a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Mathematics
Numbers: Absolute Value of Functions II

Science and Mathematics
Education Research Group

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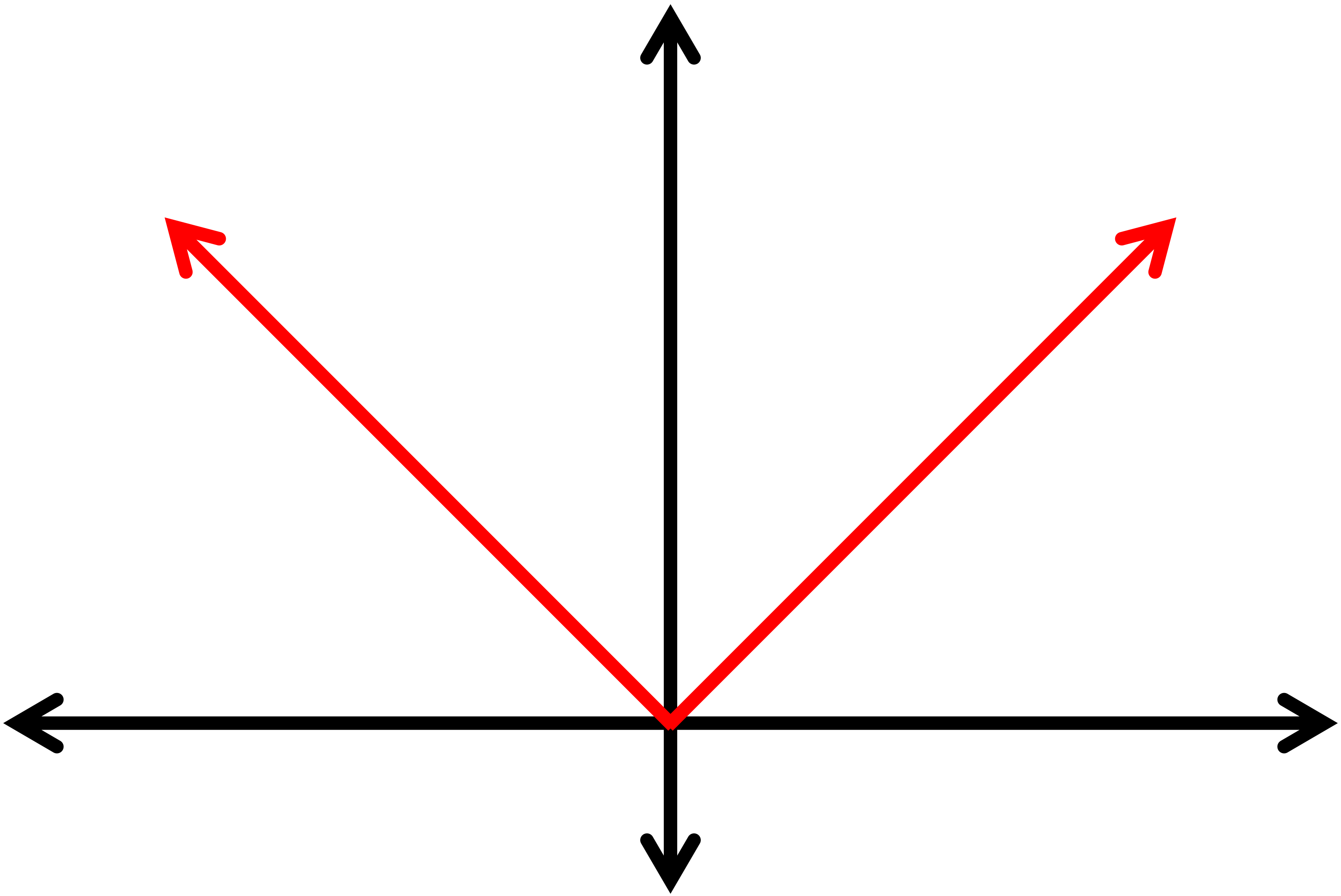
Absolute Value of Functions II



Absolute Values I

Evaluate Completely:

$$\left|-2 \times 2 + (18 - 3 \times 2)^2 \div \sqrt{144}\right|$$

A. -71

B. 71

C. 27

D. 8

E. 41

Solution

Answer: D

Justification: Within the operation of the absolute value, we must follow the order of operation as we would in any other calculations.

