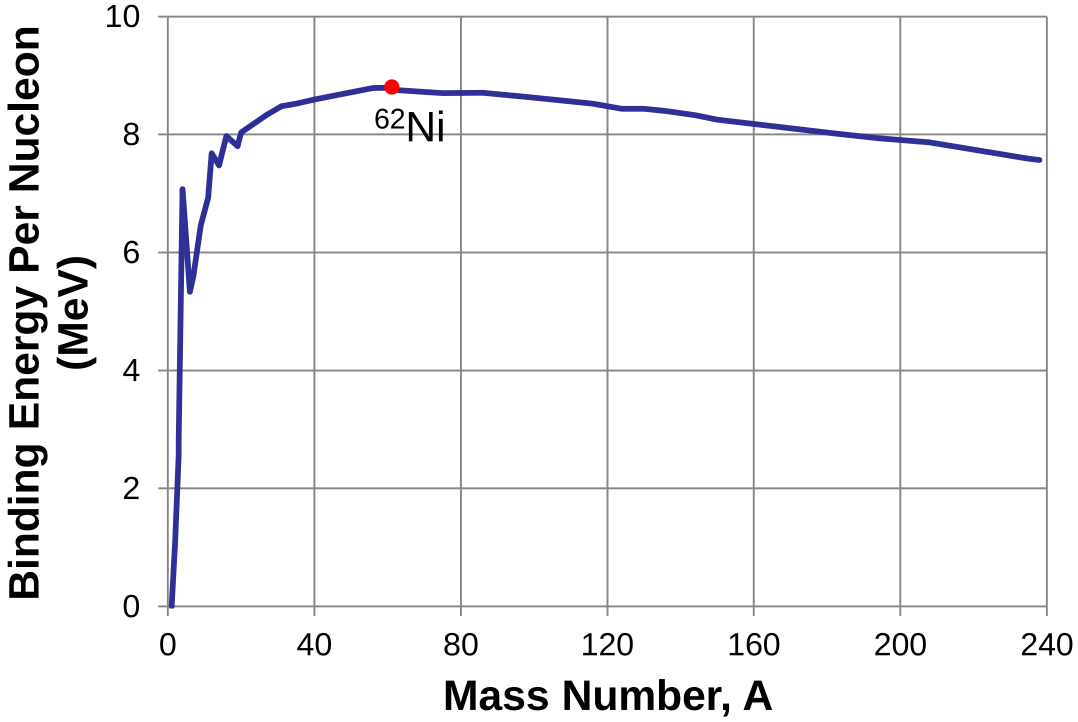
The Binding Energy Curve



The graph above shows the binding energy per nucleon of various isotopes. The maximum of the graph occurs at ^{62}Ni. Most nuclei have a binding energy per nucleon of about $8\ \mathrm{MeV}$.

Binding Energy XIII

The maximum of the binding energy curve occurs at ^{62}Ni. What can we conclude about ^{62}Ni?



A. ^{62}Ni is very stable

B. A ^{62}Ni nucleus has unusually large mass

C. ^{62}Ni tends to decay into lighter products

D. A ^{62}Ni nucleus contains many neutrons

E. ^{62}Ni is often found with high kinetic energy

Solution

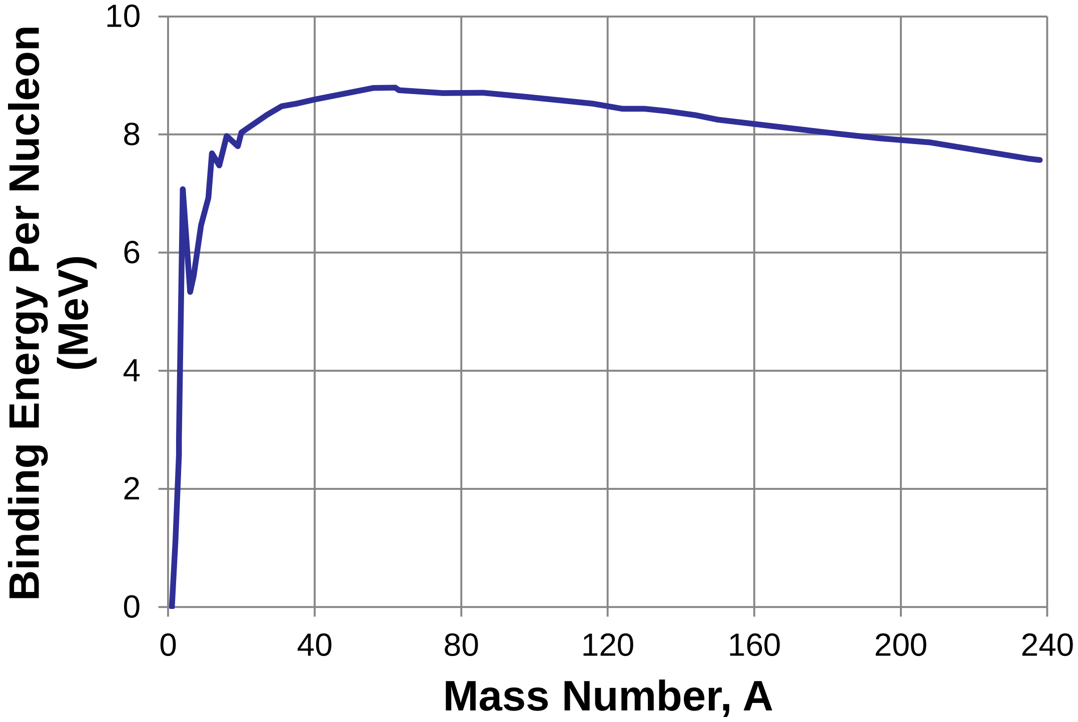
Answer: A

Justification: It takes a large amount of energy to separate each nucleon from the nucleus of ^{62}Ni. This makes ^{62}Ni very stable.

Due to the stability of nickel, many nuclear reactions end with nickel as a final product. It is not common for nickel to break up into smaller components, or fuse together to form heavier products.

Binding Energy XIV

Which of the following isotopes has the largest binding energy?



A. ^{1}H

B. ^{4}He

C. ^{62}Ni

D. ^{194}Pt

E. ^{235}U

Solution

Answer: E

Justification: Uranium-235 has the largest binding energy. Even though ^{62}Ni has the largest binding energy per nucleon, ^{235}U has a lot more nucleons and greater total binding energy.

Isotope	# of Nucleons	Binding Energy Per Nucleon	Total Binding Energy
^{1}H	1	0	0
^{4}He	4	7.1 MeV	28.4 MeV
^{62}Ni	62	8.8 MeV	546 MeV
^{194}Pt	194	7.9 MeV	1533 MeV
^{235}U	235	7.6 MeV	1786 MeV

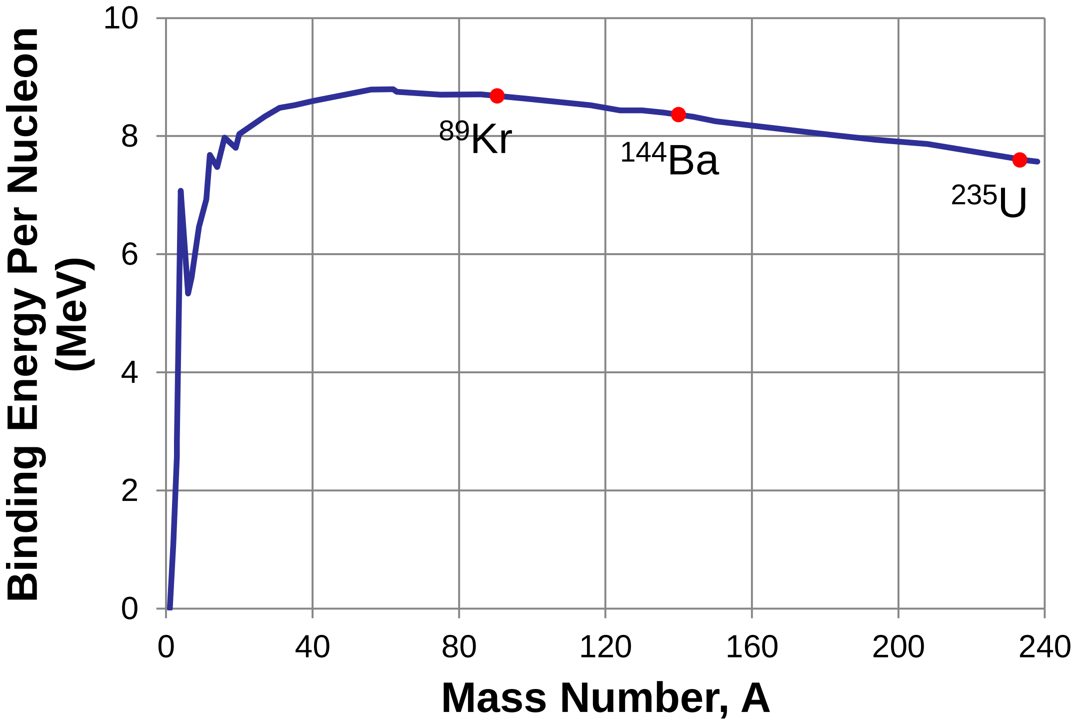
# of nucleons × binding energy per nucleon = total binding energy

Binding Energy XV

Consider the following nuclear fission reaction:

$$^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{144}_{56}\mathrm{Ba} + {}^{89}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n}$$

Which side of the reaction will have more mass?