Therefore, within $\left|-2 \times 2 + (18 - 3 \times 2)^2 \div \sqrt{144}\right|$, we must evaluate:

1. Brackets -> $(18 - 3 \times 2)$ = (18 – 6) = 12

2. Exponents -> $(12)^2$ = 144

3. Division or Multiplication: -2×2 = -4,

   $144 \div \sqrt{144}$ = 144 ÷ 12 = 12

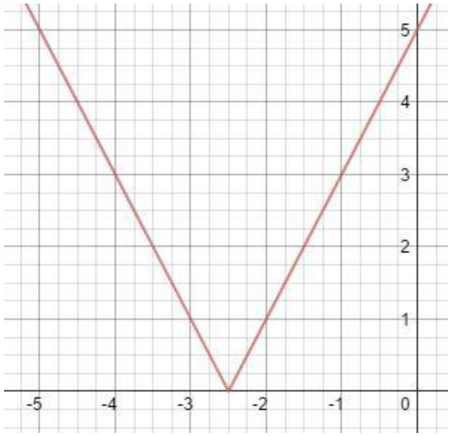
4. Addition or Subtraction: $-4 + 12 = 8$

Taking the absolute value of 8, we get $|8|$ = 8. Our Answer is D.

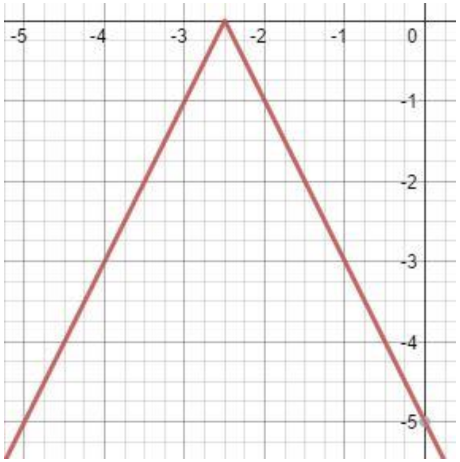
Absolute Values II

Which of the following graphs corresponds to $f(x) = |-2x-5|$

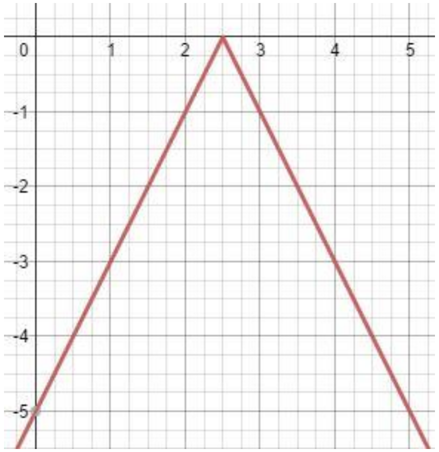
A.



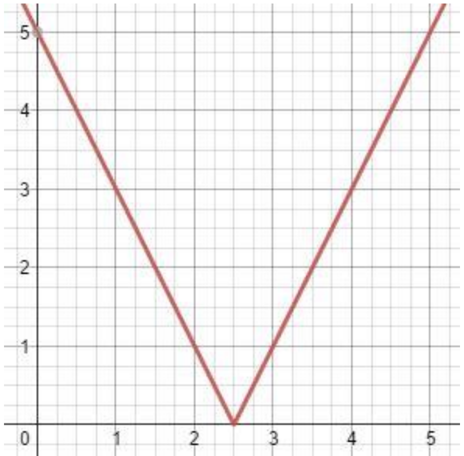
B.



C.



D.



Solution

Answer: A

Justification: First, you want to find your x-intercept and y-intercept of f(x) = $|-2x-5|$

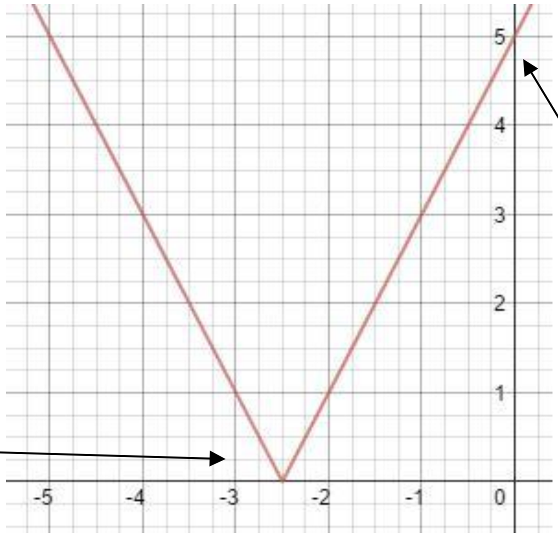
x-intercept

$$f(x) = |-2x-5| = 0$$

$$-2x-5=0$$

$$x=-\frac{5}{2}$$

$(-\frac{5}{2}, 0)$



y-intercept

$$f(0) = |-2(0)-5|$$

$$= |-5|$$

$$= 5$$

(0, 5)

The option that satisfies both of these intercepts is A. Thus, our answer is A

Solution Continued

It is also worth noting that for this question you can narrow down the answer **conceptually** before you do any calculations.

$f(x) = |-2x - 5|$, so we know that the y-value can only be a **positive** number (or zero), no matter what value we give x

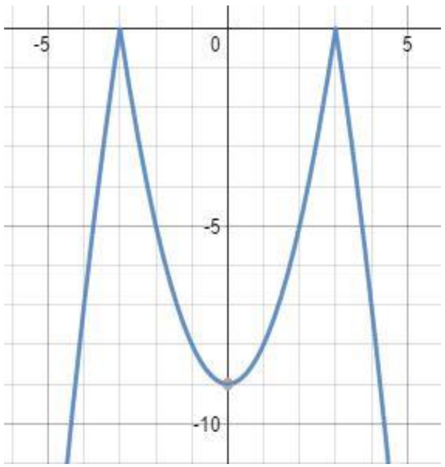
We can therefore immediately eliminate options B and C (with **negative** y-values).

From there we only need to calculate the x-intercept to find out which of the remaining options is the correct one (A or D)

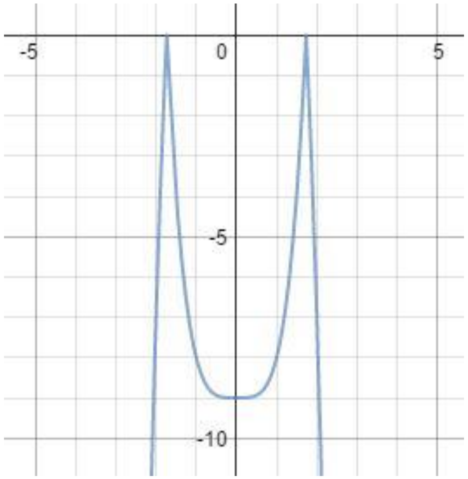
Absolute Values III

Which of the following graphs corresponds to $f(x) = \left|9 - x^2\right|$

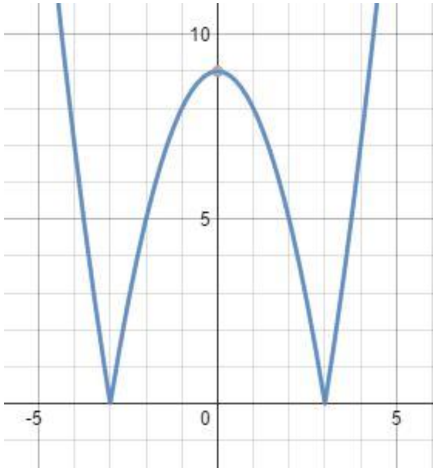
A.



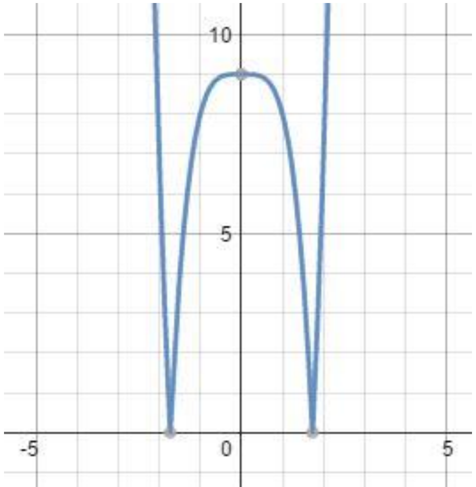
B.



C.



D.



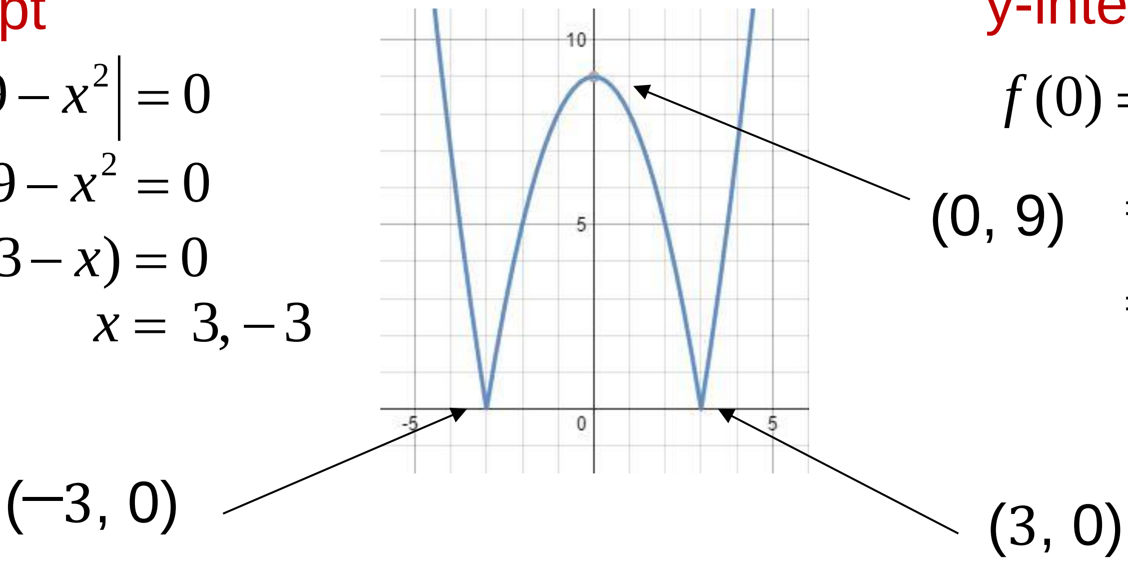
Solution

Answer: C

Justification: First, you want to find your x-intercept and y-intercept of f(x) = $|9 - x^2|$

x-intercept

$$f(x) = |9 - x^2| = 0$$
$$9 - x^2 = 0$$
$$(3 + x)(3 - x) = 0$$
$$x = 3, -3$$



y-intercept

$$f(0) = |9 - (0)^2|$$
$$= |9|$$
$$= 9$$

The option that satisfies both of these intercepts is C.
Thus, our answer is C.