A. Left side

B. Right side

C. Both sides have the same mass

Solution

Answer: A

Justification: From the binding energy curve, we can conclude that both ^{144}Ba and ^{89}Kr have higher binding energy per nucleon than ^{235}U. The lower binding energy per nucleon of ^{235}U means that it has less mass defect per nucleon compared to ^{144}Ba and ^{89}Kr.

$$E_{binding} = m_{defect} c^2$$

Since the number of nucleons are conserved in the reaction, ^{235}U should be heavier than the products:

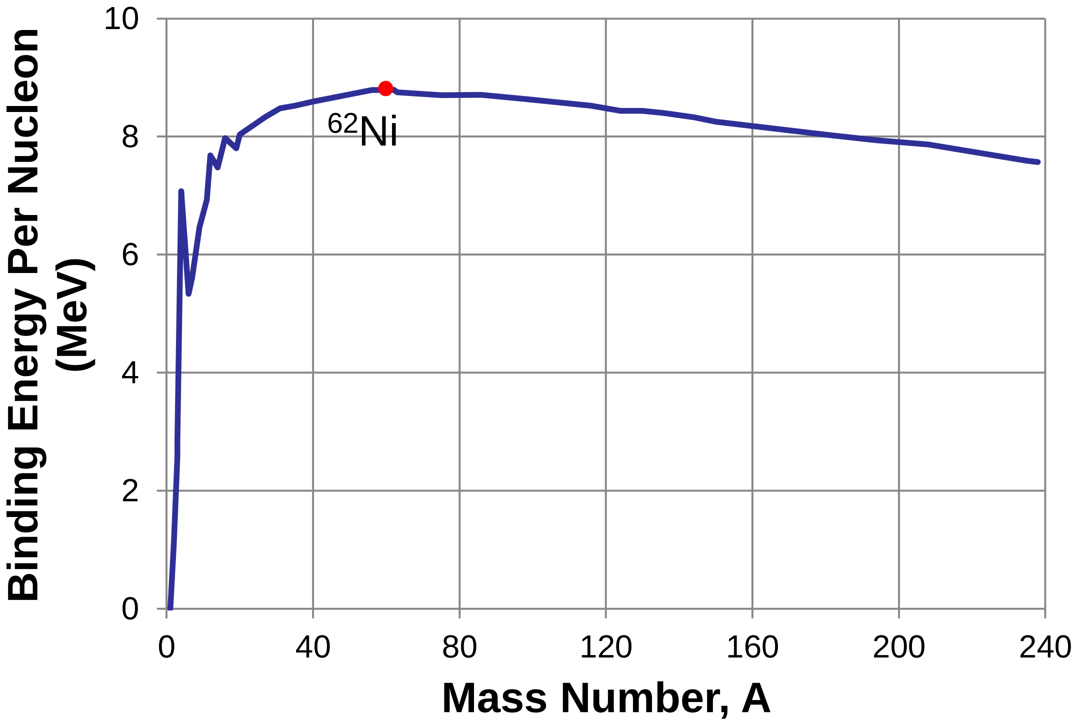
$$m_U + m_n = 235.0439\,\mathrm{u} + 1.0087\,\mathrm{u} = 236.0526\,\mathrm{u}$$

$$m_{Ba} + m_{Kr} + 3m_n = 143.9229\,\mathrm{u} + 88.9176\,\mathrm{u} + 3 \cdot 1.0087\,\mathrm{u} = 235.8666$$

Solution continues on the next slide

Solution Continued

The fact that ^{235}U decays into products with higher binding energy per nucleon forms the basis for nuclear fission. When the products have less mass than the reactants, the excess mass is released in the form of energy.



Isotopes to the left of ^{62}Ni may undergo nuclear fission to become more stable, and release energy in the process.

Summary

Particle	Mass (kg)	Mass (u)
Proton	$1.67262 \cdot 10^{-27}$ kg	1.0073 u
Neutron	$1.67492 \cdot 10^{-27}$ kg	1.0087 u
Electron	$9.10938 \cdot 10^{-31}$ kg	0.0005486 u

$$1\,\text{u} = 1.6605 \cdot 10^{-27}\ \text{kg}$$

$$1\,\text{u} = 1.4923 \cdot 10^{-10}\ \text{J}$$

$$1\,\text{u} = 931.5\ \text{MeV}$$

Mass defect: The difference in mass between the mass of a nucleus and the mass of individual protons and neutrons

Binding Energy: The energy required to remove separate the nucleons from a nucleus

$$E_{binding} = m_{defect} c^2$$