Solution Continued

You will notice once again that for this question the answer can be narrowed down conceptually before applying any calculations

f(x) = $|9 - x^2|$, so we know that the y-values can only be **positive** (or zero), for any value of x

We can therefore immediately eliminate options A and B (with **negative** y-values).

From there we only need to calculate the x-intercept to find out which of the remaining options is the correct one (C or D)

Absolute Values IV

Which of the following are piecewise functions of $f(x) = |x^2 + 5x - 6|$?

A. $f(x) = |x^2 + 5x - 6| = \begin{cases} x^2 + 5x - 6 \geq 0, & x \geq 1, x \leq -6 \\ x^2 + 5x - 6 < 0, & -6 < x < 1 \end{cases}$

B. $f(x) = |x^2 + 5x - 6| = \begin{cases} x^2 + 5x - 6 \geq 0, & -6 \leq x \leq 1 \\ x^2 + 5x - 6 < 0, & x > 1, x < -6 \end{cases}$

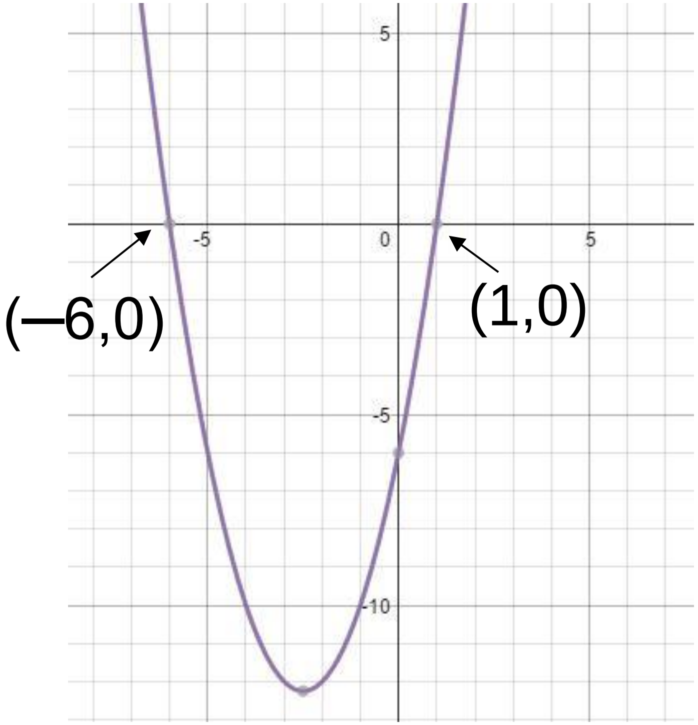
C. $f(x) = |x^2 + 5x - 6| = \begin{cases} x^2 + 5x - 6 \geq 0, & -1 \leq x \leq 6 \\ x^2 + 5x - 6 < 0, & x < -1, x > 6 \end{cases}$

D. $f(x) = |x^2 + 5x - 6| = \begin{cases} x^2 + 5x - 6 \geq 0, & x \leq -1, x \geq 6 \\ x^2 + 5x - 6 < 0, & -1 < x < 6 \end{cases}$

Solution

Answer: A

Justification: The graph of g(x) = $x^2 + 5x - 6$ (no absolute value) looks like the graph below:



Notice that for g(x) = $x^2 + 5x - 6$, it factors into $(x + 6)(x - 1)$. Then, our x-intercept will be:

$(x + 6)(x - 1) = 0$

$x = -6, 1$. -> (– 6,0), (1,0)

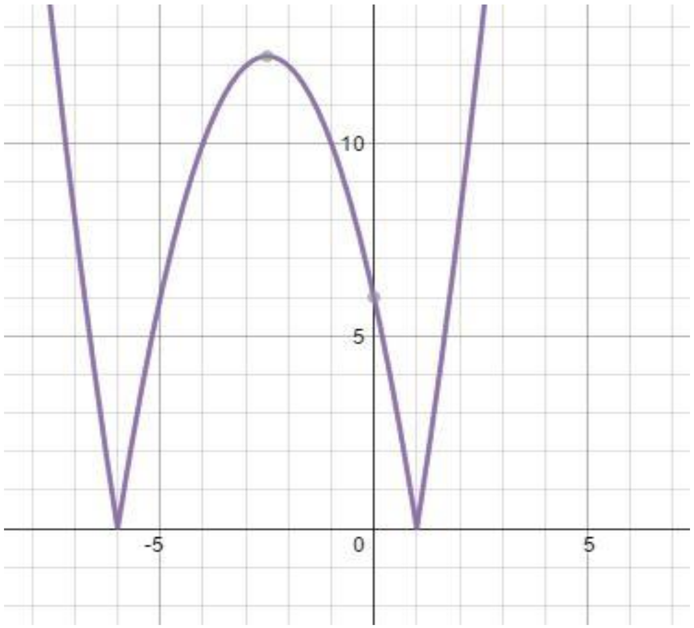
These will be points that wouldn't change when we have

f(x) = |g(x)| = $|x^2 + 5x - 6|$

They are called the invariant points.

Solution Continued

Knowing that f(x) = $\left| x^2 + 5x - 6 \right|$ will always be above the x-axis (the y-values are always **positive**), we get the graph below:



Notice that the middle part of g(x) = $x^2 + 5x - 6$ has been reflected by the x-axis when $-6 < x < 1$.

Thus, when $-6 < x < 1$, we know that the inside of f(x) = $\left| x^2 + 5x - 6 \right|$ is negative, Therefore, when we write it as piecewise function, we get:

$$f(x) = \left| x^2 + 5x - 6 \right| = \begin{cases} x^2 + 5x - 6 \geq 0, & x \geq 1, x \leq -6 \\ x^2 + 5x - 6 < 0, & -6 < x < 1 \end{cases}$$

Absolute Values V

Solve for x:

$$|x^2 - 2x - 8| = 8$$

A. $x = 0, 2$

B. $x = 4, -2$

C. $x = 2, 1+\sqrt{17}, 1-\sqrt{17}$

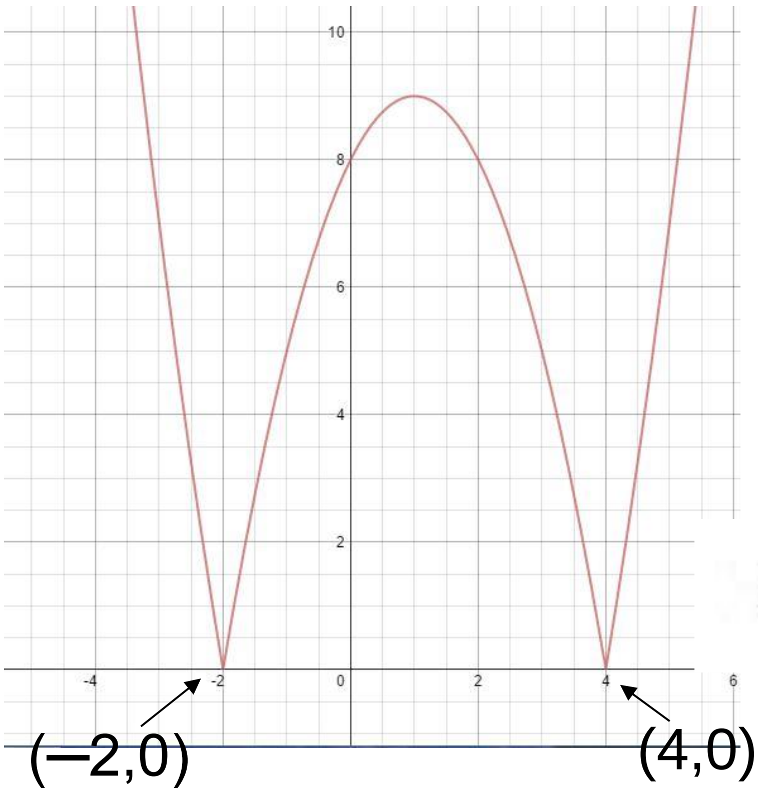
D. $x = 0, 2, 1+\sqrt{17}$

E. $x = 0, 2, 1+\sqrt{17}, 1-\sqrt{17}$

Solution I

Answer: E

Justification: The graph of f(x) = $\left|x^2 - 2x - 8\right|$ looks like the graph below:



Notice that for f(x) = $x^2 - 2x - 8$, it factors into $(x-4)(x+2)$. Then, our x-intercept will be:

$(x-4)(x+2)$ = 0

$x = 4, -2$. -> (4,0), (-2,0)

When we write it as piecewise, we get:

$$f(x) = \left|x^2 - 2x - 8\right| = \begin{cases} x^2 - 2x - 8 \geq 0, & x \geq 4,\ x \leq -2 \\ x^2 - 2x - 8 < 0, & -2 < x < 4 \end{cases}$$

Solution I Continued

Having piecewise functions, we can have two cases

Case 1: $x^2 - 2x - 8 \geq 0$

$x^2 - 2x - 8 = 8$

$x^2 - 2x - 16 = 0$

x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm 2\sqrt{17}}{2} = 1 \pm \sqrt{17}$

x = $1 \pm \sqrt{17}$ = 5.123, -3.123

Since both x = $1 \pm \sqrt{17}$ are within the restriction (x ≥ 4, x ≤ − 2), both of the answers are valid.

Case 2: $x^2 - 2x - 8 < 0$

$-(x^2 - 2x - 8) = 8$

$(x^2 - 2x - 8) = -8$

$x^2 - 2x = 0$

$x(x - 2) = 0$

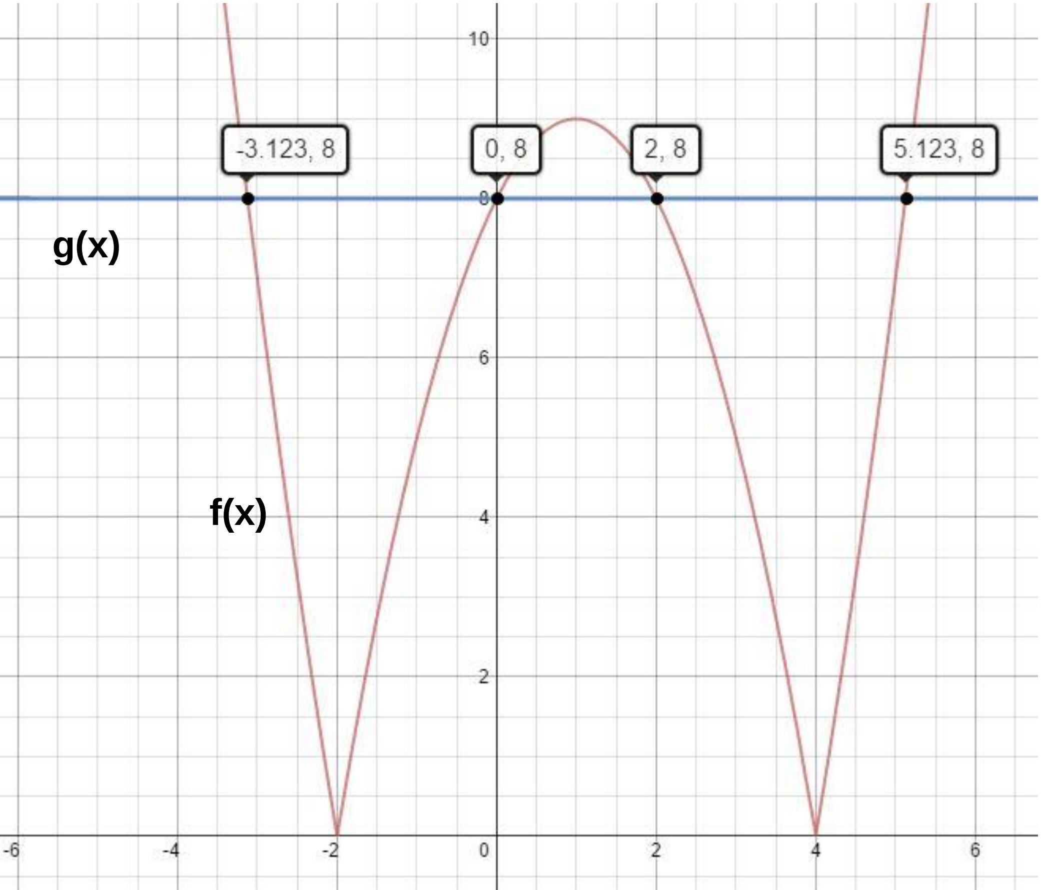
x = 0,2

Since both x = 0,2 within the restriction (− 2 < x < 4), both of the answers are valid.

Thus, our answer is E.

Solution II

We let f(x) = $\left|x^2 - 2x - 8\right|$, g(x) = 8, and draw both f(x) and g(x) graphs as below (using technology such as **GeoGebra**):



Then, we have to find the points of intersection between the graphs.

Thus, for our answers, we have x = -3.123, 0, 2, and 5.123.

For our decimal answers, if we evaluate $1 + \sqrt{17}$, and $1 - \sqrt{17}$,we get x = 5.123 and -3.123

Thus, our answer is E.

Absolute Values VI

Solve for x:

$$|2(x-2)| = \left|\frac{1}{2}x + 2\right|$$

A. $x = 0$

B. $x = 4$

C. $x = \frac{4}{5}$

D. $x = 4, \frac{4}{5}$

E. $x = \pm 4, \pm \frac{4}{5}$

Solution I

Answer: D

Justification: let f(x) = $|2(x-2)|$, and g(x)=$|\frac{1}{2}x+2|$

If both f(x) and g(x) are written in piecewise functions, they look as such:

$$f(x)=|2(x-2)|=\begin{cases}|2(x-2)| \geq 0,\ x \geq 2\\ |2(x-2)| < 0,\ x < 2\end{cases}$$

$$g(x)=\left|\frac{1}{2}x+2\right|=\begin{cases}\left|\frac{1}{2}x+2\right| \geq 0,\ x \geq -4\\ \left|\frac{1}{2}x+2\right| < 0,\ x < -4\end{cases}$$

Combining these inequalities, we create 4 different inequalities:

Case 1. $x \geq 2$, $x \geq -4$

Case 2. $x \geq 2$, $x < -4$

Case 3. $x < 2$, $x \geq -4$

Case 4. $x < 2$, $x < -4$

Solution I Continued

Case 1: x ≥ 2, x ≥ -4 = x ≥ 2

$$2(x-2) = \frac{1}{2}x + 2$$

$$2x - 4 = \frac{1}{2}x + 2$$

$$\frac{3}{2}x = 6$$

$$x = 4$$

Since 4 ≥ 2, this answer is valid.

Case 2: x ≥ 2, x < -4

$$2(x-2) = -(\frac{1}{2}x + 2)$$

$$2x - 4 = -\frac{1}{2}x - 2$$

$$\frac{5}{2}x = 2$$

$$x = \frac{4}{5}$$

Since $\frac{4}{5} \ngeq$ 2 and $\frac{4}{5} \nless -4$, this answer is invalid.

Solution I Continued

Case 3: x < 2, x ≥ – 4 = $-4 \leq x < 2$

$$-2(x-2) = \frac{1}{2}x + 2$$

$$-2x + 4 = \frac{1}{2}x + 2$$

$$-\frac{5}{2}x = -2$$

$$x = \frac{4}{5}$$

Since $-4 \leq \frac{4}{5} < 2$, this answer is valid.

Case 4: x < 2, x < -4 = $x < -4$

$$-2(x-2) = -(\frac{1}{2}x + 2)$$

$$-2x + 4 = -\frac{1}{2}x - 2$$

$$-\frac{3}{2}x = -6$$

$$x = 4$$

Since $4 \not< -4$, this answer is invalid.

Combining the results from valid cases, we get x = 4, $\frac{4}{5}$.

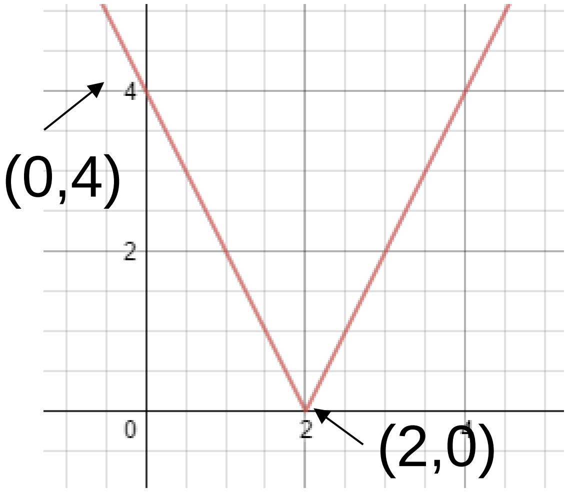
Thus our answer is D.

Solution II

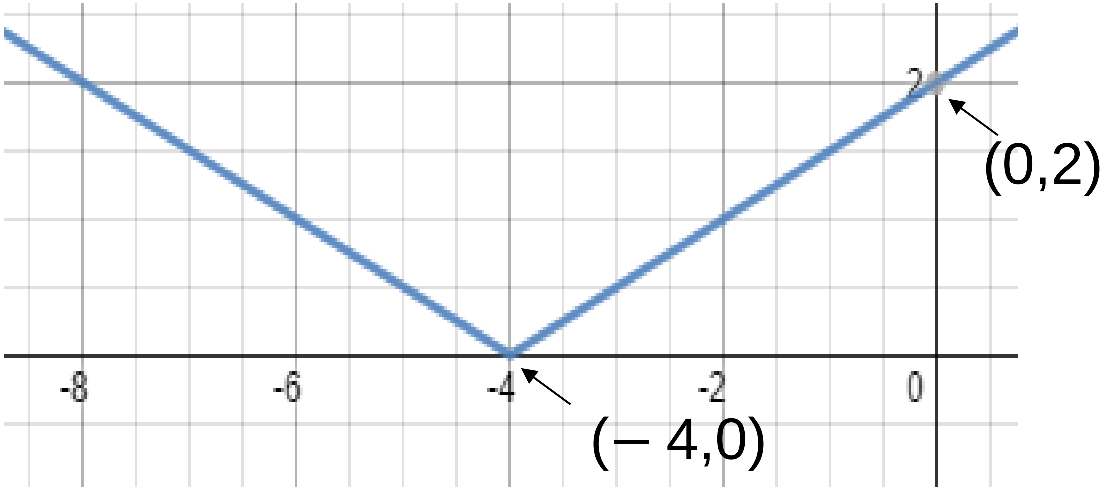
Answer: D

Justification: The graph of f(x) = $\left|2(x-2)\right|$ and g(x) = $\left|\frac{1}{2}x+2\right|$ look like the graphs below:

f(x) = $\left|2(x-2)\right|$

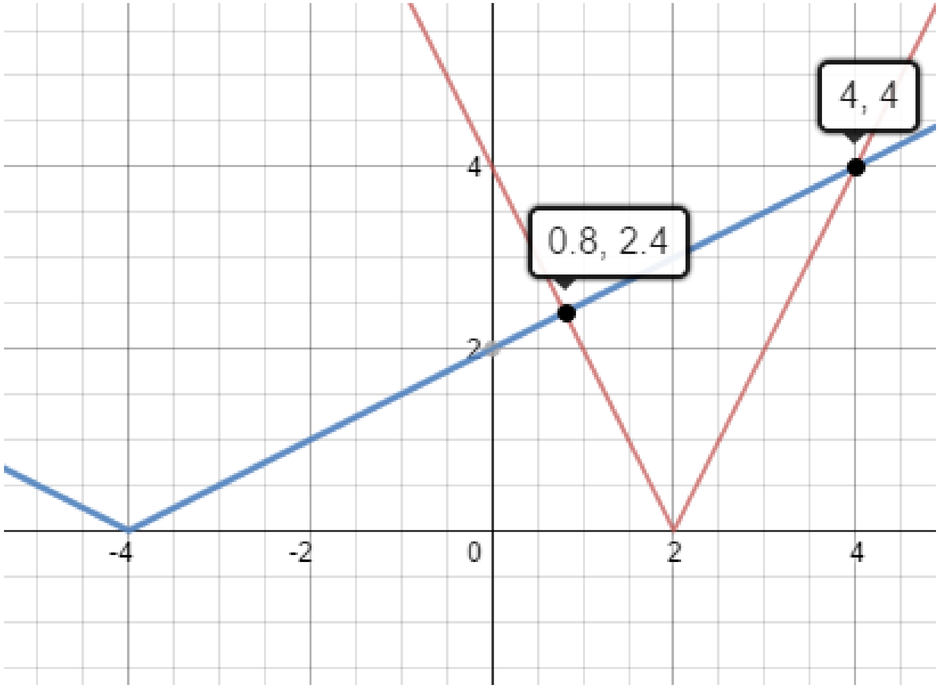


g(x) = $\left|\frac{1}{2}x+2\right|$



Solution II Continued

Graphing both together, we can find the intersection(s) between those two graphs (using technology such as **Desmos**)



Our x values of intersections are 4 and 0.8 $\left(\frac{4}{5}\right)$. Thus our answer is D.

Absolute Values VII

In Playland, there is a ride called KC’s Raceway for kids, which has an average rider’s height of 48”. For the riders’ safety, the difference between the height of the guardian and their accompanied kid should not exceed 12”. Which option best describes the relationship of permitted height for this ride?
(x is the person’s height in inches)

A. $|x| \leq 12$

B. $|x-48| = 12$

C. $|x-48| \leq 12$

D. $|x-36| = 12$

E. $|x-36| \leq 12$

Retrieved from http://www.pne.ca/playland/rides-games-food/kidsplayce.html

Solution

Answer: C

Justification: From the question, it says: *the average height of the riders is 48".*

Since x can be greater than 12, option A is knocked off.

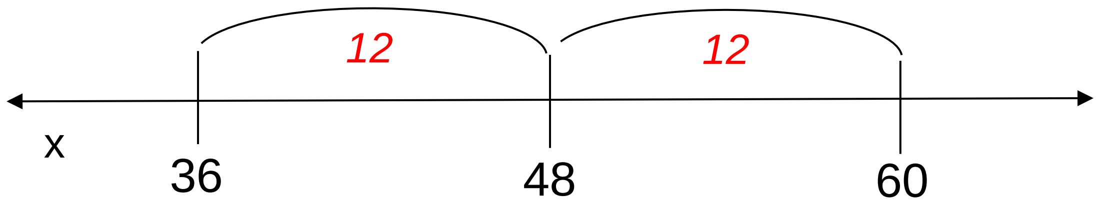
As well, from the question, it says: *the difference between the height of the guardian and their accompanied kid shouldn't exceed 12*". This implies that all difference values should be less than or equal to 12. Thus, our answer will have to be expressed in an inequality.

Since option B and D have only used equalities (=), they cannot be our best answer.

Solution Continued

We can use the absolute value equation to model this question. By definition, the absolute value represents the distance away from 0.

Since our difference in height should be less than or equal to 12, our possible difference in height could range from -12 to 12, where all the negative values will become positive due to the absolute value sign. If we use 48'' as the mid-point (average) height, our x values range from 36 to 60 (from *48 - 12* to *48 + 12*). Since 48 is the mid-point, and we want the distance away from 48 to be less than or equal to twelve, we can do the following:



Instead of having the mid-point at 0, we need to shift it to be at 48.

Therefore, our equation should be: $|x - 48| \leq 12$. Our answer is C.